Operations Management in Short-term Power Markets

Ditte Mølgård Heide-Jørgensen

PhD Thesis

August 2016

DEPARTMENT OF MATHEMATICAL SCIENCES
FACULTY OF SCIENCE
UNIVERSITY OF COPENHAGEN

Ditte Mølgård Heide-Jørgensen
Department of Mathematical Sciences
University of Copenhagen
Universitetsparken 5
2100 København Ø
dihj@math.ku.dk // dittemn@gmail.com

This PhD thesis was submitted to the PhD School of The Faculty of Science, University of Copenhagen, Denmark on August 24th 2016.
Before publication in October 2016 a number of minor corrections were made.

Supervisor: Trine Krogh Boomsma, Associate Professor,
University of Copenhagen, Denmark

Co-supervisors: Nina Detlefsen,
Dansk Fjernvarme, Denmark
Salvador Pineda, Associate Professor,
University of Copenhagen, Denmark
Pierre Pinson, Professor,
Technical University of Denmark, Denmark

Assesment Committee: Rolf Poulsen (Chairman), Professor,
University of Copenhagen, Denmark
Kim Allan Andersen, Professor,
Aarhus University, Denmark
Miguel F. Anjos, Professor,
Polytechnique Montréal, Canada

ISBN: 978-87-7078-951-6

PREFACE

In September 2011 a new government took over the political power in Denmark and their ambitious goals of renewable energy penetrations was a great motivation for me to dive head first into the world of renewable energy and electricity markets by taking on a position as PhD student at the Intelligent Energy Systems Programme at Risø, DTU, under the supervision of Peter Meibom. Unfortunately, for me, Peter soon left for a job in the industry, but luckily Trine Krogh Boomsma took over on my supervision, and she brought me back "home" to the Department of Mathematical Sciences at the University of Copenhagen, where my mathematical education had taken place. Here, I continued working on my first project on an intra-hour model for the balancing market in a system with a high wind penetration. This work was done in collaboration with Nina Detlefsen who at the time was employed at the Danish transmission system operator (TSO), Energinet.dk, and Jeanne Aslak Petersen from the University of Aarhus. It resulted in the paper, *Short-term balancing of supply and demand in an electricity system: forecasting and scheduling* which is published in Annals of Operations Research, see Petersen et al. (2016). I had my first daugther in the summer 2012 and after a 9 months maternity leave I brought my family along on my visit to Andy Philpott at the University of Auckland from April through June 2013. Our discussions kick-started my academic machinery after its maternity leave outage. These discussions continued with Trine when I came home and the second project on a dynamic programming model for the stochastic single-unit commitment problem formed. I continued working on the project with my co-supervisor Pierre Pinson at DTU Elektro while Trine was on maternity leave in the summer of 2014 and I presented it at the CMS and IFORS conferences that summer. From November 2014 until the beginning of December 2015 I was on leave with my second daughter. Upon my return I wrapped up the second project resulting in the paper *A dynamic programming approach to the ramp-constrained intra-hour stochastic single-unit commitment problem* which is submitted to an operations research and management science journal. Towards the end of my PhD I worked with equilibrium problems of the electricity markets, but still with emphasis on stochastic models with high time resolution. This work lead to my third project *Open- and closed-loop equilibrium models for the day-ahead and balancing markets* which was done in collaboration with both Trine and my co-supervisor Salvador Pineda. I presented the project at the CMS 2016 conference, however, this is still work in progress.

The thesis consists of an introduction to electricity markets and modelling thereof, including paragraphs on the influence of renewables and uncertainty. Furthermore, I briefly introduce some aspects of game theory in the context of electricity markets. Then three chapters follow – one for each of the three projects I have worked on during my time as a PhD student.

Acknowledgement

My most grateful thanks go to Trine, who took me in when I had no supervisor. Trine has been the best supervisor I could imagine. I want to thank her for believing in me, encouraging me, letting me make my own mistakes and get wiser, helping me out and sharing her extensive knowledge of and passion for modelling of energy problems with me. Thanks to my co-supervisors, Pierre and Salva, for taking over on all of this while Trine was on maternity leave, giving me new views on energy problems. Also thanks to Pierre for inviting me into his group at DTU Elektro. Thanks to my other co-supervisor, Nina, for sharing her knowledge and insight in the energy sector, for many interesting discussions, and for her spoken support along the way. Thanks to Peter Meibom for giving me the opportunity to do the PhD in the first place and supervising me in the first months of my PhD. Thanks to Andy Philpott for letting me visit his department and for discussions and comments during my conference presentations.

I also wish to thank my close friend and collaborator on my first project, Jeanne Aslak Petersen, for sharing her knowledge on electricity markets, systems and problems with me, for demanding solid arguments for my decisions, encouraging me these last few months and for not giving up on me when our project was too much hard work.

Thanks to colleagues both at MATH and DTU Elektro and especially my office mates in 04.3.03 and the other PhD students for making office life interesting and for discussing everything and nothing over lunch or coffee.

Thanks to the administration for making all the formalities go as smooth as possible by e.g. helping me out with rules of the PhD school, maternity leave, teaching formalities etc. etc., and to the secretaries in particular for always being interested in my doings.

I'm very thankful for all the practical, everyday help from my family on several tight spots (some might say continuously in the end), and I want to thank both family and friends (and helpless train passengers) for listening with patience during my passionate talks about energy markets, renewable energy and mathematical modelling.

To my husband Thomas I want to give special thanks. He has been a vital support all the way through the ups and downs and all the hard work. I want to thank him for listening, understanding, turning things upside down, providing perspective and for trying to ensure that I got enough sleep.

Finally, thanks to our daughters Emilie and Mathilde for joining us along the way, teaching me how many things you can do literally single-handed (like coding) and how to appreciate quiet Monday mornings in the office, for helping me prioritize my time, ensuring that I got more smiles than compilation errors every day, and underlining the importance of establishing sustainable energy systems for the future generations.

To T, E and M – for all the time that flew
I dedicate with love and pride my "book" and dreams to you

Til T, E og M – for tiden vi måtte forsømme
dedikerer jeg med kærlighed min "bog" og mine drømme

Summary

Electricity market models have often been modelled as deterministic or two-stage stochastic models with an hourly time resolution. This thesis looks into possible ways of extending such models and formulating new models to handle both higher time resolution than hourly and stochastics without losing computational tractability. The high time resolution is crucial to correctly describe renewables, such as wind power, and capture how they affect the system and the system costs, since they are often fluctuating and hard to predict, also within the hour.

The thesis consists of four chapters. The first is an introduction to the background for the work with stochastic electricity market models with a high time resolution. It is followed by three self-contained chapters.

The second chapter *Short-term balancing of supply and demand in an electricity system: forecasting and scheduling* is on a balancing market model like in the Nordic countries with high time resolution, and it takes extensive balancing rules into consideration. We look into how wind forecast errors can be handled in a system with a large and increasing amount of wind power and at what costs. The project was done in collaboration with Jeanne Aslak Petersen, a PhD student at Aarhus University and the Danish TSO Energinet.dk, and the chapter is identical to the published paper Petersen et al. (2016) except that the reference list is part of the common reference list for the thesis.

The third chapter *A dynamic programming approach to the ramp-constrained intra-hour stochastic single-unit commitment problem* considers a real-time market setup. We describe two stochastic multi-stage single-unit commitment models in which commitment decisions are made on an hourly basis and dispatch decisions are made on a higher time resolution, e.g. 5 minutes. The stochastic input is the electricity price modelled as a time-inhomogenous Markov chain that the power producer uses to maximise profits. To maintain computational tractability with such high time resolution and stochastics the models are solved with dynamic programming. The two models differ in the way the dynamic programming algorithm handles the integer variables leading to two different non-anticipativity assumptions.

In the fourth chapter *Open- and closed-loop equilibrium models for the day-ahead and balancing markets* we look into how power producers act in a market which is not perfectly competitive. Specifically, we look into the possibility of exercising market power when the electricity market consists of two sequential markets

– the day-ahead market and the balancing market – and some power producers have access to both markets whereas others only can participate in the first market. The model is formulated with both an open-loop and a closed-loop approach, and we find that the solution to the more realistic, but also computationally harder closed-loop model differs substantially from the open-loop solution. Again the day-ahead market is assumed to have hourly time resolution, but the balancing market has a higher time resolution.

Resumé

Elmarkedsmodeller er ofte formuleret som deterministiske eller som to-stadie stokastiske modeller med data og beslutninger taget på timebasis. Denne afhandling ser på mulighederne for at udvide modellerne eller formulere nye modeller, så de kan håndtere højere tidsopløsning og stokastik uden at den øgede kompleksitet ødelægger beregningstiden. Den høje tidsopløsning er afgørende for at kunne beskrive vedvarende energikilder såsom vindkraft, samt hvordan de påvirker systemet og systemomkostningerne, korrekt, da de ofte er fluktuerende og svære at prognosticere, også inden for timen.

Afhandlingen består af fire kapitler. Det første er en introduktion til baggrunden for arbejdet med stokastiske modeller af elmarkedet med høj tidsopløsning. Herefter følger tre selvstændige kapitler.

Det andet kapitel *Short-term balancing of supply and demand in an electricity system: forecasting and scheduling* omhandler en balancemarkedsmodel som i de nordiske lande. Modellen har høj tidsopløsning og tager højde for avancerede balancemarkedsregler. Vi undersøger hvordan vindprognosefejl kan håndteres i et elsystem med en høj og stigende vindkraftsandel, samt hvilke omkostninger dette medfører. Projektet blev udført i samarbejde med PhD studerende Jeanne Aslak Petersen fra Aarhus Universitet og den danske systemansvarlige (TSO), Energinet.dk. Kapitlet er identisk med den publicerede artikel Petersen et al. (2016) på nær at der i denne afhandling er én fælles litteraturliste.

I det tredie kapitel *A dynamic programming approach to the ramp-constrained intra-hour stochastic single-unit commitment problem* betragter vi elmarkedet som et realtidsmarked. Her beskriver vi to stokastiske multi-stadie single-unit commitment modeller, hvor commitment beslutningerne tages på timebasis og dispatch beslutningerne bliver taget med en højere tidsopløsning, fx 5 minutter. Den stokastiske del af modellen er elprisen, der modelleres som en tidsinhomogen Markovkæde, som elproducenten bruger for at maksimere sin profit. For at holde beregningstiden nede selv med en høj tidsopløsning og stokastisk data, bliver modellen løst vha. dynamisk programmering. De to modeller adskiller sig fra hinanden ved den måde den dynamiske programmeringsalgoritme håndterer de binære variable, hvilket fører til to forskellige non-anticipativitetsbetingelser.

I det fjerde kapitel *Open- and closed-loop equilibrium models for the day-ahead and balancing markets* undersøger vi, hvordan elproducenter agerer i elmarkedet, når der ikke er perfekt konkurrence. Specielt ser vi på muligheden for at udøve

markedsmagt, når elmarkedet er sammensat af to markeder – spot- og balancemarkedet – og nogle producenter har adgang til begge markeder mens andre kun har adgang til det første marked. Modellen formuleres både med en open-loop og en closed-loop tilgang og vi finder, at løsningen til den mere realistiske, men også mere komplekse og beregningstunge closed-loop model adskiller sig væsentligt fra open-loop løsningen. Vi lader igen spot markedet være på timebasis, mens balancemarkedet har højere tidsopløsning.

CONTENTS

– 1 –

INTRODUCTION

This chapter provides an overview over the three problems addressed in Chapters 2, 3 and 4 including the relevant background, related literature and the applied methodology. First, I explain the functioning of electricity markets as well the electricity system challenges that need to be addressed and the theoretical background. This serves as a basis for the presentation of the social welfare maximising, deterministic balancing market model in Chapter 2, the profit maximising, stochastic, perfect competition single-unit commitment models in Chapter 3 and finally the profit maximising, stochastic, but strategic equlibrium models of Chapter 4. The three chapters are self contained, except for the common reference list at the end of the thesis.

1.1 Electricity markets and unit commitment

In this thesis I consider the electricity wholesale market. The buyers in the market are electricity consumers or more often retailers serving as a link between the market and the individual households or companies, all referred to as consumers in the following. I assume the sellers are electricity producers, with some physical and/or contractual constraints on their production and some costs related to the production that will affect the price at which the producers are willing to produce and sell electricity.

If we consider sequential markets, under some circumstances I would also like to consider a third player, an arbitrageur, someone who buys electricity from the producers or sells it to the consumers, in the first market and then sells the electricity to the consumers or buys it from the producers, respectively, in the second market while trying to make a profit via potential price differences.

Electricity markets work in many ways as other physical markets. Consumers and producers submit buying and selling offers to the market, and then the market clearing settles the price and the volume e.g. by matching an aggregated supply curve with an aggregated demand curve. However, there are some characteristics of electricity that differentiate it from other commodities. First of all electricity cannot be efficiently stored, and since deficit of electricity leads to undesired blackouts and surplus of electricity may lead to melt downs of e.g. cables which likewise results in blackouts, the supply and demand much match both on an hour-to-hour, minute-to-minute and even second-to-second basis.

In the 1990ies the electricity markets were liberalised to avoid the monopoly like settings that caused high prices, but still electricity systems have system operators (SOs) responsible for maintaining the balance between supply and demand. Some markets, e.g. the NEM in Australia[1] and the APX UK in the UK, see Weber (2010), are real time markets that clear close to the hour of operation and several times a day. In other places a day-ahead spot market is cleared the day ahead of production, to allow for the use of slow units that need several hours to turn on/off or change production level. Then a subsequent intra-day or balancing market is run several times closer to the hour of operation to handle imbalances that have occurred in the meantime, due to outages of power lines, unplanned shut downs of units, changes in demand or wind power production etc. This is the way the markets function in e.g. Germany and the Nordic Countries, see Weber (2010). For both electricity market setups, expensive automatic reserves are ready to handle differences between supply and demand that may occur after the markets have cleared.

The socio-economics of balancing supply and demand in an electricity system from the SOs point of view, can be modelled as the *economic dispatch problem* (ED). For a given time horizon, $\mathcal{H} = \{1, \ldots, H\}$, and a set of power production units, $\mathcal{I} = \{1, \ldots, I\}$ the aim is to determine the optimal production level q_{ih} of each power producing unit, $i \in \mathcal{I}$ in each time period $h \in \mathcal{H}$ given a minimum and maximum production level, $q_i^{\min}$ and $q_i^{\max}$, for each unit $i \in \mathcal{I}$, while ensuring that the exogenously given demand in the system d_h is matched by the sum of the production of all the units and minimising the sum of the production costs $c_i(q_{ih})$. Thus assuming that producers offer their production at marginal costs, which is equivalent to assuming perfect competition in the market. If the cost function c_i is linear, this can be formulated as a *linear programming problem* (LP), which is easy to solve with a stadard solver utilizing the simplex algorithm.

$$
\begin{aligned}
\min_{q_{ih}} & \sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} c_i(q_{ih}) \\
\text{S.t.} & \sum_{i \in \mathcal{I}} q_{ih} = d_h, \;\; h \in \mathcal{H} \\
& q_i^{min} \leq q_{ih} \leq q_i^{max}, \;\; h \in \mathcal{H}, \;\; i \in \mathcal{I}.
\end{aligned}
$$

The problem can be extended in several ways, e.g. to cover multiple areas connected via transmission lines. Moreover, additional technical constraints can be included in the problem, e.g. ramp rates and constraints representing market instruments like reserve capacity constraints ensuring that there is enough idle

[1] See Chapter 3.

capacity to maintain system reliability in the case of outages of units or cables. If the cost function is not linear, but still convex, this is a *convex programming problem* (CP) and can still be readily solved.

More often the problem includes decisions on when to start up and shut down the production units. Then it is called the *unit commitment problem* (UC), see Garver (1962); Sheble and Fahd (1994); Gollmer et al. (2000); Hobbs et al. (2001); Anjos (2013). The online/offline status of the units is modelled with binary variables so the problem (with linear costs) is no longer an LP, but a *mixed integer linear programming problem* (MIP).

For every $i \in \mathcal{I}$ and $h \in \mathcal{H}$, the online status is represented by u_{ih}, which is 1 when the unit is online, and 0 otherwise. The binary start-up variable v_{ih} is 1 if unit i is started up at time h, and 0 otherwise. Let c_i^o denote the cost of being online and c_i^s represent the start-up cost for unit i. Sometimes offline costs and shut-down costs are defined similarly to the online and start-up costs, but I will omit them here. Now, we obtain the objective

$$\min_{q_{ih},u_{ih},v_{ih}} \sum_{i\in\mathcal{I}} \sum_{h\in\mathcal{H}} \left(c_i(q_{ih}) + c_i^o u_{ih} + c_i^s v_{ih}\right).$$

With the balance constraint as above and minimum/maximum capacity constraints now applied only when the unit is online

$$\begin{aligned}
&\sum_{i\in\mathcal{I}} q_{ih} = d_h, \quad h \in \mathcal{H} \\
&q_i^{min} u_{ih} \le q_{ih} \le q_i^{max} u_{ih}, \quad h \in \mathcal{H}, \quad i \in \mathcal{I}.
\end{aligned}$$

Minimum up- and down-time constraints ensure that a unit is online in at least T_i^{up} consecutive time intervals before it shuts down and offline in at least T_i^{down} consecutive time intervals before it turns on. Finally, logical constraints ensure the start-up variables are set by the commitment variables.

$$\begin{aligned}
&u_{ih} - u_{ih-1} \le u_{ik}, \quad k = h+1, \ldots, \min\{h + T_i^{up} - 1, H\}, \quad h \in \mathcal{H}, \quad i \in \mathcal{I} \\
&u_{ih-1} - u_{ih} \le 1 - u_{ik}, \quad k = h+1, \ldots, \min\{h + T_i^{down} - 1, H\}, \quad h \in \mathcal{H}, \quad i \in \mathcal{I} \\
&u_{ih} - u_{ih-1} \le v_{ih}, \quad h \in \mathcal{H} \backslash \{1\}, \quad i \in \mathcal{I} \\
&u_{ih}, v_{ih} \in \{0,1\}, \quad h \in \mathcal{H}, \quad i \in \mathcal{I}.
\end{aligned}$$

The MIP formulation is a hard problem to solve with realistic sized data. Several solution methods including *Lagrangian relaxation* and *dynamic programming* (DP) has been proposed in the literature, see Zhuang and Galiana (1988); Muckstadt and Koenig (1977); Frangioni et al. (2008); Rong et al. (2008).

1.2 Challenges for the electricity systems

Pollution and climate changes have encouraged political decisions to integrate renewable energy in the electricity systems. For instance, national and EU goals for renewable energy have resulted in a wind power production increase in Denmark, such that up to 50% of the electricity will be from wind power production in 2020 and 35% of the total energy consumption will be from renewable energy resources[2]. In the EU the goal is that 20% of the energy consumption comes from renewables in 2030[3], and also here electricity consumption will be a large part of the renewable energy consumption. These political decisions have led to massive investments in renewable energy, of which especially the large amounts of wind power challenge the security of supply. Wind power is variable, also within the hour, to a large degree uncontrollable and hard to predict, making it hard for the SOs to maintain balance in systems with a lot of wind power. In fact, Weber (2010) finds that the total imbalance in the system and the wind forecast error are strongly correlated in systems with a high wind penetration. The SOs mainly handle the imbalances by adjusting production of the conventional power producing units, known as activation of reserves. Hence, there is a growing need for flexible conventional power producing units in the electricity systems.

