Fundamentals of Mathematical Statistics — Errata

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Abstract

This note identifies typographical and other errors in my book on *Fundamentals of Mathematical Statistics*, CRC Press, April 2023. I am grateful to teachers and students at the University of Copenhagen for spotting most of these.

Errors identified after March 2024

- **p9 line 4** $f_x(\theta)$ should be $f_{\theta}(x)$.
- **p16 Example 1.27** The parameters (α, β) should vary in \mathbb{R}^2_+ .
- p31 Proof of Proposition 2.10 First line in displayed formula should read

$$\varphi_Y(u) = \mathbf{E}(e^{i\langle u, Y \rangle_W}) = e^{i\langle u, a \rangle_W} \mathbf{E}(e^{i\langle u, BX \rangle_W}) = e^{i\langle u, a \rangle_W} \mathbf{E}(e^{i\langle B^*u, X \rangle_V})$$

(a subscript was missing for an inner product).

- **p31 Theorem 2.11** could be formulated better as: Let $(V, \langle \cdot, \cdot \rangle)$ be a Euclidean space, $\xi \in V$, and $\Sigma \in \mathcal{L}(V, V)$ a positive semidefinite and self-adjoint linear map. Then there is a random variable X with $X \sim \mathcal{N}_V(\xi, \Sigma)$ if and only if there is a linear map $A \in \mathcal{L}(V, V)$ with $AA^* = \Sigma$. Then X has the same distribution as $Y = \xi + AZ$ where $Z \sim \mathcal{N}_V(0, I)$ and I is the identity map on V.
- **p36 Lemma 2.22** There are too many commas in the inner products here. It should read: $(V, \langle \cdot, \cdot \rangle)$, $(W_1, \langle \cdot, \cdot \rangle_1)$, and $(W_2, \langle \cdot, \cdot \rangle_2)$.
- **p46 line -8** $\mathbf{V} \setminus \{0\} \times \mathbb{R}$ should be $V \setminus \{0\} \times \mathbb{R}$ (no bold faced V)

p66 line -11 The Jacobian should read:

$$J(\beta) = \begin{pmatrix} -\beta^{-2} \\ -2\beta^{-3} \end{pmatrix}$$

p111 line -7 Sign mistake; should read $K(\theta, \theta) = -\kappa(\theta) = -i(\theta)$.

p111 line -5 should read

$$\Lambda_n = 2(\ell_n(\hat{\theta}_n) - \ell_n(\theta)) = -n(\hat{\theta}_n - \theta)^\top K(\theta, \hat{\theta}_n)(\hat{\theta}_n - \theta).$$

p111 line -4 last formula in line should read $K(\theta, \hat{\theta}_n) \xrightarrow{P} -i(\theta)$. **p115 line -8** should read

$$K(\hat{\hat{\theta}}_n, \hat{\theta}_n) = 2 \int_0^1 (1-t)\kappa(\hat{\theta}_n + t(\hat{\hat{\theta}}_n - \hat{\theta}_n)) dt.$$

p124 line 3 should read

$$\Lambda_n = -2\left(\ell_n(\hat{\beta}_n) - \ell_n(\beta)\right) \xrightarrow{\mathcal{D}} \chi^2(m)$$

 $\mathbf{p124}\ \mathbf{line}\ \mathbf{-4}\ \mathbf{should}\ \mathbf{read}$

$$\operatorname{plim}_{n \to \infty} \tilde{K}(\beta, \hat{\beta}_n) = \operatorname{plim}_{n \to \infty} 2 \int_0^1 (t-1)i(\beta + t(\hat{\beta}_n - \beta)) dt = -i(\theta).$$

p136 last line should read:

$$(\xi, \sigma^2) \in C_{\delta}(x) \iff x - \sigma z_{1-(1-\delta)\alpha} < \xi < x - \sigma z_{\delta\alpha}$$

p137 line 4 should read:

$$C_{\delta}(x) = \{(\xi, \sigma^2) : x + \sigma z_{(1-\delta)\alpha} < \xi < x - \sigma z_{\delta\alpha}\}.$$

p197, first displayed formula should read:

$$G^{2} = 2\sum_{i=0}^{r-1}\sum_{j=0}^{s-1} OBS_{ij} \log \frac{OBS_{ij}}{EXP_{ij}}, \quad X^{2} = \sum_{i=0}^{r-1}\sum_{j=0}^{s-1} \frac{(OBS_{ij} - EXP_{ij})^{2}}{EXP_{ij}}.$$

