# Fundamentals of Mathematical Statistics - Errata 

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#### Abstract

This note identifies typographical and other errors in my book on Fundamentals of Mathematical Statistics, CRC Press, April 2023. I am grateful to teachers and students at the University of Copenhagen for spotting most of these.


## Errors identified after 2024

p9 line $4 f_{x}(\theta)$ should be $f_{\theta}(x)$.
p16 Example 1.27 The parameters $(\alpha, \beta)$ should vary in $\mathbb{R}_{+}^{2}$.
p31 Proof of Proposition 2.10 First line in displayed formula should read

$$
\varphi_{Y}(u)=\mathbf{E}\left(e^{i\langle u, Y\rangle_{W}}\right)=e^{i\langle u, a\rangle_{W}} \mathbf{E}\left(e^{i\langle u, B X\rangle_{W}}\right)=e^{i\langle u, a\rangle_{W}} \mathbf{E}\left(e^{i\left\langle B^{*} u, X\right\rangle_{V}}\right)
$$

(a subscript was missing for an inner product).
p31 Theorem 2.11 could be formulated better as: Let $(V,\langle\cdot, \cdot\rangle)$ be a Euclidean space, $\xi \in V$, and $\Sigma \in \mathcal{L}(V, V)$ a positive semidefinite and self-adjoint linear map. Then there is a random variable $X$ with $X \sim \mathcal{N}_{V}(\xi, \Sigma)$ if and only if there is a linear map $A \in \mathcal{L}(V, V)$ with $A A^{*}=\Sigma$. Then $X$ has the same distribution as $Y=\xi+A Z$ where $Z \sim \mathcal{N}_{V}(0, I)$ and $I$ is the identity map on $V$.
p36 Lemma 2.22 There are too many commas in the inner products here.
It should read: $(V,\langle\cdot, \cdot\rangle),\left(W_{1},\langle\cdot \cdot \cdot\rangle_{1}\right)$, and $\left(W_{2},\langle\cdot, \cdot\rangle_{2}\right)$.
p46 line -8 $\mathbf{V} \backslash\{0\} \times \mathbb{R}$ should be $V \backslash\{0\} \times \mathbb{R}($ no bold faced $V$ )

## Errors in the first printing

p5 Example 1.9 The Cauchy density should be

$$
f_{\alpha, \beta}(x)=\frac{1}{\pi} \frac{\beta}{\left((x-\alpha)^{2}+\beta^{2}\right)}, \quad x \in \mathbb{R}
$$

p15 middle It would be more correct to write the Fisher information as

$$
i(\theta)=\mathbf{V}_{\theta}\left\{S(X, \theta)^{\top}\right\}=\mathbf{E}_{\theta}\{I(X, \theta)\}
$$

since $S(X, \theta)$ is defined as an $1 \times k$ matrix rather than a vector in $\mathbb{R}^{k}$.
p17 middle Similarly, as on p15, the correct expression for the Fisher information should be

$$
i(\alpha, \beta)=\mathbf{E}_{\alpha, \beta}\{I(X, \alpha, \beta)\}=\mathbf{V}_{\alpha, \beta}\left\{S(X, \alpha, \beta)^{\top}\right\}=\mathbf{V}_{\alpha, \beta}\left\{\left(\log X, X / \beta^{2}\right)^{\top}\right\}
$$

p46 Exercise 2.7 c) "is of" should be "of"
p50 Example 3.5 The expression for the rewritten density should be

$$
f_{\theta}(x)=\frac{e^{\theta_{1} x+\theta_{2}\left(-x^{2} / 2\right)}}{c(\theta)}
$$

p55 Theorem 3.11 The $n$-fold direct product $\mathcal{P}^{\otimes n}$ is also minimally represented.
p57 line -9 should read:
Now, since $\Theta$ was an open and convex subset of $V, \tilde{\Theta}$ is an open and convex subset of $L \ldots$
p57 line -6 should read:
and thus, if $\langle\lambda, \tilde{t}(x)\rangle$ is a.e. constant with respect to $\tilde{\mu}$, then also $\langle\lambda, t(x)\rangle$ is a.e. ...
p57 last line should read:
but then the representation would not be regular since $\tilde{\Theta}$ is not an open subset of $V$.
p64 Example 3.27, line -10 $B=\mathbb{R}^{m}$ should be $B=\phi^{-1}(\Theta)$
p65 line 12 should read

$$
\ell_{x}(\theta)=\theta^{\top} x-\frac{1}{2}\|\theta\|^{2}
$$

p65 (3.10) should read

$$
\ell_{x}(\theta)=\phi(\beta)^{\top} x
$$

p66 line - 10 'for alle $\beta$ ' should read 'for all $\beta$
p70 Exercise 3.6, line $4 \lambda x$ should be $\beta x$
p70 Exercise 3.8 Poisson's should be Poisson
p71 Exercise 3.10 Lebesque should be Lebesgue
p75 line -8 should read:

$$
\mathbf{B}\left(\hat{\theta}_{n}\right)=\frac{n \theta}{n+1}-\theta=-\frac{\theta}{n+1}
$$

p77 Theorem 4.4 $\hat{\lambda}=t(x)$ should be $\hat{\lambda}=t(X)$
p88 Example 4.16 The moment functions are

$$
m_{1}(\xi)=\xi, \quad m_{2}(\xi)=P_{\xi}(X>0)=\Phi(\xi), \quad m_{3}(\xi)=3 \xi+\xi^{3}
$$

p104 Theorem 5.6 The asymptotic covariance should read:

$$
\Sigma(\theta) / n=\operatorname{Dm}(\theta)^{-1} \mathbf{V}_{\theta}(t(X)) D m(\theta)^{-\top} / n .
$$

p104 Example 4.7 last line on page should read:

$$
m_{1}(\xi)=\xi, \quad m_{2}(\xi)=P_{\xi}(X>0)=\Phi(\xi), \quad m_{3}(\xi)=3 \xi+\xi^{3}
$$

p107 Corollary 5.12 Last line would be clearer as:
where $A=\sqrt{n i\left(\hat{\theta}_{n}\right)}$ is the unique positive definite matrix $A$ satisfying $A^{2}=n i\left(\hat{\theta}_{n}\right)$.
p111 last line before proof delete 'obtain a'
p132 Exercise 5.9 c) Compare the asymptotic distribution of $\hat{\lambda}_{n}$ and $\tilde{\lambda}_{n}$.
p169 First displayed formula should read:

$$
\hat{p}_{N}=p_{\mathrm{MC}}(d(x))=\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{(d(x), \infty)}\left(d\left(X_{i}^{*}\right)\right)=\frac{1}{N} \sum_{i=1}^{N} Y_{i}
$$

p177 Exercise 7.7 e) Derive the quadratic score test statistic for $H_{1}$ under the assumption of $H_{0}$.
p195 line -3 "the the" should be "the"
p201 line -8 "Pearson's ${ }^{2}$ evaluates" should be "Pearson's $\chi^{2}$ statistic evaluates"
p229, line -6 $\mu(d(x)$ should be $\mu(d x)$