To model these challenges for electricity systems with a high share of renewables appropriately, we need

1. Models with a higher time resolution than hourly to capture the variations of wind power and the accompanying ramping requirements precisely.

2. Multi-stage stochastic models to make it possible to represent the uncertainty in the wind power production and updated information between e.g. the day-ahead market and the balancing market.

The first requirement can be accommodated simply by introducing a more fine grained time horizon, $t \in \mathcal{T} = \{1, \ldots, T\}$ such that $\mathcal{H}$ can be interpretted as a number of hours and $\mathcal{T}$ a number of intra-hour time periods. In the following I denote by $\mathcal{T}_h = \{T_{h-1} + 1, \ldots, T_h\} \subseteq \mathcal{T}$ the intra-hour time periods in hour h and let $\tau_h = 1/(T_h - T_{h-1})$ be the reciprocal value of the number of intra-hour time periods in hour h. Usually $\tau_h = \tau_k$, for $h, k \in \mathcal{H}$. The second requirement is e.g. handled by letting d_t represent the net demand (demand minus uncertain renewable production) in time period t and assuming it is a discrete random variable on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The realization is denoted $d_{t\omega}$. Commitment decisions and an initial production schedule are made before knowing the realized net demand and then adjusted afterwards. This corresponds to the day-ahead and balancing market in a two stage setup where each market is cleared once. Let $q^+_{it\omega}$ denote the production increase on unit i in time period t and scenario ω and let $q^-_{it\omega}$ denote the production decrease. Then we have the two-stage *stochastic unit commitment problem* (SUC) which is a *stochastic programming problem*

[2] From http://www.ens.dk/en/policy/danish-climate-energy-policy on July 21st 2016.
[3] DIRECTIVE 2009/28/EC.

(SP) with recourse, see Birge and Louveaux (2011).

$$
\begin{aligned}
&\min_{q_{ih\omega},q^+_{it\omega},q^-_{it\omega},u_{ih},v_{ih}} \sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}}\left(c^o_i u_{ih} + c^s_i v_{ih} + \sum_{t\in\mathcal{T}_h}\mathbb{E}[c_i(\tau_h q_{ih} + q^+_{it\omega} - q^-_{it\omega})]\right)\\
&\text{S.t. } \sum_{i\in\mathcal{I}} q_{ih} = \mathbb{E}[d_{t\omega}],\ \ h\in\mathcal{H}\\
&\sum_{i\in\mathcal{I}}(\tau_h q_{ih} + q^+_{it\omega} - q^-_{it\omega}) = d_{t\omega},\ \ \omega\in\Omega,\ \ t\in\mathcal{T}_h,\ \ h\in\mathcal{H}\\
&q^{min}_i u_{ih} \le q_{ih} \le q^{max}_i u_{ih},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I}\\
&q^{min}_i u_{ih} \le q_{ih} + q^+_{it\omega} - q^-_{it\omega} \le q^{max}_i u_{ih},\ \ \omega\in\Omega,\ \ t\in\mathcal{T}_h,\ \ h\in\mathcal{H},\ \ i\in\mathcal{I}\\
&u_{ih} - u_{ih-1} \le u_{ik},\ \ k = h+1,\dots,\min\{h+T^{up}_i - 1, H\},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I}\\
&u_{ih-1} - u_{ih} \le 1 - u_{ik},\ \ k = h+1,\dots,\min\{h+T^{down}_i - 1, H\},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I}\\
&u_{ih} - u_{ih-1} \le v_{ih},\ \ h\in\mathcal{H}\backslash\{1\},\ \ i\in\mathcal{I}\\
&q^+_{it\omega}, q^-_{it\omega} \ge 0,\ \ t\in\mathcal{T},\ \ i\in\mathcal{I}\\
&u_{ih}, v_{ih} \in \{0,1\},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I}.
\end{aligned}
$$

Some work has already been done on variants of such two-stage SUC models, but mainly on hourly models ($\mathcal{T} = \mathcal{H}$), e.g. Papavasiliou and Oren (2013); Bouffard and Galiana (2008); Morales et al. (2009a); Pritchard et al. (2010). However, the balancing market is usually cleared several times a day with updated information each time, which can be represented by multi-stage reformulations of the SUC. Since the problem already contains multiple time periods, it can be reformulated as a multi-stage SP with recourse by introducing explicit non-anticipativity constraints in the problem:

$$
\begin{aligned}
q^+_{it\omega} &= q^+_{it\omega'} \text{ if } (d_{1\omega},\dots,d_{t\omega}) = (d_{1\omega'},\dots,d_{t\omega'}),\ \ \omega,\omega'\in\Omega,\ \ t\in\mathcal{T},\ \ i\in\mathcal{I}\\
q^-_{it\omega} &= q^-_{it\omega'} \text{ if } (d_{1\omega},\dots,d_{t\omega}) = (d_{1\omega'},\dots,d_{t\omega'}),\ \ \omega,\omega'\in\Omega,\ \ t\in\mathcal{T},\ \ i\in\mathcal{I}.
\end{aligned}
$$

Considering the multi-stage SUC from the point of view of a production planner rather than a market modeller, the multi-stage problem looks slightly different. Here, the production is not split into two markets with separate sets of production variables, but rather a single variable, $q_{it\omega}$, represents the production at each time period $i\in\mathcal{T}$, $i\in\mathcal{I}$ and $\omega\in\Omega$. The multi-stage production planners SUC is

$$
\begin{aligned}
&\min_{q_{ih\omega},u_{ih},v_{ih}} \sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}} \left(c_i^o u_{ih} + c_i^s v_{ih} + \sum_{t\in\mathcal{T}_h} \mathbb{E}[c_i(q_{it\omega})] \right) \\
&\text{S.t. } \sum_{i\in\mathcal{I}} q_{it\omega} = d_{t\omega},\ \ \omega\in\Omega,\ \ t\in\mathcal{T} \\
&q_i^{min} u_{ih} \le q_{it\omega} \le q_i^{max} u_{ih},\ \ \omega\in\Omega,\ \ t\in\mathcal{T}_h,\ \ h\in\mathcal{H},\ \ i\in\mathcal{I} \\
&u_{ih} - u_{ih-1} \le u_{ik},\ \ k = h+1,\dots,\min\{h+T_i^{up}-1, H\},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I} \\
&u_{ih-1} - u_{ih} \le 1 - u_{ik},\ \ k = h+1,\dots,\min\{h+T_i^{down}-1, H\},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I} \\
&u_{ih} - u_{ih-1} \le v_{ih},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I} \\
&u_{ih}, v_{ih} \in \{0,1\},\ \ h\in\mathcal{H},\ \ i\in\mathcal{I} \\
&q_{it\omega} = q_{it\omega'} \text{ if } (d_{1\omega},\dots,d_{t\omega}) = (d_{1\omega'},\dots,d_{t\omega'}),\ \ \omega,\omega'\in\Omega, t\in\mathcal{T},\ \ i\in\mathcal{I}.
\end{aligned}
$$

The stochastic extensions – and in particular the multi-stage SUC – make the already computationally challenging UC problem even harder to solve, since both the high time resolution and the stochastic setting will increase the number of binary variables considerably. Hence only few multi-stage models can be found in the literature, e.g. Nowak and Römisch (2000), but still the number of stages is rather low.

1.2.1 A social welfare maximising, deterministic, intra-hour balancing market model under perfect competition

To overcome 1. Chapter 2 presents the balancing market model OPTIBA, which is coupled in a rolling planning fashion to a UC model for the day-ahead market. After the UC model has run for a day, the production and transmission plan is converted to a high time resolution, e.g. 5 minutes, and wind forecast errors are simulated via the wind forecast module. OPTIBA subsequently runs for every hour of the day optimising a pro-active reserve activation like a balancing market. Here, the production levels from the UC model can be adjusted by increasing production, called activation of up-regulation or by decreasing production called activation of down-regulation. To better represent the actual reserve activation we consider a number of technical rules for the balancing market. The rolling planning setup enables us to model frequent updates of the wind power forecasts even though the model is deterministic. In this way, there is updated information between the run of the day-ahead market and the balance market. The high time resolution (5 minutes compared to an hour) reveals intra-hour imbalances due to wind forecasts errors, ramping of power producing units and of transmission lines. Further, we investigate the effect of different ramping speeds to and from regions. The model finds the most cost effective way to handle these imbalances as a trade off between the up- and down-regulation (restricted by the technical balancing rules) and the expensive automatic reserves. Our main result is that in spite of the increase in wind power production in the system from the 2010 level to the expected 2020 level the increased balancing costs do not outweigh the benefits of the inexpensive wind power. Finally, we consider the costs of simply leaving the imbalances to the expensive automatic reserves, which shows that there are considerable cost savings in a pro-active reserve activation already in the current system, and that these savings only increase for the 2020 wind power production. The natural extension of the work on OPTIBA is to consider a two-stage setup with several scenarios for the wind forecast errors and then to quantify the impact on the system balance, production schedules and balancing costs taking into account uncertainty. This has been done in Andersen (2015) for a simplified version of OPTIBA.

1.2.2 A multi-stage stochastic, profit maximising, intra-hour single-unit commitment problem under perfect competition

As mentioned above, Lagrangian relaxation can be used to solve the UC problem, by relaxing the balance constraint that couples all the production units together by matching supply and demand. This decouples the units, so the problem can be solved one unit at a time or even in parallel. Furthermore, it allows for the use of solution methods that can solve the single-unit commitment problem (1UC) effectively. The 1UC subproblem now consists of deciding the online status of a single unit as well as the production level, given the electricity price in each time period and maximising profits. After the liberalisation of the electricity markets this problem is also interesting in itself from a producers point of view. To handle both 1. and 2. at the same time we propose in Chapter 3 two dynamic programming (DP) approaches to solve the stochastic, intra-hour 1UC.

As mentioned above, DP has previously been used in the literature to solve the system-wide unit commitment problem, but also the single-unit commitment see Frangioni and Gentile (2006); Tseng and Barz (2002). Usually, commitment and dispatch is decided for a given state consisting of the time and possibly the online status of the units, etc., and then moving forwards or backwards in time, at each state determining the optimal solution for this state given the optimal solution for previous states respectively future states. The advantage of using DP is that we can handle all the binary variables separately as state variables. Only the scheduling of the power producing unit remains, which, as earlier mentioned, is an convex programming problem and hence easy to solve. The drawback is that DP cannot handle constraints that couple continuous variables over time, e.g. constraints that couple the continuous production variables, unless the state space is discretized. This is because the state space for the dynamic programming algorithm is discrete. However, Frangioni and Gentile (2006) overcome this by defining the states of the DP algorithm such that the time in a state is not just one time period, but a whole online or offline period of a unit. This way, they can handle the ramping within each state. The model is a forward moving deterministic dynamic programming model. We extend this to a backwards moving, stochastic, dynamic programming model with high time resolution, which we name the multi-hour model, since each state covers several hours. This can be seen as a stronger non-anticipativity condition in stochastic programming and it provides a lower bound on the original problem. Tseng and Barz (2002) propose a backwards moving dynamic programming algorithm to handle both the deterministic and stochastic single-unit commitment problem. They handle ramping heuristically and propose to discretise the production levels. We extend this model to a high time resolution model, which we name the single-hour model, that handles ramping within the hour continuously and discretise only the production level in the last time period in each hour. Numerical results show that the discretisation is fine grained enough to represent the solution accurately. Furthermore, we find for both models that the high time resolution reveals profit opportunities that are invisible on the hourly level. The savings are relevant for producers optimising their production schedule, but also for SOs to evaluate the total costs of running the electricity system. Future work could look into utilizing the single-hour model for the UC problem solved by Lagrangian relaxation.

1.3 Competition in electricity markets

A large amount of the literature on electricity markets (e.g. all of the above) implicitly assumes that the markets are perfectly competitive. This means that actors in the markets cannot influence the market price, i.e. they cannot exercise market power and hence offer production at their marginal costs. However, there are several circumstances in electricity systems that enable the possibility of exercising market power. Some markets have actors of a size superior to the others enabling them to exercise market power. Furthermore, congestion issues in the electricity system can isolate areas with only a few actors propragating this problem. Actors participating in markets in several areas can thus profit from setting marginal price in (at least) some of the areas. Also, in setups with sequential markets like the day-ahead market followed by the balancing market, in which some actors may only participate in one of the markets, and in which production in the first market directly influences the available capacity in the second market, there may be opportunities to exercise market power and alter prices. Due to these issues, many game theoretic models of mono-, duo- and oligopolistic electricity markets exist in the literature.

1.3.1 Types of competition

There are several ways of modelling imperfect competition in electricity markets. A large part of the literature is concerned with *Cournot competition* in which actors bid in a quantity they are willing to produce with knowledge of the demand curve and anticipating the bids of the other actors, see e.g. Murphy and Smeers

(2005). Then the actors are paid the market price for the volume they offered to the market. This is called competition in quantity. It is also possible to compete in price, which is known as *Bertrand competition*. As this involves offering a price (again knowing the demand curve) and then adjusting production to the market demand, it is, however, seldomly used in electricity markets since generators often operate close to their capacity limits.

If one player is dominant and the rest of the players await the action of the dominant player before acting themselves, the competition can be described as a *Stackelberg game*, von Stackelberg (1934); Gabriel et al. (2013). Here, the dominant player, also called the leader, makes decisions while anticipating the response from the other players, also called the followers, and then they subsequently make their decisions while competing in e.g. a Cournot setup, once the decision of the leader is known.

The concept of Cournot competition can be generalized by *conjectural variations*. The conjectural variations govern how a producer expects the other producers to react to a change in the system price, see Song et al. (2003); García-Alcalde et al. (2002); Wogrin et al. (2013). In a setup with a linear demand function this can be reformulated to a conjectured price response, in which a parameter captures how a producer believes a change in its production can influence the system price. When this parameter varies it is possible to describe oligopolistic settings ranging from perfect competition over Bertrand and collusion to Cournot.

Finally, it is also possible to describe the offers in the market by supply functions, i.e. offers consisting of both prices and corresponding capacities. The concept of *supply function equilibrium* (SFE) models was first introduced in electricity market modelling by Klemperer and Meyer (1989). The supply provides the volume the power producer is willing to produce at a given price. With uncertainty in demand or price, the supply function provides a way of hedging. Still, the Cournot setting is often preferred due to its computational tractability Ventosa et al. (2005).

1.3.2 Open-loop models

The modelling of sequential markets raises the question of whether to use an open-loop or a closed-loop model. In the open-loop model the two markets are co-optimised, i.e. it is assumed that all decisions are made simultaneously, see Murphy and Smeers (2005). For a Cournot or conjectural variations based model this will result in an equlibrium problem that can be formulated as a mixed complementarity problem (MCP) via the Karush Kuhn Tucker (KKT) conditions, see Gabriel et al. (2013).

In the following simple one market MCP example, we consider an oligopoly with a number of producers, $i \in \mathcal{I}$ optimising their production level, x_i, given a production cost function $c_i(x_i)$ and the inverse demand function for the market, $p = \beta - \alpha d$, where p is the market price, d is the demand and $\alpha > 0$ and $\beta > 0$

are inverse demand function parameters. The market equilibrium problem can be formulated as

$$\max \left\{ px_i - c_i(x_i) : x_i \overset{(\sigma_i)}{\geq} 0 \right\}, \;\; i \in \mathcal{I}$$
$$p = \beta - \alpha d$$
$$d = \sum_{i \in \mathcal{I}} x_i,$$

where σ_i is the dual variable of the non-negativity constraint. Assuming $px_i - c_i(x_i)$ is concave, the KKT MCP reformulation is

$$\frac{\partial p}{\partial x_i} x_i + p - \frac{\partial c_i}{\partial x_i} + \sigma = 0, \;\; i \in \mathcal{I}$$
$$0 \leq x_i \perp \sigma_i \geq 0, \;\; i \in \mathcal{I}$$
$$p = \beta - \alpha d$$
$$d = \sum_{i \in \mathcal{I}} x_i.$$

Here, the difference between Cournot, perfect competition or more general conjectural variations lies in the partial derivative of the price. Perfect competition is obtained by letting $\frac{\partial p}{\partial x_i} = 0$ and Cournot by $\frac{\partial p}{\partial x_i} = -\alpha$. See also Wogrin et al. (2013).

1.3.3 Closed-loop models

In a closed-loop model, on the other hand, it is assumed that decisions in each market are made sequentially in time, and that decisions in the first market (denoted the upper level) are known, when decisions in the second market (denoted the lower level) are made, while the upper level decisions are made anticipating the lower level effects, see again Murphy and Smeers (2005). An example is a multi-leader Stackelberg game, where the leaders are the participants in the first market and the followers the participants in the second market. To solve this problem it can be formulated as an equilibrium problem for the upper level constrained by the e.g. the KKT equilibrium conditions from the market in the lower level, resulting in an equilibrium problem with equilibrium constraints (EPEC). Sequential markets models, like the forward spot markets, the day-ahead and balancing markets and also the capacity expansion and spot market models, all have the property that the decisions are taken sequentially in time after new information has arrived as opposed to simultaneously. Therefore closed-loop models is the natural choice for the modelling of these problems. However, the EPECs are much harder to solve than the MCPs, and existence and uniqueness of solutions can in many cases only be established under restrictive assumptions, see Shanbhag et al. (2011); Zhang et al. (2010). The first use of closed-loop modelling of sequential electricity markets was in Allaz (1992), in which reflections on the differences from assumptions in the open-loop modelling framework is also discussed. Some work has been done on comparing the solutions for the two different kinds of models, see Wogrin et al. (2013) for the capacity expansion problem. The authors find that when considering only one time period in the spot market, the open- and closed-loop solutions are the same, but otherwise the solutions may differ. Similarly, for the forward spot problem, Shanbhag et al. (2011) establish a setup in which the open- and closed-loop solutions are the same. Ito and Reguant (2015) present a closed-loop model resembling both the forward spot and the day-ahead balancing markets, considering a monopolist and fringe suppliers. The authors do not consider the open-loop formulation. In Chapter 4, we consider the sequential day-ahead and balancing markets, however we assume a duopolistic setting. We assume a high time resolution in the balancing market and formulate a stochastic model with update of the intercept of the inverse demand function corresponding to changes in net demand, i.e. demand minus wind power production. The upper level variables, i.e. the day-ahead production, are linked to the lower level variables via constraints on total capacity. Costs and profits are functions of total production as well. This way the model differs from the capacity expansion problem and the forward spot problem in which the upper level variables, capacity respectively forward trade, are not constrained by total capacity limits binding them to lower level variables. Also, the models usually have separate profits and separate (if any) costs for variables in each market. With these differences in the sequential market setup, we investigate whether the open- and closed-loop solutions are still the same for the day-ahead and balancing markets.