Errors in the first printing

p5 Example 1.9 The Cauchy density should be

$$f_{\alpha,\beta}(x) = \frac{1}{\pi} \frac{\beta}{((x-\alpha)^2 + \beta^2)}, \quad x \in \mathbb{R}$$

p15 middle It would be more correct to write the Fisher information as

$$i(\theta) = \mathbf{V}_{\theta} \{ S(X, \theta)^{\top} \} = \mathbf{E}_{\theta} \{ I(X, \theta) \}$$

since $S(X, \theta)$ is defined as an $1 \times k$ matrix rather than a vector in \mathbb{R}^k .

p17 middle Similarly, as on p15, the correct expression for the Fisher information should be

$$i(\alpha,\beta) = \mathbf{E}_{\alpha,\beta}\{I(X,\alpha,\beta)\} = \mathbf{V}_{\alpha,\beta}\{S(X,\alpha,\beta)^{\top}\} = \mathbf{V}_{\alpha,\beta}\{(\log X, X/\beta^2)^{\top}\}$$

p46 Exercise 2.7 c) "is of" should be "of"

p50 Example 3.5 The expression for the rewritten density should be

$$f_{\theta}(x) = \frac{e^{\theta_1 x + \theta_2(-x^2/2)}}{c(\theta)}.$$

- **p55 Theorem 3.11** The *n*-fold direct product $\mathcal{P}^{\otimes n}$ is also minimally represented.
- p57 line -9 should read:

Now, since Θ was an open and convex subset of V, $\tilde{\Theta}$ is an open and convex subset of L...

p57 line -6 should read:

and thus, if $\langle \lambda, \tilde{t}(x) \rangle$ is a.e. constant with respect to $\tilde{\mu}$, then also $\langle \lambda, t(x) \rangle$ is a.e. ...

p57 last line should read:

but then the representation would not be regular since $\tilde{\Theta}$ is not an open subset of V.

- p64 Example 3.27, line -10 $B = \mathbb{R}^m$ should be $B = \phi^{-1}(\Theta)$
- p65 line 12 should read

$$\ell_x(\theta) = \theta^\top x - \frac{1}{2} \|\theta\|^2.$$

p65 (3.10) should read

$$\ell_x(\theta) = \phi(\beta)^\top x.$$

p66 line -10 'for alle β ' should read 'for all β

p70 Exercise 3.6, line 4 λx should be βx

- p70 Exercise 3.8 Poisson's should be Poisson
- p71 Exercise 3.10 Lebesque should be Lebesgue
- p75 line -8 should read:

$$\mathbf{B}(\hat{\theta}_n) = \frac{n\theta}{n+1} - \theta = -\frac{\theta}{n+1}$$

p77 Theorem 4.4 $\hat{\lambda} = t(x)$ should be $\hat{\lambda} = t(X)$

p88 Example 4.16 The moment functions are

 $m_1(\xi) = \xi, \quad m_2(\xi) = P_{\xi}(X > 0) = \Phi(\xi), \quad m_3(\xi) = 3\xi + \xi^3$

p104 Theorem 5.6 The asymptotic covariance should read:

$$\Sigma(\theta)/n = Dm(\theta)^{-1} \mathbf{V}_{\theta}(t(X)) Dm(\theta)^{-\top}/n.$$

p104 Example 4.7 last line on page should read:

$$m_1(\xi) = \xi, \quad m_2(\xi) = P_{\xi}(X > 0) = \Phi(\xi), \quad m_3(\xi) = 3\xi + \xi^3$$

p107 Corollary 5.12 Last line would be clearer as: where $A = \sqrt{ni(\hat{\theta}_n)}$ is the unique positive definite matrix A satisfying $A^2 = ni(\hat{\theta}_n)$.

- p111 last line before proof delete 'obtain a'
- **p132 Exercise 5.9 c)** Compare the asymptotic distribution of $\hat{\lambda}_n$ and $\tilde{\lambda}_n$.
- p169 First displayed formula should read:

$$\hat{p}_N = p_{\mathrm{MC}}(d(x)) = \frac{1}{N} \sum_{i=1}^N \mathbf{1}_{(d(x),\infty)}(d(X_i^*)) = \frac{1}{N} \sum_{i=1}^N Y_i$$

- **p177 Exercise 7.7 e)** Derive the quadratic score test statistic for H_1 under the assumption of H_0 .
- p195 line -3 "the the" should be "the"
- **p201 line -8** "Pearson's 2 evaluates" should be "Pearson's χ^2 statistic evaluates"
- **p229, line -6** $\mu(d(x)$ should be $\mu(dx)$