## Fundamentals of Mathematical Statistics — Errata

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## Abstract

This note identifies typographical and other errors in my book on *Fundamentals of Mathematical Statistics*, CRC Press, April 2023. I am grateful to teachers and students at the University of Copenhagen for spotting most of these.

## Errors identified after 2024

**p9 line 4**  $f_x(\theta)$  should be  $f_{\theta}(x)$ .

**p16 Example 1.27** The parameters  $(\alpha, \beta)$  should vary in  $\mathbb{R}^2_+$ .

p31 Proof of Proposition 2.10 First line in displayed formula should read

$$\varphi_Y(u) = \mathbf{E}(e^{i\langle u, Y \rangle_W}) = e^{i\langle u, a \rangle_W} \mathbf{E}(e^{i\langle u, BX \rangle_W}) = e^{i\langle u, a \rangle_W} \mathbf{E}(e^{i\langle B^*u, X \rangle_V})$$

(a subscript was missing for an inner product).

- **p31 Theorem 2.11** could be formulated better as: Let  $(V, \langle \cdot, \cdot \rangle)$  be a Euclidean space,  $\xi \in V$ , and  $\Sigma \in \mathcal{L}(V, V)$  a positive semidefinite and self-adjoint linear map. Then there is a random variable X with  $X \sim \mathcal{N}_V(\xi, \Sigma)$  if and only if there is a linear map  $A \in \mathcal{L}(V, V)$  with  $AA^* = \Sigma$ . Then X has the same distribution as  $Y = \xi + AZ$  where  $Z \sim \mathcal{N}_V(0, I)$  and I is the identity map on V.
- **p36 Lemma 2.22** There are too many commas in the inner products here. It should read:  $(V, \langle \cdot, \cdot \rangle)$ ,  $(W_1, \langle \cdot, \cdot \rangle_1)$ , and  $(W_2, \langle \cdot, \cdot \rangle_2)$ .

**p46 line -8 V** \  $\{0\} \times \mathbb{R}$  should be  $V \setminus \{0\} \times \mathbb{R}$  (no bold faced V)

## Errors in the first printing

p5 Example 1.9 The Cauchy density should be

$$f_{\alpha,\beta}(x) = \frac{1}{\pi} \frac{\beta}{((x-\alpha)^2 + \beta^2)}, \quad x \in \mathbb{R}$$

p15 middle It would be more correct to write the Fisher information as

$$i(\theta) = \mathbf{V}_{\theta} \{ S(X, \theta)^{\top} \} = \mathbf{E}_{\theta} \{ I(X, \theta) \}$$

since  $S(X, \theta)$  is defined as an  $1 \times k$  matrix rather than a vector in  $\mathbb{R}^k$ .

**p17 middle** Similarly, as on p15, the correct expression for the Fisher information should be

$$i(\alpha,\beta) = \mathbf{E}_{\alpha,\beta}\{I(X,\alpha,\beta)\} = \mathbf{V}_{\alpha,\beta}\{S(X,\alpha,\beta)^\top\} = \mathbf{V}_{\alpha,\beta}\{(\log X, X/\beta^2)^\top\}$$

p46 Exercise 2.7 c) "is of" should be "of"

p50 Example 3.5 The expression for the rewritten density should be

$$f_{\theta}(x) = \frac{e^{\theta_1 x + \theta_2(-x^2/2)}}{c(\theta)}.$$

**p55 Theorem 3.11** The *n*-fold direct product  $\mathcal{P}^{\otimes n}$  is also minimally represented.

**p57 line -9** should read:

Now, since  $\Theta$  was an open and convex subset of V,  $\tilde{\Theta}$  is an open and convex subset of L...

**p57 line -6** should read:

and thus, if  $\langle \lambda, \tilde{t}(x) \rangle$  is a.e. constant with respect to  $\tilde{\mu}$ , then also  $\langle \lambda, t(x) \rangle$  is a.e. . . .

p57 last line should read:

but then the representation would not be regular since  $\tilde{\Theta}$  is not an open subset of V.

**p64 Example 3.27, line -10**  $B = \mathbb{R}^m$  should be  $B = \phi^{-1}(\Theta)$ 

p65 line 12 should read

$$\ell_x(\theta) = \theta^\top x - \frac{1}{2} \|\theta\|^2.$$

**p65** (3.10) should read

$$\ell_x(\theta) = \phi(\beta)^\top x.$$

**p66 line -10** 'for alle  $\beta$ ' should read 'for all  $\beta$ 

**p70 Exercise 3.6, line 4**  $\lambda x$  should be  $\beta x$ 

p70 Exercise 3.8 Poisson's should be Poisson

p71 Exercise 3.10 Lebesque should be Lebesgue

p75 line -8 should read:

$$\mathbf{B}(\hat{\theta}_n) = \frac{n\theta}{n+1} - \theta = -\frac{\theta}{n+1}$$

**p77 Theorem 4.4**  $\hat{\lambda} = t(x)$  should be  $\hat{\lambda} = t(X)$ 

p88 Example 4.16 The moment functions are

$$m_1(\xi) = \xi$$
,  $m_2(\xi) = P_{\xi}(X > 0) = \Phi(\xi)$ ,  $m_3(\xi) = 3\xi + \xi^3$ 

p104 Theorem 5.6 The asymptotic covariance should read:

$$\Sigma(\theta)/n = Dm(\theta)^{-1} \mathbf{V}_{\theta}(t(X)) Dm(\theta)^{-\top}/n.$$

p104 Example 4.7 last line on page should read:

$$m_1(\xi) = \xi$$
,  $m_2(\xi) = P_{\xi}(X > 0) = \Phi(\xi)$ ,  $m_3(\xi) = 3\xi + \xi^3$ 

p107 Corollary 5.12 Last line would be clearer as:

where  $A = \sqrt{ni(\hat{\theta}_n)}$  is the unique positive definite matrix A satisfying  $A^2 = ni(\hat{\theta}_n)$ .

p111 last line before proof delete 'obtain a'

**p132 Exercise 5.9 c)** Compare the asymptotic distribution of  $\hat{\lambda}_n$  and  $\tilde{\lambda}_n$ .

p169 First displayed formula should read:

$$\hat{p}_N = p_{\text{MC}}(d(x)) = \frac{1}{N} \sum_{i=1}^N \mathbf{1}_{(d(x),\infty)}(d(X_i^*)) = \frac{1}{N} \sum_{i=1}^N Y_i$$

**p177 Exercise 7.7 e)** Derive the quadratic score test statistic for  $H_1$  under the assumption of  $H_0$ .

p195 line -3 "the the" should be "the"

**p201 line -8** "Pearson's  $^2$  evaluates" should be "Pearson's  $\chi^2$  statistic evaluates"

**p229, line -6**  $\mu(d(x))$  should be  $\mu(dx)$