1.3.4 Profit maximising, stochastic, strategic open- and closed-loop models of the day-ahead and balancing markets

We consider a duopolistic Cournot setup with symmetric players, with high time resolution and stochastic demand. We solve both the open-loop and the closed-loop model with varying parameters for the inverse demand function. Our results show that solutions differ substantially, due to the relation between the day-ahead and balancing market inverse demand function and due to the difference in time resolutions or equivalently the different scenarios in the stochastic setup. Assuming that arbitrage is explicitly handled, the differences are even larger. It is evident that for proper representation of the day-ahead and balancing markets it is necessary to consider the computionally harder closed-loop models. When comparing perfect competition with Cournot we see that market power is exercised by holding back production in the day-ahead market to increase production in the balancing market. Finally, we address the problem of limited access to the balancing market. Surprisingly, we find that in a duopoly it is not always advantageous to be the producer with access to the balancing market. Rather, it depends on the parameters of the inverse demand functions whether the producers with or without access to the balancing market can exploit the situation to make the largest profit gain compared to a single-market setup. The next step is to consider the effect on the social welfare of the mentioned observations rather than just the profit of the units. Further it would be interesting to extend the models to general conjectural variations, introduce more technical constraints like ramp rates and carry out larger case studies with stochastic parameters.

– 2 –

SHORT-TERM BALANCING OF SUPPLY AND DEMAND IN AN ELECTRICITY SYSTEM: FORECASTING AND SCHEDULING

Jeanne A. Petersen[1], *Dep. of Economics and Business, Aarhus University*,
Ditte M. Heide-Jørgensen[1], *Dep. of Math. Sciences, University of Copenhagen*,
Nina K. Detlefsen, *Dansk Fjernvarme*,
Trine K. Boomsma, *Dep. of Math. Sciences, University of Copenhagen*

ABSTRACT Until recently, the modelling of electricity system operations has mainly focused on hour-by-hour management. However, with the introduction of renewable energy sources such as wind power, fluctuations within the hour result in imbalances between supply and demand that are undetectable with an hourly time resolution. Ramping restrictions on production units and transmission lines contribute further to these imbalances. In this paper, we therefore propose a model for optimising electricity system operations within the hour. Taking a social welfare perspective, the model aims at reducing intra-hour costs by optimally activating so-called manual reserves based on forecasted imbalances. Since manual reserves are significantly less expensive than automatic reserves, we expect a considerable reduction in total costs of balancing. We illustrate our model in a Danish case study and investigate the effect of an expected increase in installed wind capacity. We find that the balancing costs do not outweigh the benefits of the inexpensive wind power, and that the

[1]Contributed equally to this work

savings from activating manual reserves are even larger for the high wind capacity case.

KEYWORDS *OR in energy; Scheduling; Forecasting; Power system balancing*

2.1 Introduction

For more than a century, electricity supply has been highly dependent on fossil fuels such as coal and oil, and even today, fossil fuels continue to make up a large part of the global supply chain for electricity. Due to depletion of fossil fuels and looming climate changes, development of sustainable energy, not least electricity, has received considerable attention over the last two decades. Sustainable electricity development concentrates on three major aspects: savings on the demand side, efficiency improvements on the supply side, and replacement of fossil fuel based production by various sources of renewable production (Lund (2007)).

With the introduction of renewable production, security of supply has become an issue of utmost importance. Unlike most commodities, electricity cannot be efficiently stored. Furthermore, major imbalances between supply and demand are extremely undesirable, since deficit or excess in production may result in blackouts. In fact, production and consumption have to balance constantly. However, whereas conventional production can be controlled, renewable production is often intermittent and non-controllable. Therefore, increasing shares of renewables significantly challenge the ability of an electricity system to balance supply and demand.

In addition to the fluctuating nature of intermittent production, several other aspects affect the balancing of supply and demand in the electricity system. Substantial transmission distances between regions and different ramping speeds to and from a region may result in large imbalances between supply and demand. Ramping restrictions on conventional power units, failures of power units and transmission lines, and fluctuations in the demand for electricity likewise impact the balance of the system.

The monitoring and maintenance of the balance between supply and demand is controlled by System Operators (SOs), who each have their own control area. How to balance between control areas is agreed upon by the SOs or by international cooperation. To maintain the balance within a control area, an SO manages so-called reserves. Reserves are activated or deactivated by either increasing or decreasing conventional production.

In general, reserves can be categorised into automatic and manual reserves. Automatic reserves are fast (operational within seconds) and flexible, hence expensive, and used for controlling the system frequency. Manual reserves are slower (operational e.g. within minutes) and less flexible, hence less expensive, and usually activated in order to release automatic reserves. In the Nordic countries, manual reserves are provided by producers submitting bids to the SO close to real-time. These bids are activated on a least cost basis. In contrast, automatic reserves are negotiated between producers and the SO on long-term contracts. With the integration of renewables, a major problem for the SO is how to manage the system in the most cost efficient way.

Common practice of many SOs for managing supply and demand is a reactive approach in which they first leave the observed imbalance to be handled by the fast automatic reserves, and second let the slower manual reserves take over. Nevertheless, by activating slow reserves prior to the observation of the imbalance and rather on the basis of an expected imbalance, the SO could utilise slower reserves much more efficiently. In particular, such proactive activation of manual reserves would reduce the need for automatic reserves and thereby the costs of balancing.

This paper establishes a framework for short-term management of electricity system operations. The purpose of the presented model is to analyse the operational consequences of proactive activation of manual reserves under different configurations of the electricity system. Furthermore, it may be used in advance of negotiating contracts on automatic reserves. We consider a system in which wind power is the renewable source. Still, the methodology applies equally well to many other renewable sources such as solar power and run-of-river hydropower. We assume that the associated electricity market includes a day-ahead market for hourly dispatch and an intra-hour balancing market like the electricity markets in the Nordic countries. However, the set-up of the paper can easily be adapted to other market designs. Our contribution is the following:

We develop a mixed integer linear programming model for the optimal management of a future electricity system with significant shares of wind power. We use it to analyse system operation from a social welfare perspective. This model is a multi-area economic dispatch (ED) model that proactively activates manual reserves such as to minimise balancing costs, while taking operational constraints into account, including detailed ramping restrictions on transmission lines and conventional generation units. In taking the social welfare perspective, we assume that conventional units make their idle capacity available to the SO, and so the optimal bids of manual reserves are determined endogenously by our model. To appropriately represent intra-hour system operations, the model has a time resolution of a few minutes. However, to ensure consistency with hour-by-hour day-ahead planning, it is integrated with a unit commitment (UC) model that has an hourly time resolution.

Furthermore, we generate a representative forecast of wind power production that serves as input to the intra-hour model. To capture the lower fat tail of wind power production at a given point in time we assume marginal Beta distributions. In constructing the joint distribution, we model the temporal correlation structure using the Gaussian copula.

In the following, we refer to the intra-hour model and the accompanying framework with the unit commitment and forecasting as OPTIBA (OPTimisation of the Intra-hour BAlance).

We illustrate the intra-hour model with a number of case studies based on the Danish electricity system for varying seasons in a year and for the current and planned future level of wind power penetration. Furthermore, in each case we run our model in a rolling planning fashion to incorporate updated information and thereby simulate the results of optimal intra-hour balancing over time.

2.2 Related literature

The hour-by-hour management of an electricity system is handled by solving the UC problem. The first mixed-integer linear programming approach to this problem dates back to Garver (1962), and a review of the early work on UC models and corresponding solution methods can be found in Sheble and Fahd (1994). These models consider a regulated electricity sector, whereas the deregulation of the electricity markets in the 1990s motivated the development of new state-of-the-art models, see Hobbs et al. (2001). These usually take the viewpoint of either a price-taker or a central market operator. A price-taker maximises profit by optimally operating the power generating units subject to the market structure and prices, e.g. in Heredia et al. (2012). In contrast, the central market operator optimises the system operation while maximising social welfare, or with inflexible demand, minimising total production costs, see Doorman and Nygreen (2003).

Many UC models incorporate uncertainty in some form. Already in Dillon et al. (1978), unit outages and the resulting capacity shortage were addressed in a stochastic programming setting. More recent contributions also consider uncertain demand, e.g. see Bunn and Paschentis (1986); Takriti et al. (1996); Caröe and Schultz (1998); Nowak and Römisch (2000); Gollmer et al. (2000).

With the improvement of demand forecasts and the large scale penetration of sustainable energy, renewable production has become the major source of uncertainty and again new models have emerged. One such example is found in Weber et al. (2009). This UC model is likewise a stochastic programming model, but accounts for the variability and unpredictability of wind power. To reflect occasional updating of wind power forecasts, limit problem size, and reduce computation time, the model is run in a rolling planning fashion with re-optimisation every few hours.

Others handle the uncertainty by solving the UC and the subsequent intraday ED problem as a two-stage stochastic mixed-integer program. An example is Papavasiliou and Oren (2013) who handle unit commitment of slow units in the first stage and commitment of fast units as well as dispatch of all units in the second stage. This model takes into account ramp rates, contingencies, and transmission network constraints. A wind scenario generation and selection algorithm is included as well as a subgradient algorithm to solve the stochastic programming problem. Similar two-stage models are found in Bouffard and Galiana

(2008); Zheng et al. (2012); Morales et al. (2009a); Pritchard et al. (2010). However, in all of these references, the time resolution is hourly, and being only two-stage models, they do not capture the frequent update of wind forecasts carried out in reality. Moreover, the models do not include the technical constraints on reserve power that are presented in this paper.

For UC models in general, it is the mixed-integer nature of the problem combined with a large number of generating units and time intervals throughout the optimisation horizon that makes problem complexity and computation time drastically increase. Several attempts to tighten the constraints and limit the number of binary variables have therefore been made, e.g. see Carrión and Arroyo (2006); Ostrowski et al. (2012); Morales-Espana et al. (2013). Furthermore, considerable attention has been devoted to specifically designed solution methods, most of them based on a decoupling of units by Lagrangian relaxation, see Takriti et al. (1996); Nowak and Römisch (2000). The inclusion of uncertainty makes computational intractability even worse. In spite of this, to fully account for variations in renewable production, the need for models with higher time resolutions is indisputable.

Our model attempts to overcome this need by optimising balancing decisions with a time resolution of a few minutes. To make the problem computationally feasible, we decouple hourly and intra-hourly scheduling and carry out the optimisation in a sequential fashion (which provides an upper bound on the costs of joint optimisation). Since UC is often made ahead of operation, this decoupling allows us to formulate the intra-hour optimisation as an ED problem and thereby it facilitates the use of a higher time resolution. By considering a short time horizon of only one or two hours, predictability increases and we are able to formulate a deterministic problem.

Related models for intra-hour management include Lindgren and Söder (2008), who present a mixed-integer multi-area optimisation model based on the Northern European system. This paper simulates wind forecasts over time and models the reserve market with re-optimisation when new forecasts become available, taking frequency controls and transmission into consideration, but ignoring operational constraints such as ramping. Reserve activation is done via predetermined bid lists. Other models replace such exogenously given bid lists by endogenous activation of reserve power, see Jaehnert and Doorman (2012); Farahmand and Doorman (2012). As in our model, these models are based on sequential optimisation of day-ahead and balancing operations. Focus is likewise on cost savings in an integrated Northern European power market, but with emphasis mainly on hydro-power resources. Furthermore, the authors do not account for ramping restrictions and market rules for the activation of reserves. Finally, Ela and O'Malley (2012) address the sequence of day-ahead planning, intra-hour balancing, and second-to-second automatic generation control. The idea is to investigate how variability and unpredictability of wind power affect system costs and reliability. Although this model includes technical constraints such as ramp rate restrictions, it likewise does not account for the reserve activation rules of the market.

As the intra-hour models mentioned above, our ED model has a sufficiently high time resolution to represent the variability of wind, while it in addition considers the complex market rules and restrictions on ramping and activation of reserve power. By receiving output from a UC model, it optimises the operation of the system with a two-hour time horizon based on adjustments to the day-ahead schedule via activation of reserves. The model is run in a rolling planning fashion to accommodate hourly updates of wind forecasts.

The rest of the paper is divided into sections as follows: The interface between hour-by-hour and intra-hour scheduling is outlined in Section 2.3. The generation of wind power forecasts can be found in Section 2.4, and our model for optimisation of intra-hour balancing is presented in Section 2.5. Data and assumptions of the case study are provided in Section 2.6, and the results are discussed in Section 2.7. Finally, a conclusion is provided in Section 3.6.

2.3 From hour-by-hour to intra-hour scheduling

We aim at short-term balancing of supply and demand through forecasting of wind power production and scheduling of conventional generation. In doing so, we decouple hourly and intra-hourly scheduling. This decoupling allows us to make an hour-by-hour UC plan, reflecting day-ahead market clearing, and on the basis of this plan, make an ED plan with an intra-hour resolution. In doing so, we assume that the hour-by-hour production schedule is made by an existing model using a wind power forecast, and that our intra-hour model carries out a rescheduling of the generating units with an updated forecast.

The development of an intra-hour model requires detailed information. We assume power producers provide hourly generation schedules while the SO updates consumption and wind power production forecasts close to real-time operation. To reflect this, we generate input to our model on the basis of an hour-by-hour UC model. The time horizon may be different depending on the market design, but here we refer to the hour-by-hour UC as day-ahead planning. The link between this model and our intra-hour model, OPTIBA, is illustrated by the framework in Fig. 2.1.

The first module, UC, collects hourly values of predicted consumption, forecasted wind power production, planned production schedules for conventional units, and allocated flow on transmission lines from the UC model. OPTIBA reads this data into three modules that convert it to τ-minute time resolution[2].

The module HA_cons (Hour Ahead consumption) converts the predicted hourly consumption level to an hour-ahead prediction with τ-minute time resolution.

[2]Depending on the frequency with which activation of manual reserves is permitted by the market rules or technical restrictions of the units, and taking into account the running time of the model, τ can for example be taken to be five or ten minutes.

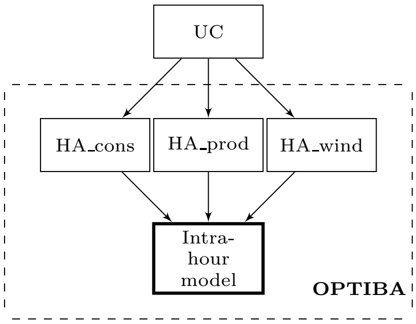


Figure 2.1: Our modelling framework, including the link between the hour-by-hour UC model (run a day ahead of operation) and our intra-hour model (run an hour ahead of operation)

For simplicity, and assuming demand is a smooth function of time, we do this using a third order spline interpolation between the hourly values.

HA_prod (Hour Ahead conventional production) likewise converts the schedule for hourly production, and transmission between internal areas as well as to and from external areas (import/export) into τ-minute data. For the conventional power generating units, we assume that ramping is scheduled in the first and last $\tau^{\max}$ time intervals of an hour, where $2 \cdot \tau^{\max}$ is the number of time intervals per hour a unit is allowed to ramp according to the technical restrictions of the generating unit or the power system. The same ramping patterns are used for the transmission lines to both internal and external areas.

HA_wind (Hour Ahead wind power production) collects the hourly values of wind power production and updates the forecast in τ-minute intervals. A more detailed description can be found in Section 2.4.

The information generated by the modules, HA_cons, HA_prod, and HA_wind, is used as input to the intra-hour model. The model then re-schedules the generating units that are online according to the hourly UC model in order to cover imbalances between supply and demand, which is what we refer to as the activation of manual reserves.

2.4 Wind power forecasts

For scheduling in power systems with high wind penetration, an accurate wind power forecast is crucial. The aim of this section is therefore to generate a representative wind power production forecast that will serve as input to the intra-hour model presented in Section 2.5.

For both forecasting and scheduling, we discretise the time horizon into τ-minute intervals, and by $\mathcal{T} = \{1, \ldots, T\}$ we denote the set of these intervals. In the case of hours, we use the notation $\mathcal{H} = \{1, \ldots, H\}$. Finally, we denote the τ-minute intervals within hour h by $\mathcal{T}_h$, where $h \in \mathcal{H}$.

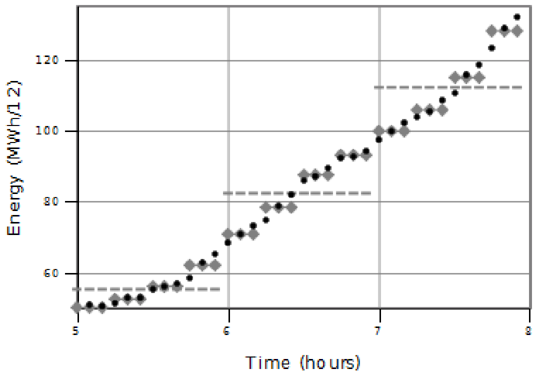


Figure 2.2: Measured energy from wind power production in Denmark from 5 a.m. to 8 a.m., January 21st 2012. The figure shows the measured 5-minute values (dots), as well as aggregated 15-minute values (diamonds) and hourly values (dashes)

We assume that an hour-by-hour forecast $\{\bar{w}_h\}_{h\in\mathcal{H}}$ is already known, but we wish to make a forecast with higher time resolution. As seen from the historical data in Fig. 2.2, variations within the hour are significant and the 5-minute wind power production deviates substantially from values obtained with a lower time resolution. Thus, we aim to generate a τ-minute forecast $\{w_t\}_{t\in\mathcal{T}}$. Without further intra-hour information, one could use linear interpolation between the hourly values $\bar{w}_h$ and $\bar{w}_{h+1}$ to determine w_t for $t \in \mathcal{T}_h$. Here, we use a slightly different approach.

The idea is to approximate the linear interpolation forecast by a simulation mean. The advantage of the simulation approach is its ability to describe the entire distribution of wind power production. The simulation mean may not fully agree with the values obtained by linear interpolation. In reality, however, the SO receives more detailed information, including an updated wind power forecast, close to real-time operation. As a result, deviations occur between the hour-by-hour and intra-hour forecasts. We assume that the simulation mean represents an updated forecast, and interpret the deviations between the simulation and interpolation as forecast errors.

The simulation approach is based on the use of copulas to describe the temporal dependence structure of the stochastic process, $\{W_t\}_{t\in\mathcal{T}}$, of wind power production. Whereas a low-dimensional joint distribution may be explicitly specified by a parametric model, joint parametric modelling becomes cumbersome in higher dimensions. The use of copulas allows us to independently model the marginal distributions and the multivariate dependence structure. We assume marginal Beta distributions to capture the lower fat tail of wind power production, and choose the Gaussian copula, which offers a simple sample procedure to capture the dependence structure. To the best of our knowledge, the modelling of non-Normal marginal distributions and a temporal correlation structure for wind power production has not previously been suggested in the literature.

2.4.1 Marginal Beta distributions

Whereas many forecasting studies are based on the assumption of a (symmetric) Normal distribution, we assume marginal Beta distributions to appropriately reflect that high wind speeds are less frequent than low wind speeds. Further justifications of the choice of Beta distribution can be found in Louie (2010).

For wind power production at a given point in time, we let $W_t \sim \text{Be}(\alpha_t, \beta_t)$, where $\text{Be}(\alpha_t, \beta_t)$ is the Beta distribution with parameters $\alpha_t > 0, \beta_t > 0$. The distribution function of W_t is then

$$F_t(w; \alpha_t, \beta_t) = \frac{\Gamma(\alpha_t + \beta_t)}{\Gamma(\alpha_t)\Gamma(\beta_t)} \int_0^w v^{\alpha_t - 1}(1-v)^{\beta_t - 1} dv, \;\; 0 \leq w \leq 1,$$

where $\Gamma(\cdot)$ is the gamma function. In the following, we denote this function simply by $F_t(\cdot)$. For parameter estimates, see the supplementary material in Appendix 2.A.

2.4.2 Gaussian copula

Although wind power production at a given point in time can be described by a Beta distribution, we have disregarded the temporal correlation structure in the specification of the marginal distributions. The Gaussian copula allows us to construct a joint distribution that captures this dependence.

Let Σ be a $T \times T$-correlation matrix. For how to estimate this, see the supplementary material in Appendix 2.A. The Gaussian copula is then

$$F(w_1, \ldots, w_T; \Sigma) = \Phi_T(\Phi^{-1}(F_1(w_1)), \ldots, \Phi^{-1}(F_T(w_T)); \Sigma),$$

where the term $\Phi^{-1}(\cdot)$ is the inverse distribution function of a standard Normal distribution and the term $\Phi_T(\cdot; \Sigma)$ is the distribution function of a T-variate Normal distribution with correlation matrix Σ.

Now, let $(U_1, \ldots, U_T)^\top \sim \mathcal{N}_T(0, \Sigma)$, where $\mathcal{N}_T(0, \Sigma)$ is the T-variate Normal distribution, and for $t = 1, \ldots, T$, let

$$W_t = F_t^{-1}(\Phi(U_t)),$$

where $F_t^{-1}(\cdot)$ is the inverse of the marginal distribution function $F_t(\cdot)$. We then obtain that $W_t \sim \text{Be}(\alpha_t, \beta_t)$ with marginal distribution function $F_t(\cdot)$ and that $(W_1, \ldots, W_T)^\top$ have joint distribution function $F(\cdot; \Sigma)$.

2.4.3 Simulation

We can use the above observations for sampling. If we generate a number of sample paths $(u_1^s, \ldots, u_T^s)$, $s = 1, \ldots, S$ from the T-variate Normal distribution

$\mathcal{N}_T(0, \Sigma)$, and for each such sample path compute

$$w_t^s := F_t^{-1}(\Phi(u_t^s)),$$

then $(w_1^s, \ldots, w_T^s)^\top, s = 1, \ldots, S$ can be viewed as samples from a joint distribution with marginal Beta distributions $\text{Be}(\alpha_t, \beta_t)$ and temporal correlation matrix Σ.

To generate samples from the T-variate Normal distribution $\mathcal{N}_T(0, \Sigma)$, we make the following observations. Let $(V_1, \ldots, V_T)^\top \sim \mathcal{N}_T(0, I)$, where I denotes the $T \times T$-identity matrix. The marginals V_t are independent and all have a standard Normal distribution. Apply Cholesky decomposition of the correlation matrix such that

$$\Sigma = LL^\top,$$

where L is a lower triangular $T \times T$-matrix and $L^\top$ its transpose. We now have that $(U_1, \ldots, U_T)^\top := L(V_1, \ldots, V_T)^\top \sim \mathcal{N}_T(0, \Sigma)$. Hence, if we generate independently a number of samples $v_t^s, s = 1, \ldots, S$ from the standard Normal distribution, then

$$(u_1^s, \ldots, u_T^s)^\top := L(v_1^s, \ldots, v_T^s)^\top, s = 1, \ldots, S$$

can be viewed as samples from the T-variate Normal distribution with correlation matrix Σ.

The generated sample paths $(w_1^s, \ldots, w_T^s)^\top, s = 1, \ldots, S$ describe the distribution of wind power production and allow us to investigate the sensitivity of the model to uncertainty. Since our model is deterministic, however, we also compute the simulation mean

$$w_t = \frac{1}{S} \sum_{s=1}^{S} w_t^s, \;\; t \in \mathcal{T},$$

and use $\{w_t\}_{t \in \mathcal{T}}$ as the wind power forecast.

2.4.4 Forecasting results

In the above, we work on wind power production normalised by capacity. In our case study, we therefore multiply the forecast with a given wind power capacity for each geographical area. Fig. 2.3 illustrates 15 of 5000 sample paths used for computing the simulation mean. The data is based on expected wind penetration in Western Denmark in 2020, and the generated forecast has a five minute time resolution. Clearly, wind power production shows large variations over time. Moreover, Fig. 2.4 displays the simulation mean computed on the basis of 500 and 5000 samples. It is likewise clear that our updated forecast shows significant intra-hour variations around the linear interpolation between the hourly values, as would be the case in reality.

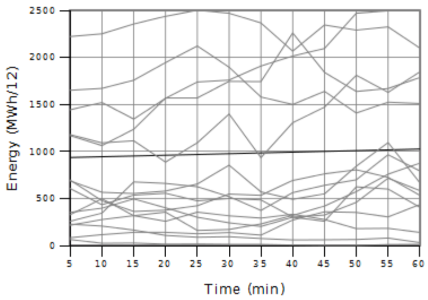


Figure 2.3: 15 wind power production sample paths (grey) and their mean (black)

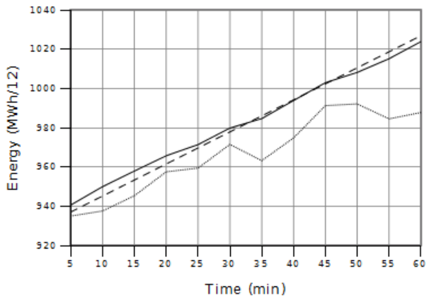


Figure 2.4: Three wind power production forecasts: The intra-hour wind power production forecasts based on linear interpolation between hourly values (dashed) and simulation means of 500 (dotted) and 5000 (black) samples, respectively

2.5 The intra-hour model

The aim of the intra-hour model is efficient short-term balancing of supply and demand in the electricity system. We consider balancing close to real-time operation when a production plan has already been made by day-ahead market clearing. On the basis of this plan, our model re-dispatches the generating units, which is what we refer to as intra-hour scheduling of manual reserves. In doing so, we assume that the UC schedule has been fixed and consider only the re-dispatch of already committed units[3]. As already mentioned, short-term balancing is the responsibility of the SO, and therefore we base our model on social welfare maximisation, or equivalently, assuming inflexible demand, minimisation of balancing costs.

We take the scheduling horizon to be a few hours, which we discretise into τ-minute intervals. For the notation, see Section 2.4. To formulate the intra-hour model, we further introduce the following notation.

The transmission grid is modelled as a network $(\mathcal{N}, \mathcal{A})$ with nodes $\mathcal{N}$ and arcs $\mathcal{A} = \{a : a = (n, n'), n, n' \in \mathcal{N}, n < n'\}$ representing transmission lines. We denote by $\delta^{out}(n) = \{a : a = (n, n'), n' \in \mathcal{N}\}$ and $\delta^{in}(n) = \{a : a = (n', n), n' \in \mathcal{N}\}$ the sets of arcs originating from and terminating in node $n \in \mathcal{N}$, respectively.

[3]This assumption can be justified for power generation units with start-up times in excess of a few hours. However, some power generation units may have start-up times less than an hour, in which case this is a simplifying assumption.

For $a \in \mathcal{A}$, we let the capacity of transmission line a be $L_a^{\max}$ and the maximum ramping rate be R_a. Moreover, we let the flow allocated day-ahead to line a in time interval t be L_{at}. There is a net import from n to n', or equivalently a net export from n' to n, if $a = (n, n')$ and $L_{at} > 0$. We represent the intra-hour scheduled import on the transmission line in the same time interval by the variable Δl_{at} using the same conventions regarding its sign.

The set of conventional units is denoted by $\mathcal{I}$, the set of units online in the time interval t is denoted by $\mathcal{I}_t$, and the units located in node n by $\mathcal{I}_n$. For $i \in \mathcal{I}$, we let the minimum and maximum generation level of unit i be $P_i^{\min}$ and $P_i^{\max}$, respectively, and the maximum rates for ramping up and down be R_i^+ and R_i^-. We denote by C_i the variable generation cost of unit $i \in \mathcal{I}$. Note that whereas activation of reserves generates an additional cost, deactivation of reserves results in cost savings. We therefore let $C_i^+ := (1+\gamma)C_i$ and $C_i^- := (1-\gamma)C_i$ be the cost of activating manual reserves and savings of deactivating manual reserves, respectively, where $\gamma \in [0,1]$ is a mark-up or a mark-down. The idea is to allow the costs and savings to reflect the additional stress imposed on the unit when using it for balancing purposes. We assume that the mark-up and mark-down are the same, although this may not always be the case in the market. The costs and savings of activating or deactivating automatic reserves in node n are denoted by C_n^+ and C_n^-, where most likely, $C_n^+ > \max_i C_i^+$ and $C_n^- < \min_i C_i^-$.

We now let day-ahead planned generation on unit i in time interval t be given by the parameter P_{it}. The variables $\Delta p_{it}^+ \geq 0$ and $\Delta p_{it}^- \geq 0$ represent the intra-hour activation and deactivation of manual reserves on the unit, respectively. Likewise, the variables $q_{nt}^+ \geq 0$ and $q_{nt}^- \geq 0$ represent the generation shortage and surplus in node n during time interval t. To keep track of whether reserves are activated or deactivated, we introduce two binary variables, z_{it}^+ and z_{it}^-. Hence, $z_{it}^+ = 1$ if we have activated reserves in time interval t and zero otherwise. Likewise, $z_{it}^- = 1$ if we have deactivated reserves and zero otherwise.

To model detailed ramping restrictions, we let the binary variables u_{it}^+ and u_{it}^- be one if unit i is ramping in the direction of or in the opposite direction of target generation level, respectively, in time interval t and zero otherwise. The binary variable v_{it} is set to one when the ramping in the direction of target generation level has finished. Activated and deactivated reserves at time t are represented by the variable $g_{it} \geq 0$, and the minimum level of reserves that can be activated at a given time is given by the parameter $G_i^{\min}$. This modelling resembles the bidding practice of the balancing market, but allows for optimising the use of reserves as opposed to using bid lists.

Finally, we assume that wind power production and demand is inflexible, and denote their values by the parameters w_{nt} and D_{nt} in node n and time interval t.

In the following section, we present the model. For an illustrative example of it, see also Section 2.5.2.

2.5.1 Economic dispatch

We schedule the activation of manual reserves to cover any imbalance between supply and demand. Manual reserve capacity is offered to the SO by producers submitting bids. In the presence of market power, producers may hold back capacity to influence market prices. The model can easily account for this by adjusting the reserve capacity accordingly. However, in taking a social welfare perspective, we assume that idle capacity is always offered to the SO. Occasionally, however, reserve activation may be technically infeasible or it may be feasible only at very high cost, in which case imbalances are left to automatic reserves. The optimal schedule is therefore determined by a trade-off between the activation costs of manual and automatic reserves. The objective is

$$\sum_{t\in\mathcal{T}}\Big(\sum_{i\in\mathcal{I}_t}(C_i^+\Delta p_{it}^+ - C_i^-\Delta p_{it}^-) + \sum_{n\in\mathcal{N}}(C_n^+ q_{nt}^+ - C_n^- q_{nt}^-)\Big),$$

which is minimised subject to the following constraints. Although some of these are purely technical restrictions, whereas others appear to be due to contractual agreement, the basis for all of them is the physical limitations of the system.

The balancing constraint ensures system balance between supply and demand. According to this constraint, if, at any point in time, scheduled production exceeds predicted consumption or vice versa, we experience generation surplus or shortage, which is left to automatic reserves. We assume that it is always possible to provide sufficient automatic reserves. Production includes day-ahead planned generation on conventional units, intra-hour activation and deactivation of manual reserves, forecasted wind power production, and finally day-ahead and intra-hour net import/export on the transmission lines. Thus, we have that

$$\begin{aligned}&\sum_{i\in\mathcal{I}_t\cap\mathcal{I}_n}(P_{it}+\Delta p_{it}^+ - \Delta p_{it}^-) + \sum_{a\in\delta^{in}(n)}(L_{at}+\Delta l_{at})\\ &-\sum_{a\in\delta^{out}(n)}(L_{at}+\Delta l_{at}) + q_{nt}^+ - q_{nt}^- = D_{nt} - w_{nt},\ \ n\in\mathcal{N}, t\in\mathcal{T}. \qquad (2.1)\end{aligned}$$

Transmission flow is restricted by the available line capacity, possibly agreed by contract. In particular, intra-hour scheduled import on the transmission lines is bounded above by the line capacity less the capacity allocated day-ahead. Thus, we have that

$$-(L_a^{\max} - L_{at}) \le \Delta l_{at} \le L_a^{\max} - L_{at},\ \ a\in\mathcal{A}, t\in\mathcal{T}.$$

Activation of reserves is bounded above by the capacity that has not already been dispatched day-ahead, whereas deactivation is bounded by the dispatched capacity in excess of the minimum capacity. Formally,

$$\begin{aligned}\Delta p_{it}^+ &\le (P_i^{\max} - P_{it})z_{it}^+,\ \ i\in\mathcal{I}_t, t\in\mathcal{T},\\ \Delta p_{it}^- &\le (P_{it} - P_i^{\min})z_{it}^-,\ \ i\in\mathcal{I}_t, t\in\mathcal{T}.\end{aligned}$$

Finally, to ensure that a unit does not activate and deactivate reserves at the same time, we have that

$$z_{it}^{+} + z_{it}^{-} \leq 1, \;\; i \in \mathcal{I}_t, t \in \mathcal{T}.$$

In the absence of these constraints, positive activated and deactivated reserve levels may occur simultaneously, as their difference produces a reserve level below the minimum activation level.

The above constraints are very similar to those of the hour-by-hour ED problem. In our formulation of the intra-hour balancing problem, however, we include much more detailed ramping restrictions.

Ramping

We assume simple ramping constraints on the transmission lines. These are restrictions on the change in allocated transmission flow from one time interval to another and apply to net import. Thus,

$$\begin{aligned} -R_a - L_{a(t+1)} + L_{at} \leq \Delta l_{a(t+1)} - \Delta l_{at} \leq R_a - L_{a(t+1)} + L_{at}, \\ a \in \mathcal{A}, t \in \mathcal{T} : t \leq T-1. \end{aligned}$$

For the generation units, we include detailed ramping restrictions,

$$\begin{aligned} -(R_i^{-} + P_{i(t+1)} - P_{it})u_{it}^{-} \leq \Delta p_{i(t+1)}^{+} - \Delta p_{it}^{+} \leq (R_i^{+} - P_{i(t+1)} + P_{it})u_{it}^{+}, \\ i \in \mathcal{I}_t, t \in \mathcal{T} : t \leq T-1, \\ -(R_i^{+} - P_{i(t+1)} + P_{it})u_{it}^{-} \leq \Delta p_{i(t+1)}^{-} - \Delta p_{it}^{-} \leq (R_i^{-} + P_{i(t+1)} - P_{it})u_{it}^{+}, \\ i \in \mathcal{I}_t, t \in \mathcal{T} : t \leq T-1. \end{aligned} \tag{2.2}$$

Here we also record if a unit is ramping in the direction of ($u_{it}^{+} = 1$) or in the opposite direction of ($u_{it}^{-} = 1$) target generation level. To prevent the ramping variables from being one when no ramping occurs, we assume a minimum ramp rate. Thus,

$$\begin{aligned} \varepsilon u_{it}^{+} + M(u_{it}^{+} - 1) \leq \Delta p_{i(t+1)}^{+} - \Delta p_{it}^{+} + \Delta p_{i(t+1)}^{-} - \Delta p_{it}^{-}, \\ i \in \mathcal{I}_t, t \in \mathcal{T} : t \leq T-1, \\ \varepsilon u_{it}^{-} + M'(u_{it}^{-} - 1) \leq \Delta p_{it}^{+} - \Delta p_{i(t+1)}^{+} + \Delta p_{it}^{-} - \Delta p_{i(t+1)}^{-}, \\ i \in \mathcal{I}_t, t \in \mathcal{T} : t \leq T-1, \end{aligned}$$

where ε is a sufficiently small number and M and M' are sufficiently large numbers.

Ramping in consecutive time intervals is restricted to a maximum time of $\tau^{\max}$, to reduce the stress imposed on the unit when using it for balancing purposes. This is enforced by the constraints

$$\sum_{t'=t}^{\min\{T,\, t+\tau^{\max}\}} (u_{it'}^{+} + u_{it'}^{-}) \leq \tau^{\max}, \;\; i \in \mathcal{I}_t, t \in \mathcal{T}. \tag{2.3}$$

Target generation level

Upon activation of reserves, a unit must be scheduled to operate at a minimum level for a fixed time interval. However, by consecutively activating or deactivating reserves at the same level, this is equivalent to considering a minimum activation time. Additional levels of reserves may be activated or deactivated on the same unit at a given point in time. This we refer to as new reserve levels[4]. For an illustration of the resulting operation schedule of a unit, see Section 2.5.2.

To ensure a minimum activation level, we have

$$G_i^{\min} v_{it} \leq g_{it} \leq M v_{it}, \;\; i \in \mathcal{I}_t, t \in \mathcal{T}, \tag{2.4}$$

where M is a sufficiently large number, e.g. $P_i^{\max}$. The minimum generation level is activated upon ramping in the direction of target level. Hence,

$$u^+_{i(t-1)} - u^+_{it} \leq v_{it}, \;\; i \in \mathcal{I}_t, t \in \mathcal{T}. \tag{2.5}$$

The ramping in the direction of target generation level, the ramping in the opposite direction of target level, and the activation of a new generation level cannot occur simultaneously, and so

$$u^+_{it} + u^-_{it} + v_{it} \leq 1, \;\; i \in \mathcal{I}_t, t \in \mathcal{T}.$$

Finally, the total amount of reserves provided is the sum of previously activated (respectively de-activated) reserve levels with a fixed time of τ^{res}. Hence,

$$\sum_{t'=\max\{1,t-\tau^{\text{res}}+1\}}^{t} g_{it'} \leq \Delta p^+_{it} + \Delta p^-_{it} \leq \sum_{t'=\max\{1,t-\tau^{\text{res}}+1\}}^{t} g_{it'} + M(u^+_{it} + u^-_{it}), \tag{2.6}$$
$$i \in \mathcal{I}_t, t \in \mathcal{T},$$

for M sufficiently large, e.g. $P_i^{\max}$.

2.5.2 Example of activation of reserves

The following provides an example of how to manage reserves in our model. For simplicity, we confine ourselves to intra-hour scheduling and activation of reserves.

Consider a time horizon of 50 minutes with a resolution of $\tau = 5$ minutes, a fixed activation time of $\tau^{\text{res}} = 6$ time intervals and a maximum ramping time of $\tau^{\max} = 2$ time intervals. Furthermore, let the minimum ramping level be $\varepsilon = 1$ MWh/12, and the minimum reserve activation level be $G_i^{\min} = 2$ MWh/12,

[4]For example, assume that 5 MWh (per τ-minute time interval) is activated at time 12:00 for 20 minutes. At 12:10, we may additionally activate 5 MWh for 20 minutes. Total amount of reserves provided is then 5 MWh at 12:00, 10 MWh at 12:10, and 5 MWh at 12:20.

corresponding to ramping 1 MW and producing at 2 MW during each five minute time interval, respectively.

From the balancing constraint (2.1), we detect the following deficit of energy compared to the day-ahead planned production:

t	1	2	3	4	5	6	7	8	9	10
Energy (MWh/12)	0	1	2	2	2	2	2	2	1	0

This deficit can be covered by a reserve activation of length $\tau^{\text{res}} = 6$ with $\Delta p_{it}^{+} = 2$ MWh/12 for some unit i in the time intervals $t = 3, \ldots, 8$. Assuming that the unit starts to ramp in the direction of this target level at time $t = 1$, and ramp in a maximum of $\tau^{\max} = 2$ intervals, the ramping constraints (2.2) set $u_{i1}^{+} = 1$. At $t = 2$, we further ramp in the direction of target level, setting also $u_{i2}^{+} = 1$. Since we can only ramp for two consecutive time intervals, $u_{i3}^{+} = 0$. With the minimum ramping of $\varepsilon = 1$ MWh/12, the unit also covers the deficit in the ramping period $t = 2$. As the unit reaches the desired level of production, ramping is finished and constraints (2.5) ensure that $v_{i3} = 1$. This means that a new activation of reserves has taken place. Constraints (2.4) now force the reserve level to be at least at the minimum level of $G^{\min} = 2$ MWh/12, and so $g_{i3} = 2$. The reserve activation variable Δp_{it}^{+} is maintained at 2 MWh/12 by (2.6) during the six time intervals; see the dark grey reserve activation in Fig. 2.5. In case no additional reserves are activated, after six time intervals, constraints (2.6) force the unit to ramp in the opposite direction of target towards the day-ahead level, which again forces $u_{i8}^{-} = u_{i9}^{-} = 1$ by (2.2). Since we can only ramp for two consecutive intervals, $u_{i10}^{-} = 0$ by (2.3). As before, the unit covers the deficit in the ramping period $t = 9$. Finally, constraints (2.6) ensure that we are back at the day-ahead planned production level at time $t = 10$.

In the case of further need for reserves, e.g. in time interval $t = 6$, we activate new reserves on top of the initial activation. Now, let the time horizon be 65 minutes and the total deficit compared to the day-ahead planned production be:

t	1	2	3	4	5	6	7	8	9	10	11	12
Energy (MWh/12)	0	1	2	2	3.5	5	5	5	4	2	2	1.5

The unit starts ramping in the direction of target level in time interval $t = 4$, forcing $u_{i4}^{+} = 1$ by (2.2) and continues to ramp in time interval $t = 5$, so that also $u_{i5}^{+} = 1$. Note that we could not have started ramping in the direction of target level at $t = 3$ due to the maximum ramping restrictions (2.3). This new activation is shown as the light grey area of Fig. 2.5. Again by (2.6) we ramp back from the initial reserve activation in time intervals $t = 8$ and $t = 9$, however, preserving our latest and still running reserve activation. This continues to run at the level $g_{i6} = 3$ MWh/12 for the rest of the activation period.

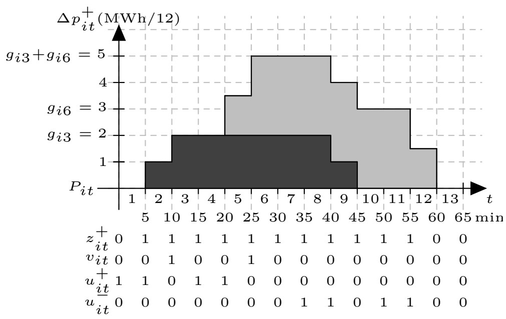


Figure 2.5: Two reserve activations on the same unit and the corresponding values of the binary variables

Should there be need for a longer reserve activation period, by (2.6) it is possible to continue at the same reserve level directly after the previous activation has finished, and without ramping in the opposite direction, by letting $g_{i9} = g_{i3}$.

2.6 Case study

In our case studies, we consider a time horizon of two hours with a five minute time resolution, corresponding to $\tau = 5$ and $T = 24$.

2.6.1 Rolling planning

We generate an hour-by-hour production plan by running the UC model from Weber et al. (2009) called WILMAR with data from the Danish electricity system of 2010[5]. When the UC model has run for a day, the intra-hour model is launched. Initially, it updates the wind forecast and converts the hourly output from the UC model (production, consumption, etc.) to intra-hour values, see Section 2.3. To reflect the frequent updating of wind forecasts, we run the model in a rolling planning fashion. Hence, the intra-hour model reschedules the generation units for the first two hours of the day, discards the second hour, and is rerun for the second and third hour of the day, etc. By using a two-hour time horizon, we avoid undesired end effects in the intra-hour optimisation such as the tendency of production units to decrease the generation level towards the end of the hour. The end values from the hour previous to the two-hour horizon serve as input to the optimisation in order to accommodate reserve activation across the shift between hours. The model is run for each hour of the day, with the final run corresponding to the time period from 10 p.m. until midnight. In this run, both the first and the second hour of the optimisation horizon are used. Finally, the UC model is run for another day. The process is repeated for each day of a week.

[5]System data has kindly been provided by Energinet.dk, which is the Danish transmission system operator (TSO).

2.6.2 Cases

We consider a winter week in January 2010 as our base case, and compare this to a week in May, July, and October of 2010 corresponding to spring, summer, and fall cases. To complement the 2010 cases, we increase the capacity of wind power in the data to fit the Danish 2020 goal according to guidelines from the Danish TSO. This results in four cases corresponding to the same weeks but with a significantly higher wind penetration. For our case studies, this represents an increase in the wind power penetration from an average of 22.4% in the current wind penetration cases to 30.4% in the high wind penetration cases.

In all eight cases, the ramping on transmission lines to areas outside the two balancing areas is handled as for the production units, see Section 2.3. We consider $\tau^{\max} = 3$ time intervals, resulting in a ramping pattern where the transmission lines are ramping from one hourly level to the next within the last 15 minutes of the previous hour and the first 15 minutes of the current hour. However, according to the Danish TSO, this is only the case for transmission lines to the Nordic countries. Ramping on transmission lines to Central Europe is done within the last five minutes of the previous hour and the first five minutes of the current. We investigate how this inconsistency, that is, different ramping patterns, leads to additional system imbalances in the case of Winter 2010.

2.6.3 Model complexity and running times

The model is run with an Intel Core 3.10GHz processor and 4 GB RAM. The model is implemented in the 64 bit GAMS framework version 24.1.3 for Windows using the CPLEX 12.5.3.0 solver.

For each case, the rolling planning setup of OPTIBA with a total time horizon of a week leads to seven runs of the UC model and a total of 168 runs of the intra-hour model. For the eight 2010 and 2020 cases, running times vary between 11 hours and 31 minutes (Winter 2010) and 22 hours and 31 minutes (Summer 2010), whereas the Winter 2010 case with different ramping patterns is somewhat slower. Considering the relatively long running times for a whole week, recall that the purpose of our model runs is analysis of the system rather than actual rescheduling. The average running time for the two hour horizon is less than six minutes which would easily be applicable for an SO wishing to reschedule proactively.

Out of the 168 model runs for each of the eight current and high wind penetration cases, at least 80.9% solve with a gap less than 2%, 89.9% solve with a gap less than 5%, and 95.2% with a gap less than 10%. For the Winter 2010 case with different ramping patterns, the gaps are slightly larger.

2.6.4 System characteristics

The system consists of two balancing areas with a total of 118 power producing units. For each of the two balancing areas the wind farms are grouped into offshore and onshore wind power producing units. Aggregated characteristics of hourly power system operations from the modules HA_cons, HA_prod, and HA_wind are displayed in Table 2.1. As expected, the higher the wind power share is, the lower the conventional production and import are, and the higher the export is.

Table 2.1: The table shows aggregated characteristics of hourly power system operations for a selected week in each season with current and high wind power share. The aggregated characteristics consist of the means (standard deviations) of demand, conventional production, conventional capacity, wind power production, wind power capacity, import, and export. Capacities are in MW and all other values in MWh/12 corresponding to the five minute intervals. Note that the wind capacity does not vary over time as we assume turbines are never shut down

	All areas – 2010 cases – current wind penetration			
	Winter	Spring	Summer	Fall
Demand (MWh/12)	4183(884)	3554(667)	3463(631)	3876(778)
Conv. prod. (MWh/12)	3110(483)	2857(301)	2670(204)	3412(493)
Online capacity (MW)	4397(492)	4015(361)	4053(255)	4605(499)
Wind power prod. (MWh/12)	1880(878)	251(242)	657(545)	586(591)
Installed wind capacity (MW)	4752	4752	4752	4752
Import (MWh/12)	1438(493)	2222(610)	2187(544)	1974(625)
Export (MWh/12)	2227(475)	1778(534)	2054 (88)	2098(330)
	All areas – 2020 cases – high wind penetration			
	Winter	Spring	Summer	Fall
Demand (MWh/12)	4183 (884)	3554(667)	3463(631)	3876(778)
Conv. prod. (MWh/12)	2945 (384)	2836(259)	2687(181)	3379(477)
Online capacity (MW)	4197 (396)	3966(311)	3949(214)	4578(481)
Wind power prod. (MWh/12)	2492(1111)	364(347)	920(747)	812(800)
Installed wind capacity (MW)	6752	6752	6752	6752
Import (MWh/12)	1245 (571)	2164(596)	1943(657)	1881(632)
Export (MWh/12)	2499 (621)	1811(524)	2076(157)	2195(407)

2.6.5 Intra-hour balancing characteristics

Current guidelines from the Danish TSO, based on physical restrictions on the generating units, lead to the following assumptions. Each reserve activation or deactivation lasts $\tau^{\text{res}} = 6$ time intervals. Longer activation times are handled in the model as a multiple of individual reserve activations. The minimum activation or deactivation level is $G_i^{\min} = 10$ MWh/12 corresponding to a production level of 10 MW in the activation or deactivation period. The maximum ramping time is $\tau^{\max} = 3$ time intervals, and the minimum ramping limit is $\varepsilon = 1$ MW.

Regarding the costs of reserves, we assume that the costs of manual activation or deactivation of reserves are the marginal cost of the unit multiplied by a mark-up or mark-down, respectively. Furthermore, we assume system-wide costs of the automatic reserves, which we estimate from the historical price of manual reserves. In particular, we set the cost of activating and deactivating automatic reserves to the 95% and the 5% quantile of the historical distribution of reserve prices in the Danish system, respectively. The costs can be found in Table 2.2.

Table 2.2: The costs of reserves. The costs of manual reserves depend on the marginal cost of the units and a mark-up or mark-down for activation and deactivation, respectively. The costs of automatic reserves are system-wide costs

	Marginal cost	Mark-down	Mark-up	Down	Up
Manual reserves (€)	3.8-216.2	0.9	1.1		
Automatic reserves (€)				20	95

In future power systems with even higher wind power penetrations, the SOs may have to install further reserves to maintain system reliability. This would be at an additional cost, which is not present in our model. However, the cost could be included as a higher cost on automatic reserves. We have investigated the influence of a change in the costs of automatic reserves in the supplementary material, see Appendix 2.B.

2.7 Results and discussion

In this section, we present numerical results from our intra-hour model and thereby aim to justify the need for such models.

2.7.1 Cost savings from using proactive reserve activation

We report the costs of proactive manual reserve activation as given by our model results. To compare, we likewise report the costs of leaving imbalances solely to automatic reserves, referred to as reactive reserve activation. The difference between these costs is the savings from proactive reserve activation. From Table 2.3, it is evident that hourly scheduling with reactive re-dispatch leads to extensive use of the expensive automatic reserves. By proactively activating and deactivating reserves, these reserve costs can be reduced. For the 2010 cases, savings vary between $2.1 - 4.9‰$ of total system costs for different seasons. In spite of the small share of total costs, the absolute savings of €$391,601 - 628,070$ are not insignificant. For the 2020 cases, the savings are slightly larger, varying between $2.4 - 9.3‰$ of total system costs corresponding to €$408,758 - 738,137$. These savings will increase if the costs of automatic reserves rise in the future.

When comparing the 2010 cases with the 2020 cases, it is clear that higher wind penetration results in lower total system costs for both the proactive and the reactive strategy. Thus, the benefits from the inexpensive wind power are

Table 2.3: Weekly costs of operating the system proactively and reactively. For each strategy, costs are divided into manual activation and deactivation of reserves and automatic activation and deactivation of reserves. Moreover, the table shows total balancing costs and total system costs including day-ahead costs. Finally, the savings from being proactive as opposed to reactive are depicted. All numbers listed are in €

		All areas – 2010 cases – current wind penetration			
Strategy		Winter	Spring	Summer	Fall
Proactive	Man. act.	414 128	283 371	279 582	396 637
	Man. deact.	−427 554	−224 730	−206 994	−309 489
	Auto. act.	5066	3280	4298	5342
	Auto. deact.	−6201	−5803	−7067	−6303
	Total	−14 560	56 118	69 820	86 188
	Total system	126 668 489	186 520 657	143 835 549	180 548 204
Reactive	Man. act.	0	0	0	0
	Man. deact.	0	0	0	0
	Auto. act.	847 870	581 497	574 898	812 584
	Auto. deact.	−234 360	−116 952	−113 478	−161 261
	Total	613 510	464 545	461 421	651 323
	Total system	127 296 558	186 929 084	144 227 150	181 113 339
Savings		628 070	408 427	391 601	565 135
		All areas – 2020 cases – high wind penetration			
Strategy		Winter	Spring	Summer	Fall
Proactive	Man. act.	616 352	299 673	270 315	395 581
	Man. deact.	−337 719	−246 961	−289 132	−331 126
	Auto. act.	14 159	4093	4380	6669
	Auto. deact.	−15 128	−6598	−6536	−6666
	Total	123 947	50 207	−20 972	64 458
	Total system	78 989 767	175 514 299	124 950 092	161 777 073
Reactive	Man. act.	0	0	0	0
	Man. deact.	0	0	0	0
	Auto. act.	1 090 133	599 378	540 179	806 828
	Auto. deact.	−228 049	−125 144	−152 392	−173 694
	Total	862 084	474 234	387 786	633 133
	Total system	79 727 904	175 938 326	125 358 850	162 345 747
Savings		738 137	424 027	408 758	568 675

not outweighed by the balancing costs. Furthermore, the savings from using the proactive approach are larger for the 2020 case, and hence, future wind penetrations of even greater magnitude will only make the intra-hour model increasingly relevant.

2.7.2 Imbalances, updating of wind power forecast and ramping

Table 2.4 shows that total balancing costs depend on total imbalances in the system. More specifically, total costs of activation and deactivation of reserves can in most cases be explained by the corresponding total deficit and surplus in the system. In all cases, except Fall 2020, deficit of power incurs positive total reserve costs, whereas surplus of power incurs negative overall reserve costs because of the need for activation and deactivation of reserves, respectively. The standard deviations of the imbalances in Table 2.4 indicate that there are more frequent and/or larger imbalances in some cases than in others. This may increase the costs of balancing the system since activation of automatic reserves is more expensive than deactivation. Thus, the large standard deviation of imbalances in the Fall 2020 case leads to positive total balancing costs, even though there is a total excess of power before the intra-hour model is run. The standard deviation of imbalances can likewise explain some of the differences in total balancing costs for the proactive approach in Table 2.3 between cases with average imbalance of the same sign. For instance, for the 2010 cases, the standard deviation is smaller in the spring, followed by summer and fall, which corresponds to the balancing costs being smaller for the spring case, followed by summer and fall.

Table 2.4: Weekly averages (standard deviation) of total imbalances and wind forecast errors in the system before re-dispatch in each five minute interval. Moreover, the table shows the average positive and negative imbalances and the average positive and negative forecast errors. The values are in MWh/12 corresponding to the five minute intervals. When the forecast error is positive, the updated forecast is higher than the original forecast

	All areas – 2010 cases – current wind penetration			
	Winter	Spring	Summer	Fall
Deficit	-52.6	-36.4	-36	-50.8
Surplus	69.2	34.8	33.8	47.9
Imbal.	16.6(189.6)	-1.6(104.5)	-2.3(110.9)	-2.9(142.3)
Neg. error	-1.7	-5.5	-19.6	-9.2
Pos. error	8	2.4	3.8	6.1
Wind error	6.3 (9.3)	-3.1 (9.1)	-6.8 (16.7)	-3.1 (17.9)
	All areas – 2020 cases – high wind penetration			
	Winter	Spring	Summer	Fall
Deficit	-67.9	-37.6	-33.8	-50.5
Surplus	67.5	37.2	45.4	51.6
Imbal.	-0.4(217.3)	-0.3(107.8)	11.5(118.1)	1.1(153.2)
Neg. error	-18.1	-4.7	-8.8	-7.2
Pos. error	5.7	3.2	15.7	8.9
Wind error	-12.4 (28.4)	-1.6 (10)	6.9 (31)	1.7 (21.8)

Table 2.4 also shows wind forecast errors. In all cases, the total system imbalance is negative (positive) when the total wind forecast error is negative (positive) as expected. In fact, the average wind forecast errors contribute substantially to total imbalances, cause activation and deactivation of reserves, and thereby account for a significant share of balancing costs.

2.7.3 Imbalances and ramping

The updating of wind power forecasts, the ramping, and the higher time resolution of demand data all affect the imbalances in the system. The imbalances are reduced by proactive deployment of reserves. These are, however, not entirely eliminated, which is evident from Fig. 2.6. This is due to the ramping restrictions on production units that impose bounds on the use of reserves, making it impossible to precisely meet demand. It would be very expensive to meet imbalances at the spikes, as this would require many of the slower units to ramp up and thereby incur large imbalances in the time intervals before and after the spikes. Hence, this would never be optimal from a cost minimisation point of view.

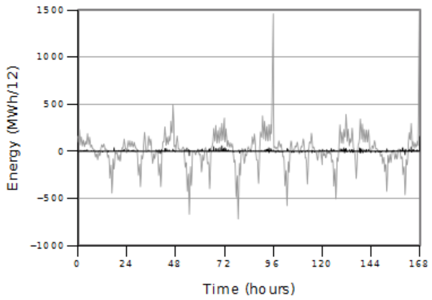


Figure 2.6: The total imbalance, i.e. the sum of imbalances in the two areas, before (grey) and after (black) activation of manual reserves for the system in the Winter 2010 case.

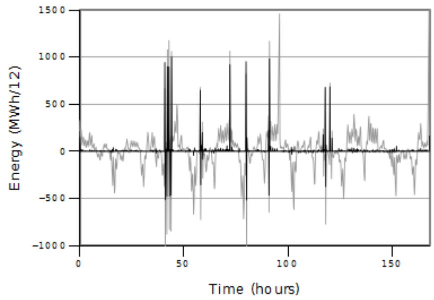


Figure 2.7: The total imbalance before (grey) and after (black) activation of manual reserves for the system in the Winter 2010 case with different ramping patterns for transmission to the Nordic Countries and transmission to Central Europe.

For the Winter 2010 case with different ramping patterns on transmission lines, the imbalances are larger than for the Winter 2010 base case. The imbalances are depicted in Fig. 2.7 which shows how the imbalances exhibit additional spikes that will not be captured in hourly UC models. These substantially larger imbalances increase balancing costs by €57,909 corresponding to 0.5‰ of the total system costs. As in the base case, the imbalances are reduced by proactive deployment of reserves. However, the imbalances are not entirely eliminated, and a higher frequency of spikes remain after re-dispatch than in the base case.

2.7.4 Reserve activations

A closer look at the Winter 2010 base case shows additional results from the intra-hour model. First, Fig. 2.8 shows a given power generating unit for a two-hour time horizon. It is seen how reserve power is activated in several overlapping blocks of the same length, i.e. $\tau^{res} = 6$ time intervals. Moreover, the figure shows how the unit ramps. This unit can ramp 70 MW from one five minute interval to the next and is seen to ramp to the limit several times. Second, deactivation of reserves is likewise utilised, as seen in Fig. 2.9. The figure shows the re-scheduling of another power generating unit for a two-hour time horizon. Here, reserve power is deactivated in overlapping blocks. The ramp rate of the unit is 125.2

MW. These two examples confirm that the ability to ramp within the hour is extensively used, and is therefore highly important in short-term scheduling.

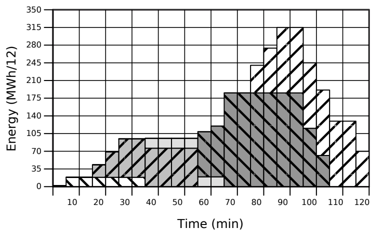


Figure 2.8: Two-hour schedule for activation of reserves on a selected unit, showing five separate reserve activations. Each column represents the amount of energy activated in a five minute interval, e.g. 70 MWh/12, corresponding to adding 70 MW to the production level in that interval.

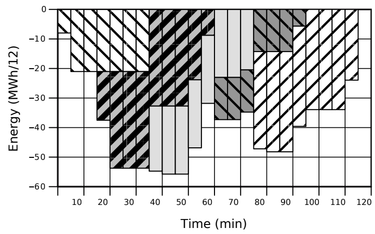


Figure 2.9: Two-hour schedule for deactivation of reserves on a selected unit, showing five separate reserve deactivations. Each column represents the amount of energy deactivated in a five minute interval, e.g. 21 MWh/12, corresponding to producing at 21 MW below the planned production level in that interval.

Finally, Fig. 2.10 shows how a unit is rescheduled over a longer time horizon. In particular, the figure displays planned production and how reserves are activated and deactivated while the total production of the unit remains inside the minimum and maximum bounds for production.

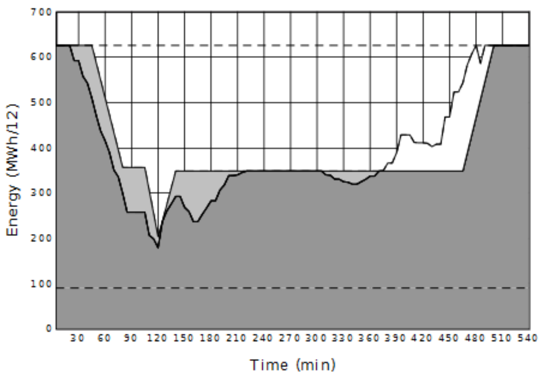


Figure 2.10: Rescheduling of a selected conventional unit. The dark grey area is the planned production, the white area is activation of reserves, and the light grey area is deactivation of reserves. The two dashed lines show the minimum and maximum production levels of the unit.

2.7.5 Sensitivity to uncertainty

For computational reasons, we formulate our model as deterministic, and accordingly, the results in Table 2.3 are based on the expected value of wind power production, assuming perfect foresight. Although predictability increases by considering a short time horizon of only one or two hours, the uncertainty in renewable production may, nevertheless, still have an impact. For this reason, we investigate the sensitivity of the model to uncertainty in the wind power production (referred to as perfect foresight) and quantify the results of using the deterministic solution in the presence of uncertainty (referred to as using expected value)[6].

We aggregate the 5000 scenarios of the Winter 2010 case into three scenarios by the use of K-means clustering. Initially, each scenario is assigned to the closest of three scenarios. The cluster averages are then computed and all 5000 scenarios are again distributed to three clusters depending on their distance to the averages. This iterative process continues until the averages are stable. The wind power production of the three final scenarios is displayed in Table 2.5. Whereas scenario 2 is close to base case, scenario 1 and 3 are relatively extreme.

Table 2.5: Mean and standard deviation in three wind power production scenarios

	Base case	Scenario 1	Scenario 2	Scenario 3
Mean	939.81	1775.12	1002.46	343.33
Std.	809.41	1156.32	862.80	395.04

In Table 2.6 we report the costs of perfect foresight in the three scenarios and the result of using the expected value solution, which is computed by fixing the levels of manual reserve and transmission in the base case, while assuming realised wind power production is given by the scenario[7]. Finally, we also display the costs of fixing the levels of manual reserve to zero, allowing only for reactive reserves.

As seen from the results, the total costs vary significantly with variations in wind power production, as does the result of using the expected value solution. When comparing the results of fixing manual reserves to the base case level and to zero, we observe that in Scenario 2, our model outperforms the reactive approach, whereas the opposite is the case for Scenarios 1 and 3. Hence, when wind power production varies in the neighbourhood of its expectation our model remains to be superior.

[6] In stochastic programming terms, this corresponds to the subproblems used in computing the expected value of the wait-and-see solutions and the expected result of using the expected value solution.

[7] In stochastic programming terms, this corresponds to a assuming a two-stage decision process, in which manual reserve and transmission decisions are made before the realisation of uncertainty and automatic reserve decision are made after the realisation.

Table 2.6: Total system costs listed in €.

	Scenario 1	Scenario 2	Scenario 3
Perfect foresight	−9 181 134	−663 448	10 812 021
Using expected value	−5 631 901	552 974	19 001 380
Reactive costs	−5 669 202	646 412	18 786 792

2.8 Conclusion

In this paper, we formulate an intra-hour model that proactively re-dispatches the power generating units scheduled by an hourly unit commitment model to account for imbalances in the system within the hour. Our model can be used by the SO as decision-support for real-time activation of manual reserve bids, as it can be used in advance of negotiating contracts on automatic reserves. However, it is particularly useful in analysing the operational consequences of different configurations of the electricity system. Contrary to existing models from the literature, our model includes complex market rules for activation of reserves. Imbalances are caused by wind power forecast errors, ramping on power units and transmission lines, as well as intra-hour variations in demand. We generate a representative forecast of wind power production that serves as input to the model. When investigating the influence of wind power forecast errors and ramping, we find that the ability to ramp within the hour is used extensively by the units. In spite of the cost of doing so, our results show that the benefits of growth in inexpensive wind power production are not outweighed by increased balancing costs. Moreover, we find that the proactive approach to re-dispatch the units is superior to the reactive approach, especially for the 2020 high wind cases. Thus, the approach is increasingly interesting for future power systems with the expected growth in wind penetrations.

Future work includes an extension of the deterministic model to include stochastic wind power production. Furthermore, different solution methods may be explored to efficiently solve stochastic intra-hour models with high time resolution.

Acknowledgements

The authors gratefully appreciate many valuable comments and suggestions from two anonymous referees, Pierre Pinson from the Technical University of Denmark and Peter Meibom from the Danish Energy Association, as well as discussions of the problem with Energinet.dk. Jeanne Andersen acknowledges support through the CFEM project and Trine Krogh Boomsma through the ENSYMORA project, both funded by the Danish Council of Strategic Research (09-067008/DSF and 10-093904/DSF, respectively). Ditte Mølgård Heide-Jørgensen acknowledges the support through the iPower project also funded by the Danish Council of Strategic Research via the DSR-SPIR program (10-095378).

Appendix

2.A Wind power forecasts: Parameters and correlations Supplementary material to *Short-term balancing of supply and demand in an electricity system: Forecasting and scheduling* in Annals of Operations Research

Parameters of the Beta distributions

Consider the Beta distribution $\text{Be}(\alpha_t, \beta_t)$ with $\alpha_t > 0, \beta_t > 0$. To determine its parameters, we use the mean-variance model of Pinson and Madsen (2009).

The intra-hour mean is given by the linear interpolation between hourly values $\bar{\omega}_h$ and $\bar{\omega}_{h+1}$, and so

$$\mu_t = ((60h - \tau t)/(60 - \tau))\bar{\omega}_h + ((\tau(t-1) - 60(h-1))/(60-\tau))\bar{\omega}_{h+1}, \quad t \in \mathcal{T}_h.$$

The variance is determined from the mean such that

$$\sigma_t^2 = \theta_1 + \theta_2 \nu_t \mu_t (1 - \mu_t),$$

where, for a constant mean, the function

$$\nu_t = \sigma_0^2 \Big(\frac{t_0 + t}{t_0 + T/\tau} \Big)^{\lambda},$$

with $\lambda \in [0, 1]$ makes the variance an increasing and concave function of time. Here, σ_0^2 is a reference variance of the forecast at time T/τ (the end of the scheduling horizon), and t_0 is the time between generating and starting the forecast.

We use the parameter values $\sigma_0^2 = 0.1$ and $\lambda \in [0.4, 0.6]$ as in Pinson (2006), and $\theta_1 = 0.02$ and $\theta_2 = 4$, since the maximum variance (i.e. for $\mu_t = 0.5$ and $t = T/\tau$) should be close to σ_0^2.

Finally, the parameters of the Beta distribution are computed using the identities[8]

$$\mu_t = \frac{\alpha_t}{\alpha_t + \beta_t}, \quad \sigma_t^2 = \frac{\alpha_t \beta_t}{(\alpha_t + \beta_t)^2(\alpha_t + \beta_t + 1)}.$$

Correlation matrix

Denote the $(T/\tau) \times (T/\tau)$-correlation matrix by $\Sigma = (\rho_{tt'})_{t,t' \in \mathcal{T}}$. With inspiration from Morales et al. (2009b); Pinson and Girard (2012), we assume exponential decay of the correlations over time such that

$$\rho_{tt'} = \exp(-\phi(t' - t)(\tau/60)).$$

We use the parameter value $\phi = 1/7$ as in Pinson and Girard (2012).

2.B The effect of the cost of the automatic reserves Supplementary material to *Short-term balancing of supply and demand in an electricity system: Forecasting and scheduling* in Annals of Operations Research

We investigate how the cost of balancing the system depends on the cost of automatic reserves. This is illustrated in Figure 2.B.1. It is seen how total costs

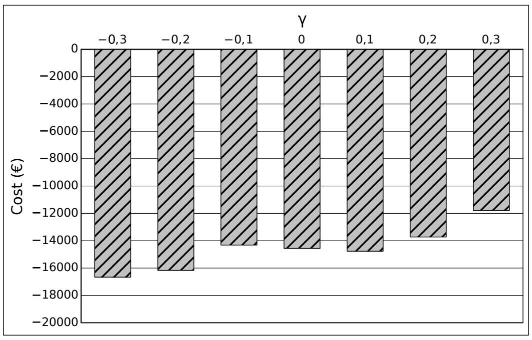


Figure 2.B.1: Balancing costs for the Winter 2010 base case. The cost of automatic reserves is €95$(1 + \gamma)$ for activating reserves and €20$(1 - \gamma)$ for deactivating reserves

increase when activation of reserves becomes more expensive and deactivation becomes less expensive. By estimating the linear trend, we find the average total

[8] For some t, we occasionally obtain $\alpha_t \leq 0$ or $\beta_t \leq 0$, which means that the Beta distribution is not an accurate model for the data. In these cases we slightly modify μ_t or σ_t^2.

balancing costs to increase by 3.9%, when the reserve costs are increased by 10% (i.e. costs of activation of reserves increase by 10% and the costs of deactivation of reserves decrease by 10%).

– 3 –

A DYNAMIC PROGRAMMING APPROACH TO THE RAMP-CONSTRAINED INTRA-HOUR STOCHASTIC SINGLE-UNIT COMMITMENT PROBLEM

Ditte M. Heide-Jørgensen, *Dep. of Math. Sciences, University of Copenhagen*,
Pierre Pinson, *Dep. of Electrical Engineering, Technical University of Denmark*,
Trine K. Boomsma, *Dep. of Math. Sciences, University of Copenhagen*

ABSTRACT We consider the problem of a profit-maximizing power producer in a liberalised market. With the increasing penetration of intermittent renewable production in many power systems, variability and uncertainty in electricity prices become major concerns and the ability to ramp becomes increasingly important. We therefore propose a multi-stage stochastic formulation of the single-unit commitment problem with a high resolution of the time horizon and ramping constraints. In particular, we assume intra-hour electricity prices are available on an hourly basis and allow for hourly binary unit commitment (UC) decisions and intra-hour continuous economic dispatch (ED) decisions. We solve the hourly UC problem by dynamic programming and the intra-hour ED problem as a convex quadratic program. To avoid excessive discretization, we suggest a compact DP formulation, which is either exact or approximates ramping across hours, whereas intra-hour ramping is handled by the quadratic program. To appropriately represent market price uncertainty and capture temporal correlations, we suggest a time-inhomogeneous first-order finite-state discrete-time Markov chain (MC). We illustrate the significant profit potential from considering the high time resolution and the ability to ramp (up to 13.88%) and the substantial impact of uncertainty in a realistic case study.

Keywords *Dynamic programming; OR in energy; Stochastic programming; Unit Commitment; Markov processes*

3.1 Introduction

In a regulated market, the unit commitment problem (UC) is solved by a central planner who operates all units in the system, while minimizing total production cost and ensuring balance between total supply and total demand. An early review on this formulation of the UC problem is found in Sheble and Fahd (1994), whereas overviews of more recent developments can be found in Hobbs et al. (2001) and Anjos (2013). Previously, extensions of the problem have mainly considered uncertain demand, e.g. Bunn and Paschentis (1986). As the quality of demand forecasts has improved, however, demand is often assumed to be known and more effort has been put into developing efficient solution methods. An example is the dynamic programming (DP) solution by Rong et al. (2008), in which the dimension of the DP problem is reduced by relaxing the integrality conditions of the on/off state variables and sequentially committing subsets of units. An alternative decomposition method is Lagrangian relaxation of the balancing constraints, see Zhuang and Galiana (1988), Muckstadt and Koenig (1977) and Frangioni et al. (2008). This decouples the units, and the UC can be solved for one unit at a time, making DP directly applicable.

During the 1990s, the electricity markets went through a liberalisation process. As a result, planning and scheduling is no longer made by a central planner, but rather by independent power producers with the objective of profit maximization, while the market now ensures the balance between demand and supply. This problem has likewise been the subject of study in the literature, both as a single-unit Lagrangian subproblem and as a profit maximization problem, see e.g. Arroyo and Conejo (2000) and Frangioni and Gentile (2006).

The increasing deployment of intermittent renewable generation in recent years challenges the balancing of supply and demand. To account for the uncertainty in supply, the literature has proposed stochastic programming formulations of the UC problem, see for example Papavasiliou and Oren (2013), Bouffard and Galiana (2008), Morales et al. (2009a) and Pritchard et al. (2010). Typically, uncertainty is represented by a so-called scenario tree that branches in each stage. The formulation implies a copying of decision variables for each scenario and, thus, the number of variables grows exponentially with the number of stages. The copying of binary variables is especially likely to make multi-stage problems computationally intractable. Consequently, most stochastic programming formulations are two-stage (as all of the above), and even when multi-stage models are considered, as in Nowak and Römisch (2000), the number of stages is rather small. As a result, these models are unable to capture the frequent update of production forecasts etc. Furthermore, many problem formulations suggested in the literature (again, as all of the above) consider hourly time intervals. Renewable generation and wind power in particular, however, shows significant variations within an hour, which makes the ramping abilities of conventional generation increasingly important. For this reason, UC problems should ideally have a higher time resolution and a sufficiently detailed representation of ramping constraints, as also demonstrated by the simulation models for sequential electricity markets in Jaehnert and Doorman (2012) and Ela and O'Malley (2012) as well as the two-stage stochastic UC with intra-hour dispatch in Wang et al. (2013). Obviously, such features further increase the complexity of the problem and thereby also the task of solving it.

In the application of Lagrangian relaxation to the stochastic central planner UC problem, variability and uncertainty of renewable generation is reflected in the shadow price of electricity (i.e. the Lagrangian multiplier of the balancing constraint). Similarly, in the UC problems of independent power producers, renewable generation yields variability and uncertainty in electricity market prices, see Tseng and Barz (2002). With renewables in the system, the intra-hour variability of prices is clearly significant, as illustrated by Australian electricity price data in Figure 3.1.

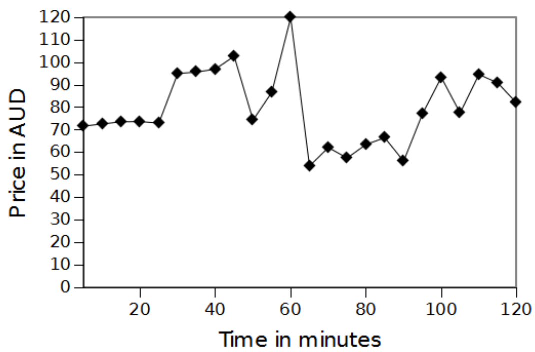


Figure 3.1: The electricity price on February 15th, 2013, from 6 to 8 AM in New South Wales, Australia, which at the time had a wind power penetration of 27%.

We refer to the single-unit problem with uncertainty in the objective function as the stochastic 1UC. The large number of binary variables in this problem has occasionally been handled by DP, see e.g. Tseng and Barz (2002) for a backward dynamic programming approach. The drawback of the DP approach is that the straightforward application cannot handle constraints on the continuous variables across time intervals, such as ramping constraints, without approximation by discretization. An example is provided by Tseng and Barz (2002), who propose to either discretize the production variables or to handle the constraints heuristically. In Frangioni and Gentile (2006) another approach is presented. Here, the authors propose a forward moving dynamic programming algorithm in which the online or offline status of the unit determines the stages. In particular, the DP subproblem in online stages consists of production scheduling between the start-up of the unit in the beginning of the stage and the shut-down in the end. This way, ramping of generating units applies only to time intervals within a stage as opposed to across stages.

Different market designs have been implemented throughout various power markets. Today, many markets such as Nord Pool[1] for the Nordic countries, clears the day-ahead market with an hourly time resolution. The intra-day market subsequently handles unplanned changes in production and consumption by a settling of the balancing market an hour ahead of operation. Other markets such as the Australian National Electricity Market (NEM) are operated as real-time spot markets that settle every half hour and determine electricity prices every 5 minutes[2]. Such markets may more easily accommodate the variations in renewable energy production, which is why this is our focus.

In this paper, we consider the stochastic 1UC problem in a real-time market with a daily time horizon and updated forecasts of 5-minute electricity prices every hour. Our contribution is threefold. First, we extend the *stochastic* but *hourly* and *approximate* DP formulation of Tseng and Barz (2002) to include intra-hour variations in market prices and ramping constraints of the generating units. To avoid excessive discretization, we suggest a more compact DP formulation. This formulation is exact under certain assumptions on planning in view of uncertainty (the multi-hour model), and otherwise approximates ramping across hours (the single-hour model), whereas intra-hour ramping is always handled by a convex quadratic program. The formulations can likewise be seen as an extension of the *exact* and *potentially intra-hourly* but *deterministic* DP formulation of Frangioni and Gentile (2006) to account for uncertainty in market prices. We propose two alternative approaches to planning in view of uncertainty. In particular, as the straightforward application is not possible if planning decisions are made on an hourly basis (the single-hour model), we also suggest a multi-hour planning formulation (the multi-hour model) in which decisions are made for several hours at once. Second, we suggest a time-inhomogeneous first-order finite-state discrete-time Markov chain (MC) to appropriately represent market price uncertainty. The result is a multi-stage stochastic ramp-constrained 1UC problem with a high time resolution and multi-hourly stages. To the best of our knowledge such a model has not previously been considered. While facilitating the application of the DP, this allows us to capture temporal correlation in prices. Third, we illustrate the significant profit potential from considering the high time resolution and the ability to ramp (up to 13.88%) and the substantial impact of

[1]See www.nordpoolspot.com.

[2]See www.aemo.com.au.

uncertainty.

The paper is structured as follows. First, the decomposition of the UC to the 1UC is explained in Section 3.2, which also contains the DP formulations. The application of DP to the stochastic ramp-constrained 1UC model is presented in Section 3.3, while details on the Markov chain for electricity prices are found in Section 3.4. Section 3.5 contains the numerical results for a representative case study. Finally, the contributions of the paper are summarized and discussed in Section 3.6.

3.2 Methodology

The unit commitment problem determines when to start up and shut down a power producing unit. We consider a given time horizon, $\mathcal{T} = \{1, \ldots, T\}$, and a set of power production units, $\mathcal{I} = \{1, \ldots, I\}$. For every $i \in \mathcal{I}$ and $h \in \mathcal{T}$, the online status is represented by a binary variable u_{ih}, which is 1 when the unit is online, and 0 otherwise. The production level is a continuous variable q_{ih}. Furthermore, v_{ih} is a binary variable, which is 1 if unit i is started up at time h, and 0 otherwise. We let c_i^o denote the cost of being online, c_i^s represent the start-up cost and $c_i(q) = aq^2 + bq + c$ the convex quadratic production cost for unit i. Now, we obtain the objective

$$\min_{q_{ih}, u_{ih}, v_{ih}} \sum_{i=1}^{I} \sum_{h=1}^{T} \left(c_i(q_{ih}) + c_i^o u_{ih} + c_i^s v_{ih} \right). \tag{3.1}$$

The problem is subject to a set of balance constraints matching supply with demand, d_h, in every time interval, h,

$$\sum_{i=1}^{I} q_{ih} = d_h, \quad h \in \mathcal{T}. \tag{3.2}$$

Further constraints include technical restrictions on every unit, usually including minimum and maximum production limits, q_i^{min} and q_i^{max},

$$q_i^{min} u_{ih} \leq q_{ih} \leq q_i^{max} u_{ih} \quad h \in \mathcal{T}, i \in \mathcal{I}, \tag{3.3}$$

ramping restrictions allowing the production level in two consecutive time intervals only to decrease by r_i^{down} or increase by r_i^{up},

$$-r_i^{down} \leq q_{ih+1} - q_{ih} \leq r_i^{up} \quad h \in \mathcal{T}, \tag{3.4}$$

and minimum up- and down-time constraints ensuring that the unit is online in at least T_i^{up} consecutive time intervals and offline in at least T_i^{down} consecutive time intervals when the online/offline status changes

$$u_{ih} - u_{ih-1} \leq u_{ik}, \quad k = h+1, \ldots, \min\{h + T_i^{up} - 1, T\}, h \in \mathcal{T}, i \in \mathcal{I}, \tag{3.5}$$

$$u_{ih-1} - u_{ih} \leq 1 - u_{ik}, \quad k = h+1, \ldots, \min\{h + T_i^{down} - 1, T\}, h \in \mathcal{T}, i \in \mathcal{I}. \tag{3.6}$$

Finally, logical constraints relate the binary start-up variable to the binary unit commitment variable

$$u_{ih} - u_{ih-1} \leq v_{ih}, \;\; h \in \mathcal{T} \backslash \{1\}, \;\; i \in \mathcal{I}. \tag{3.7}$$

This is a deterministic mixed integer programming formulation of the problem, which, as mentioned in Section 3.1, can be solved e.g. by Lagrangian relaxation. If we denote the Lagrangian multipliers of equation (3.2), also known as the shadow prices, by $\lambda_h, h \in \mathcal{T}$, the Lagrangian single unit problem can be formulated as the following maximisation problem

$$\mathcal{L}_i(\lambda) = \max_{q_h, u_h, v_h} \sum_{h \in \mathcal{T}} \left(\lambda_h q_h - c_i(q_h) - c_i^o u_h - c_i^s v_h \right), \tag{3.8}$$

subject to (3.3)-(3.7) with the unit index of the variables omitted. To accelerate the solution process these subproblems could be solved in parallel as the subproblems of the Lagrangian relaxation in Papavasiliou et al. (2015). The Lagrangian dual is

$$\min_{\lambda} \sum_{i \in \mathcal{I}} \mathcal{L}_i(\lambda) - \sum_{h \in \mathcal{T}} d_h,$$

which provides an upper bound to the UC problem.

Solving the Lagrangian 1UC subproblem corresponds to solving the profit maximising 1UC problem in a deregulated market (assuming the Lagrangian relaxation algorithm converges). Denoting by p_h the market price for every $h \in \mathcal{T}$, the problem is

$$\max_{q_h, u_h, v_h} \sum_{h \in \mathcal{T}} \left(p_h q_h - c_i(q_h) - c_i^o u_h - c_i^s v_h \right), \tag{3.9}$$

under the constraints (3.3)-(3.7), again with the unit index of the variables omitted.

We propose two formulations of the stochastic 1UC problem, which differ in how decisions are adapted to the evolution of prices. In the first formulation, we assume that the online/offline status of the unit is adapted to the price on an hourly basis (as is the production schedule). This corresponds to extending the stochastic approach of Tseng and Barz (2002) to a backwards DP programming algorithm with high time resolution and a discretization of the production variables at an hourly level to include ramping constraints. In the second formulation, we further assume the number of hours online/offline is adapted to the prices (as is the production schedule for the entire online/offline period). This results in an extension of Frangioni and Gentile (2006) likewise to a stochastic backwards dynamic programming algorithm with high time resolution, but with stages consisting of online and offline periods. The latter approach allows us to consider ramping restrictions without discretizing the continuous production variables. It, therefore, has the advantage of an exact solution, whereas the solution to the first has to be approximated. Both models enable a higher time resolution than hourly without compromising computational tractability even for a stochastic model with many binary variables. We show that the second formulation provides a lower bound to the first, as this can be seen as a stronger non-anticipativity condition in stochastic programming. We use a case study to illustrate and quantify the importance of a fine time resolution, ramping constraints, and the inclusion of uncertainty in a power system with a significant share of renewable energy.

3.3 Stochastic single-unit commitment and intra-hour dispatch

3.3.1 A dynamic programming formulation for the 1UC

To solve the 1UC problem efficiently we use a DP approach. A thorough introduction to DP can be found in Bertsekas (1987). In the DP formulation of the 1UC problem we let a state be defined by the status of the unit in the preceding hour, u_{h-1}, i.e. whether the unit was online or offline, how long the unit has been online or offline, τ, and a price vector, $\mathbf{p}_h$, of the intra-hour electricity prices in hour h. Note, that we omit the unit index as we now consider the 1UC problem. We denote the stages by $h = 1, \dots, T$, which in our case represent the hours of a $T = 24$ hour time horizon. We solve the intra-hour subproblem for each stage deciding the production level in each intra-hour time interval. These subproblems are ED problems since the binary online/offline decision variables are decided on the hourly basis in the DP formulation.

We redefine start-up (/shut-down) cost as $c_h^s(u_{h-1}, u_h)$, and online (/offline) cost as $c_h^o(u_h)$. Offline and shut-down costs may be zero as in Section 3.2, but it is not necessary for the following to work.

We denote the value function of the intra-hour ED problem by $f_h(\mathbf{p}_h)$. Now we are ready to define the DP recursion also known as the Bellman equation as follows. If the unit was offline in stage $h-1$ we denote the expected future profit from stage h and onwards by F_h^0. If, furthermore, the unit has been offline for at least T^{down} hours it can either start up or remain offline, and hence,

$$\begin{aligned} F_h^0(T^{down}, \mathbf{p}_h) = \max \Big\{ & - c_h^s(0, u_h) - c_h^o(u_h) \\ & + \big(f_h(\mathbf{p}_h) + \mathbb{E}[F_{h+1}^1(1, \mathbf{p}_{h+1}) | \mathbf{p}_h]\big) u_h \\ & + \mathbb{E}[F_{h+1}^0(T^{down}, \mathbf{p}_{h+1}) | \mathbf{p}_h](1 - u_h) : u_h \in \{0, 1\} \Big\}. \end{aligned}$$

For $\tau \in \{1, \dots, T^{down} - 1\}$ the unit has to remain offline due to the minimum down-time restrictions, and thus,

$$F_h^0(\tau, \mathbf{p}_h) = - c_h^o(0) + \mathbb{E}[F_{h+1}^0(\tau + 1, \mathbf{p}_{h+1}) | \mathbf{p}_h].$$

If the unit was online in stage $h-1$ we denote the expected future profit from stage h and onwards by F_h^1. If the unit has been online for at least T^{up} hours and the production level in the previous hour is at minimum capacity it can remain online or shut down,

$$\begin{aligned} F_h^1(T^{up}, \mathbf{p}_h) = \max \Big\{ & - c_h^s(1, u_h) - c_h^o(u_h) \\ & + \big(f_h(\mathbf{p}_h) + \mathbb{E}[F_{h+1}^1(T^{up}, \mathbf{p}_{h+1})|\mathbf{p}_h]\big)u_h \\ & + \mathbb{E}[F_{h+1}^0(1, \mathbf{p}_{h+1})|\mathbf{p}_h](1-u_h) : u_h \in \{0,1\}\Big\}. \end{aligned}$$

For $\tau \in \{1, \ldots, T^{up}-1\}$, the unit has to remain online due to the minimum up-time restrictions,

$$F_h^1(\tau, \mathbf{p}_h) = -\, c_h^o(1) + f_h(\mathbf{p}_h) + \mathbb{E}[F_{h+1}^1(\tau+1, \mathbf{p}_{h+1})|\mathbf{p}_h].$$

3.3.2 Economic dispatch with high time resolution

We solve the economic dispatch (ED) problem as a convex quadratic programming problem assuming the variable costs, $c(q_t)$, are quadratic and convex. In accordance with operational practice, production levels may be changed every few minutes, whereas start up or shut down of units takes longer and is often made on an hourly basis. We, therefore, allow for a higher time resolution in the ED problem by assuming that each time period of the UC problem, h, is divided into T_h intra-hour time intervals $t = 1, \ldots, T_h$. The production level in each time interval is subject to capacity constraints and the objective is to maximize profit. Letting $\mathbf{p}_h = (p_t)_{t=1}^{T_h}$, the ED problem is

$$\begin{aligned} f_h(\mathbf{p}_h) = \max\ & \sum_{t=1}^{T_h} (p_t q_t - c(q_t)) \\ \text{st}\ & q^{min} \le q_t \le q^{max}, \ \ t = 1, \ldots, T_h. \end{aligned}$$

3.3.3 Non-anticipativity

In the above, we assume that the online/offline status of the unit is adapted to the price on an hourly basis. More specifically, we assume that the electricity prices of the current hour are known in the beginning of the hour enabling the producer to decide whether to remain online/offline or shut down/start up. In stochastic programming terms, this corresponds to so-called non-anticipativity constraints. To formulate this, let $\mathcal{F} = \{\mathcal{F}_h\}_{h\in\mathcal{T}}$ be a filtration, where $\mathcal{F}_h$ is generated by $\mathbf{p}_h$. Then, the constraints are as follows

Assumption 1(i) u_h must be adapted to $\mathcal{F}_h$.

We likewise assume that the ED decisions are adapted to the price on an hourly basis, i.e.

Assumption 1(ii) Let $\mathbf{q}_h = (q_t)_{t=1}^{T_h}$. Then, $\mathbf{q}_h$ must be adapted to $\mathcal{F}_h$.

We refer to this model as single-hour planning as opposed to the multi-hour planning presented in the following.

3.3.4 A compact formulation

The UC problem can be formulated in a more compact fashion, which we will exploit in the following sections. Let $\mathbf{p}_{hk} = (\mathbf{p}_h, \ldots, \mathbf{p}_k)$, where $\mathbf{p}_h = (p_t)_{t=1}^{T_h}$, and let the multi-hour ED problem be

$$
\begin{aligned}
f_{hk}(\mathbf{p}_{hk}) = \max \ & \sum_{j=h}^{k} \sum_{t=1}^{T_j} (p_t q_t - c(q_t)) \\
\text{st } & q^{min} \leq q_t \leq q^{max}, \ \ t = 1, \ldots, T_j, \ \ j = h, \ldots, k.
\end{aligned}
$$

Moreover, let $c^o_{hk}(u_h) = \sum_{j=h}^{k} c^o_j(u_h)$. By iterating until the unit is no longer forced online or offline (using the law of iterated expectations), the problem is equivalent to

$$
\begin{aligned}
F^0_h(\mathbf{p}_h) = \max \Big\{ & -c^s_h(0, u_h) + \big(-c^o_{hh+T^{up}-1}(u_h) \\
& + \mathbb{E}[f_{hh+T^{up}-1}(\mathbf{p}_{hh+T^{up}-1}) + F^1_{h+T^{up}}(\mathbf{p}_{h+T^{up}}) | \mathbf{p}_h] \big) u_h \\
& + \big((-c^o_{hh}(u_h) + \mathbb{E}[F^0_{h+1}(\mathbf{p}_{h+1}) | \mathbf{p}_h] \big)(1 - u_h) : u_h \in \{0, 1\} \Big\},
\end{aligned}
$$

and

$$
\begin{aligned}
F^1_h(\mathbf{p}_h) = \max \Big\{ & -c^s_h(1, u_h) + \big(-c^o_{hh}(u_h) + f_{hh}(\mathbf{p}_{hh}) + \mathbb{E}[F^1_{h+1}(\mathbf{p}_{h+1})) | \mathbf{p}_h] \big) u_h \\
& + \big(-c^o_{hh+T^{down}-1}(u_h) \\
& + \mathbb{E}[F^0_{h+T^{down}}(\mathbf{p}_{h+T^{down}}) | \mathbf{p}_h] \big)(1 - u_h) : u_h \in \{0, 1\} \Big\}.
\end{aligned}
$$

Note that in this formulation, the multi-hour ED subproblem of F^0 is a multi-stage problem (with hourly stages) due to the non-anticipativity constraints.

3.3.5 Strong non-anticipativity

As an alternative to Assumption 1, we may assume that the decision to start up/shut down and *the number of hours* to remain offline/online depend on the prices in a given hour. In stochastic programming terms, this corresponds to a stronger non-anticipativity condition and can be formulated as

Assumption 2(i) Let $k_h = \min\{k \geq h : u_h \neq u_{k+1}\}$, i.e. the first time the unit changes status. Then, k_h must be adapted to $\mathcal{F}_h$.

We likewise assume strong non-anticipativity in the ED problem. Formally, we impose

Assumption 2(ii) Let $\mathbf{q}_k = (q_t)_{t=1}^{T_k}$ and $k_h = \min\{k \geq h : u_h \neq u_{k+1}\}$. Then, $\mathbf{q}_k$ must be adapted to $\mathcal{F}_h$ for $k = h, \ldots, k_h$.

3.3.6 Multi-hour planning

We denote the UC and ED value functions under Assumption 2 by $\tilde{F}_h^0, \tilde{F}_h^1$ and $\tilde{f}_{hk}$. Note that with a stronger non-anticipativity condition, the multi-hour ED problem becomes the deterministic problem

$$\begin{aligned} \tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) = \max \ & \sum_{k=h}^{k_h} \sum_{t=1}^{T_k} (\mathbb{E}[p_t | \mathbf{p}_h] q_t - c(q_t)) \\ \text{st} \ & q^{min} \leq q_t \leq q^{max}, \ \ t = 1, \ldots, T_k, k = h, \ldots, k_h. \end{aligned}$$

Moreover, the DP problem becomes a shortest path problem. To see this, we may further iterate until the unit changes status. As a result, the DP formulation is equivalent to

$$\begin{aligned} \tilde{F}_h^0(\mathbf{p}_h) = \max \Big\{ & - c_h^s(0,1) - c_{hk_h}^o(1) + \tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) \\ & + \mathbb{E}[\tilde{F}_{k_h+1}^1(\mathbf{p}_{k_h+1}) | \mathbf{p}_h] : k_h \in \{h + T^{up} - 1, \ldots, T\} \Big\}, \end{aligned}$$

and

$$\begin{aligned} \tilde{F}_h^1(\mathbf{p}_h) = \max \Big\{ & - c_h^s(1,0) - c_{hk_h}^o(0) \\ & + \mathbb{E}[\tilde{F}_{k_h+1}^0(\mathbf{p}_{k_h+1}) | \mathbf{p}_h] : k_h \in \{h + T^{down} - 1, \ldots, T\} \Big\}. \end{aligned}$$

Now consider a directed graph with nodes corresponding to feasible online periods (i.e. respecting minimum up-time restrictions) and arcs representing feasible offline periods going from one node corresponding to an online stage to another, while respecting minimum down-time restrictions. Furthermore, let arc costs represent start-up costs of the unit and let node costs represent the sum of online costs and the negative profits associated with each stage. Then the solution to the problem can be found as the shortest path between a source node with arcs to all nodes and a sink node with arcs coming in from all nodes (see Frangioni and Gentile (2006)). Note that in this formulation, the binary decisions to start-up/shut-down are replaced by the integral number of hours to remain online/offline.

The value function of the multi-hour planning problem provides a lower bound on the single-hour problem, i.e.

Proposition 1

$$\tilde{F}_h^j(\mathbf{p}_h) \leq F_h^j(\mathbf{p}_h), \;\; j = 0, 1.$$

For a formal proof, see 3.A.

Without the stronger non-anticipativity, intra-hour dispatch *with ramping* requires discretization of the production level on an hourly basis. This is unnecessary in the case of the stronger non-anticipativity condition, which has an exact solution. We consider the inclusion of ramping constraints in the following section.

3.3.7 Ramp-constrained single-hour planning

To accommodate the inclusion of ramping constraints, we introduce an additional state variable, q_{h-1}, in the DP formulation that accounts for the production level in the last intra-hour time interval of the preceding hour.

The ramp-constrained intra-hour ED problem is then

$$\begin{aligned} f_h(\mathbf{p}_h, q_{h-1}, q_h) = \max \;\; & \sum_{t=1}^{T_h} (p_t q_t - c(q_t)) \\ \text{st} \;\; & q^{min} \leq q_t \leq q^{max}, \;\; t = 1, \ldots, T_h \\ & - r^{down} \leq q_{t+1} - q_t \leq r^{up}, \;\; t = 1, \ldots, T_h \\ & q_0 = \begin{cases} q_{h-1}, & \text{if } \; q_{h-1} > 0 \\ q^{min} - r^{up}, & \text{if } \; q_{h-1} = 0, \end{cases} \\ & q_{T_h} = q_h, \end{aligned}$$

where the condition on q_0 ensures that the unit always starts production at q^{min} in the first time interval of the hour when it has been offline the previous hour.

We let $Q = [q^{min}, q^{max}]$ and

$$\tilde{Q}(q) = \begin{cases} Q \cap [q - T_h r^{down}, q + T_h r^{up}], & \text{if } \; q > 0, \\ Q \cap [q^{min}, q^{min} + (T_h - 1) r^{up}], & \text{if } \; q = 0, \end{cases}$$

such that the production level complies with the minimum and maximum production and can be reached within an hour (T_h intra-hour intervals) and thereby also comply with the ramping restrictions. The DP recursion is as follows. If the unit has been offline for at least T^{down} hours it can either remain offline or start up,

$$\begin{aligned} F_h^0(T^{down}, \mathbf{p}_h) = \max \Big\{ & - c_h^s(0, u_h) - c_h^o(u_h) + \max\{ f_h(\mathbf{p}_h, 0, q_h) \\ & + \mathbb{E}[F_{h+1}^1(1, \mathbf{p}_h, q_h) | \mathbf{p}_h] : q_h \in \tilde{Q}(0) \} u_h \\ & + \mathbb{E}[F_{h+1}^0(T^{down}, \mathbf{p}_{h+1}) | \mathbf{p}_h](1 - u_h) : u_h \in \{0, 1\} \Big\}. \end{aligned}$$

For $\tau \in \{1, \dots, T^{down} - 1\}$ it must stay offline,

$$F_h^0(\tau, \mathbf{p}_h) = -c_h^o(0) + \mathbb{E}[F_{h+1}^0(\tau + 1, \mathbf{p}_{h+1}) | \mathbf{p}_h].$$

If the unit has been online for at least T^{up} hours it can shut down if the production level in the previous hour ends at q^{min}, otherwise it has to remain online. Hence,

$$\begin{aligned} F_h^1(T^{up}, \mathbf{p}_h, q^{min}) = \max \Big\{ & - c_h^s(1, u_h) - c_h^o(u_h) + \max\{ f_h(\mathbf{p}_h, q^{min}, q_h) \\ & + \mathbb{E}[F_{h+1}^1(T^{up}, \mathbf{p}_{h+1}, q_h) | \mathbf{p}_h] : q_h \in \tilde{Q}(q^{min}) \} u_h \\ & + \mathbb{E}[F_{h+1}^0(1, \mathbf{p}_{h+1}) | \mathbf{p}_h](1 - u_h) : u_h \in \{0, 1\} \Big\}. \end{aligned}$$

and for $q_{h-1} > q^{min}$

$$\begin{aligned} F_h^1(T^{up}, \mathbf{p}_h, q_{h-1}) = & - c_h^o(1) + \max \Big\{ f_h(\mathbf{p}_h, q_{h-1}, q_h) \\ & + \mathbb{E}[F_{h+1}^1(T^{up}, \mathbf{p}_{h+1}, q_h) | \mathbf{p}_h] : q_h \in \tilde{Q}(q_{h-1}) \Big\}. \end{aligned}$$

For $\tau \in \{1, \dots, T^{up} - 1\}$, the unit likewise has to remain online,

$$\begin{aligned} F_h^1(\tau, \mathbf{p}_h, q_{h-1}) = & - c_h^o(1) + \max \Big\{ f_h(\mathbf{p}_h, q_{h-1}, q_h) \\ & + \mathbb{E}[F_{h+1}^1(\tau + 1, \mathbf{p}_{h+1}, q_h) | \mathbf{p}_h] : q_h \in \tilde{Q}(q_{h-1}) \Big\} \end{aligned}$$

In this problem, we have imposed Assumption 1. To solve it, we discretize the production level. Note that in spite of solving a problem with high-resolution time horizon, it is sufficient to discretize the production level in the beginning of an hour and let the ED problem handle the ramping for the remaining intra-hour intervals as continuous variables. Thus, intra-hour ramping can be handled in an exact manner, whereas we approximate hour by hour ramping.

By alternatively imposing Assumption 2 we avoid the discretization of the production level, and hence, an exact solution can be found, in spite of including ramping restrictions. Indeed, the DP recursion is the same with or without ramping restrictions since the ED covers the entire online or offline period and thus handles all the ramping.

3.4 Electricity price modelling

The modelling of electricity prices has been extensively studied, especially with the deregulation of electricity markets in the 1990s and the increasing deployment of renewable power sources in the following decade, see Weron (2014) for a comprehensive study of the literature. Here, we model the hourly sets of 5-minute electricity prices by means of an inhomogeneous, first-order, finite-state, discrete-time Markov chain. The Markov property allows for the inclusion of temporal correlation in prices while facilitating the application of the dynamic programming algorithm. Although current prices may not only depend on the immediately preceding prices but also on prices further back in time, Markov chains have been argued to be a reasonable choice for modelling of electricity prices, see for instance González et al. (2005). Finally, it is well-known that electricity prices exhibit daily patterns, Weron (2014), which is accommodated by considering a time-inhomogeneous Markov chain, see e.g. Iversen et al. (2014).

Recall that T_h is the number of intra-hour intervals in hour h. For every $h = 1, \ldots, H$, we assume that the intra-hour prices of a given hour, h, are represented by a discrete stochastic vector, $\mathbf{P}_h = (P_{ht_1}, \ldots, P_{ht_{T_h}})$, that takes values within a finite set of states $\{\mathbf{p}_h^1, \ldots, \mathbf{p}_h^B\}$ with $\mathbf{p}_h^b = (p_{ht_1}^b, \ldots, p_{ht_{T_h}}^b)$. Each of these vectors represents a high-resolution intra-hour price path corresponding to the price bin. The Markov property ensures that

$$\mathbb{P}(\mathbf{P}_{h+1} = \mathbf{p}_{h+1}^{b_{h+1}} | \mathbf{P}_1 = \mathbf{p}_1^{b_1}, \ldots, \mathbf{P}_h = \mathbf{p}_h^{b_h}) = \mathbb{P}(\mathbf{P}_{h+1} = \mathbf{p}_{h+1}^{b_{h+1}} | \mathbf{P}_h = \mathbf{p}_h^{b_h}),$$

where $b_1, \ldots, b_{h+1} \in \{1, \ldots, B\}$. Note that we may have that $\mathbf{p}_{h_1}^b \neq \mathbf{p}_{h_2}^b$, since the Markov chain is time-inhomogeneous.

We construct the price paths by clustering of historical data. For every $h = 1, \ldots, H$, we define B price bins $]\underline{p}_{hb}, \bar{p}_{hb}], b = 1, \ldots, B$ such that each price in the first time interval of hour h is in exactly one of these bins. For consecutive stages h and $h+1$, we consider the observed price paths beginning in bin $]\underline{p}_{hb}, \bar{p}_{hb}]$ in the first time period of the current stage, h, and ending in bin $]\underline{p}_{h+1b'}, \bar{p}_{h+1b'}]$, in the first time period of the next stage, $h+1$, for $b, b' \in \{1, \ldots, B\}$. As a representative price vector $\mathbf{p}_h^b = (p_{ht_1}^b, \ldots, p_{ht_{T_h}}^b)$ we choose the price path with smallest Euclidean distance to the rest of the paths (the centre). By choosing the centre of the cluster, we represent price variations throughout the hour, which enables us to assess the effect of ramping. Another possibility would be the average of the cluster, but as this would diminish variations in prices, we would underestimate the costs of ramping restrictions.

The probability, $\pi_{b_h, b_{h+1}}$, that the Markov chain is in state b_{h+1} in stage $h+1$ given that it is in state b_h in stage h is called the one-step transition probability and can be estimated as the number of times the price started in bin b_h in the beginning of hour h and ended in bin b_{h+1} in the beginning of hour $h+1$ divided the number of times the price started in bin b_h,

$$\hat{\pi}_{b_h, b_{h+1}} = \frac{n_{b_h, b_{h+1}}}{\sum_{b=1}^{B} n_{b_h, b}},$$

where $n_{b_h, b_{h+1}}$ is the number of observed price paths beginning in state b_h and ending in state b_{h+1}.

An example of the price paths and the chosen representative can be seen in Figure 3.2.

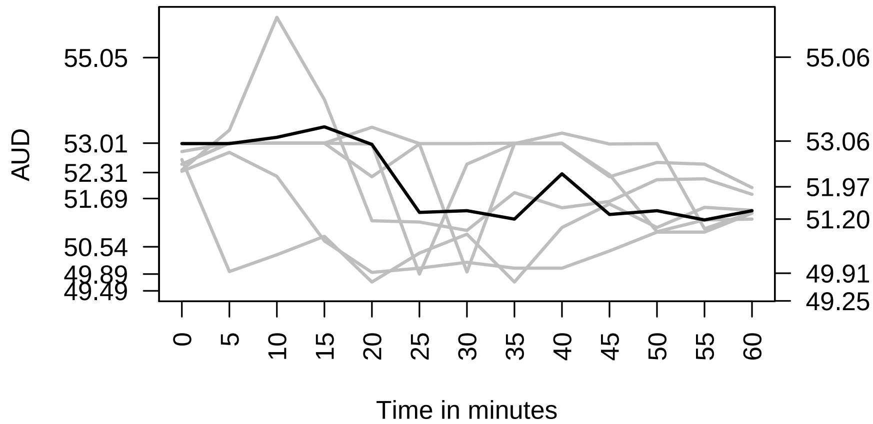


Figure 3.2: Choosing a price path between two bins. The intra-hour price path has 5-minute time resolution. The grey lines are the observed price paths and the black is the chosen representative. The left and right y-axis represent the price bin values for the current and coming hour, respectively.

3.5 Computational results

3.5.1 Price data

Price data with 5-minute resolution from July 2012 to December 2013 from New South Wales has kindly been provided by the Australian system operator. The data is separated into a set of weekdays and a set of special days (including weekends, school holidays and public holidays). Outliers are removed based on the histogram of the log-transformed data, see Appendix B in the electronic supplemental material. This gives us a total price range of 0–150 AUD (Australian dollars).

As mentioned in Section 3.4 the price range is divided into a number of bins that represent the states of the Markov chain. The upper and lower bounds of the bins are chosen such that the fraction of prices in each bin is $1/B$, where B is the number of bins. We consider 8 bins for our case studies to represent the price data properly. Increasing the number of bins would require additional data to estimate the parameters of the Markov chain appropriately. Excluding the first and last bin, this results in an average price range of 1.74 AUD per bin for the weekday data set and 1.78 AUD for the special day data set. The price range of the first and last bin is naturally somewhat larger as they contain the rare very low and very high prices, respectively.

In order to provide meaningful comparative models with 15 and 30-minute time resolution, we aggregate the price trajectories from the Markov chain model by taking the mean over 15 and 30 minutes, respectively.

3.5.2 Unit data

We study and compare the ramp constrained intra-hour stochastic 1UC problem for power producing units with different production characteristics. Table 3.1 displays their technical data regarding capacity, ramp rates, etc. For the 1a unit, the data is obtained from Conejo et al. (2010). The other units are variants of the first unit, constructed to compare the influence of ramping constraints and higher costs on the solution to the stochastic 1UC problem. Table 3.2 holds the start-up, online and production costs. These numbers are likewise based on Conejo et al. (2010), but adjusted to comply with the quadratic production cost function. Unit 1a resembles a base load unit and is as such assumed to be online at midnight which is the beginning of the time horizon. Furthermore, we assume that it produces at 103MW in the last time period of the hour previous to the beginning of the time horizon. To enable comparison we make the same initial assumptions for the rest of the units.

Table 3.1: Technical data of generating units. Capacity bounds, ramp rates and minimum up and down times for each unit. Unit 1a represents a base load unit. The other units are variants of it increasingly resembling peak load units.

Unit	q^{max} MW	q^{min} MW	r^{up} MW/min	r^{down} MW/min	T^{up} h	T^{down} h
1a & 1c	152	30.4	2.53	2.53	8	4
1b & 1d	152	30.4	6	6	8	4
1e	152	30.4	6	6	4	2

Table 3.2: Unit costs. Start-up cost and coefficients of the quadratic production cost function for the units.

Unit	c^{su} $	c^{on} $	b $/MWh	a $/(MWh)^2
1a & 1b	1430.4	250	13	0.002
1c & 1d & 1e	1430.4	300	52.9	0.002

3.5.3 Implementation

The dynamic programming algorithm for solving the UC problem is implemented in Java. The convex quadratic programming formulation ED problem is likewise solved in Java with the Cplex callable library using Cplex 12.6. Both are run with a 2.7GHz processor and 4GB RAM.

3.5.4 Hourly benchmark

In order to evaluate the models and in particular the benefit of the high time resolution we implement a simplified model to obtain an hourly benchmark. We assume that the market continues to have 5-minute prices, but as for the hourly commitment decisions, the dispatch decisions are hourly. To represent ramping we let production increase or decrease linearly throughout the hour between the production level at the end of the previous hour and the production level at the end of the current hour.

3.5.5 Profit gains from intra-hour dispatch

The results from the single-hour planning model can be found in Table 3.3. Obviously, the expensive units have lower profits than less expensive units. For all time resolutions, the fast ramping units, 1b and 1d, have at least as high profits as their slower counterparts, 1a and 1c. Moreover, a doubling of the ramp rate results in a profit increase of only 0.12%, comparing 1b with 1a in the 5-minute stochastic model, but 2.12% when comparing 1d with 1c. In contrast, the corresponding 60-minute resolution profits show no difference between the fast and slow ramping units. Thus the high time resolutions reveal a profit opportunity for the expensive fast ramping units. On the contrary, when comparing the profit of 1e to that of unit 1d, the 60-minute stochastic model shows a difference of 1.59% whereas the 5-minute stochastic model shows a 1.64% difference. Hence, the high time resolution reveals only little profit opportunity from the reduction of up- and down-times.

Obviously, there is a positive difference between the high resolution profits and the 60-minute benchmark. Especially the units 1d and 1e show a positive profit difference of 2.8% and 2.84%, respectively, for the stochastic 5 minute model, compared to the 60-minute benchmark. This is higher than for the 1a, 1b and 1c units. The reason is that 1d and 1e have high costs and at the same time have high ramping ability, and thus exploit the possibility of adjusting production to the price.

From a deterministic model with expected prices we notice the same pattern with even greater differences than in the stochastic model. Hence, if all parameters are deterministic, and in particular the price is known in advance of planning, there is a substantial profit gain from considering the intra-hour variations. In spite of this, the stochastic model generally produces a higher profit in absolute terms.

Finally, the table shows that the profit difference when compared with the 60 minute benchmark for unit 1e in the stochastic model decreases from 2.84% (13.88%) with 5 minute resolution to 2.05% (12.38%) and 0.55% (7.72%) in the stochastic (respectively deterministic) case as the time resolution decreases to 15 and 30 minutes, respectively. Hence, the 30-minute resolution does not add much value whereas the 15-minute resolution may be sufficient for intra-hour manage-

Table 3.3: Unit profits for the single-hour model and the weekday data set. The profits for 5-, 15- and 30-minute time resolutions are listed along with their corresponding 60-minute benchmarks. Finally, the difference between high time resolution profit and 60-minute benchmark profit relative to the high time resolution profit is shown. Except for the relative difference all values are in AUD.

		Deterministic			Stochastic		
	Unit	Solution	60-min bench.	Diff.	Solution	60-min bench.	Diff.
5-min res.	1a	145691.71	145056.64	0.44%	147328.63	146668.68	0.45%
	1b	145858.90	145056.64	0.55%	147500.55	146668.68	0.56%
	1c	2085.52	1973.53	5.37%	6863.12	6812.39	0.74%
	1d	2291.53	1973.53	13.88%	7008.84	6812.39	2.80%
	1e	2291.53	1973.53	13.88%	7123.52	6921.00	2.84%
15-min res.	1a	145627.87	145029.56	0.41%	147268.14	146647.98	0.42%
	1b	145734.04	145029.56	0.48%	147377.25	146647.98	0.49%
	1c	1905.82	1825.22	4.23%	6758.14	6717.85	0.60%
	1d	2120.89	1858.22	12.38%	6904.92	6767.86	1.98%
	1e	2120.89	1858.22	12.38%	7018.59	6875.03	2.05%
30-min res.	1a	145456.79	144988.59	0.32%	147100.01	146615.55	0.33%
	1b	145456.79	144988.59	0.32%	147100.01	146615.55	0.33%
	1c	1608.58	1514.63	5.84%	6569.13	6479.05	1.37%
	1d	1815.48	1675.39	7.72%	6727.06	6693.13	0.50%
	1e	1815.48	1675.39	7.72%	6846.11	6808.17	0.55%

ment.

Table 3.4, in general, shows the same patterns for the multi-hour model. The intra-hour planning allows fast ramping units to exploit their ramping abilities, which results in higher profits than for the slow ramping units, both in absolute terms and the relative difference from the hourly benchmarks. Finally, we see again that relative profit difference between the high time resolution cases and the 60-minute benchmarks decreases as the time resolution decreases except for some cases in which profits are very low. What is also evident, however, is that for the stochastic cases and the fast ramping units the multi-hour model has in general a much lower profit than its single-hour counterpart, indicating that the lower bound is not very tight. For the slow and inexpensive units 1a and 1b we see that the profits are almost the same for the multi-hour model and the single-hour model, even a little higher for the multi-hour model. In this case the lower bound is in fact so tight that the discretization of the production levels for the single-hour model yields a slightly worse result. Thus, a production planner could benefit from planning only one hour ahead, when planning for fast ramping units with high variable costs, but we may use the exact and computationally less expensive multi-hour planning for inexpensive slow ramping units.

Tendencies are the same for the weekday and special days data sets, although the lower prices in the special days data result in reduced profits overall. The results can be found in the supplementary material, see Appendix 3.C.

Table 3.4: Unit profits for the multi-hour model and weekday data set. The profits for 5-, 15- and 30-minute time resolutions are listed along with their corresponding 60-minute benchmarks. Finally, the difference between high time resolution profit and 60-minute benchmark profit relative to the high time resolution profit is shown. Except for the relative difference all values are in AUD.

		Deterministic			Stochastic		
	Unit	Solution	60-min bench.	Diff.	Solution	60-min bench.	Diff.
5-min res.	1a	145691.71	145056.64	0.44%	147330.80	146670.85	0.45%
	1b	145858.90	145056.64	0.55%	147502.72	146670.85	0.56%
	1c	2049.61	1962.49	4.25%	3130.16	3064.79	2.09%
	1d	2190.05	1962.49	10.39%	3267.21	3064.79	6.20%
	1e	2190.05	1962.49	10.39%	3687.65	3441.69	6.67%
15-min res.	1a	145627.87	145029.56	0.41%	147270.31	146650.14	0.42%
	1b	145734.04	145029.56	0.48%	147379.42	146650.14	0.49%
	1c	1882.13	1824.03	3.09%	2969.09	2925.72	1.46%
	1d	2027.94	1852.45	8.65%	3107.60	2956.01	4.88%
	1e	2027.94	1852.45	8.65%	3530.10	3354.58	4.97%
30-min res.	1a	145456.79	144988.59	0.32%	147102.17	146617.72	0.33%
	1b	145456.79	144988.59	0.32%	147379.42	146650.14	0.49%
	1c	1619.65	1525.56	5.81%	2724.16	2636.27	3.23%
	1d	1773.80	1675.39	5.55%	2866.46	2788.90	2.71%
	1e	1773.80	1675.39	5.55%	3274.62	3219.78	1.67%

3.5.6 Precision

Since we have discretized the hourly production levels to exploit the DP solution of the stochastic 1UC problem with ramping, we briefly consider the effect of the number of production levels. We find that increasing the number of production levels from 16 to 31 generates no profit increase for unit 1a and only 0.00021% profit increase for unit 1e which would be most vulnerable to price volatility and thus have the greatest risk of having suboptimal hourly production levels. Finally, we consider an extra unit, 1f, which corresponds to 1e, but with lower minimum up- and down-time and lower start-up costs to see if a unit more susceptible to shutting down and starting up would change this picture. Here the profit increase remains insignificant at 0.00025%. We conclude that for these units the 16 production values are sufficient to appropriately represent the hourly production levels.

3.5.7 Running times

The running times of the model heavily depends on the number of states in the Markov chain and for the single-hour model the number of production levels. In general, the multi-hour model solves quickly with running times around 350 seconds for the stochastic model and less than 10 seconds for the deterministic model. Due to the increased number of states for the single-hour model the running times are higher, around 1000 seconds for the stochastic 5-minute model and 30 seconds for the deterministic model. For a multi-stage model with 24 stages this is very reasonable. The running times for the special days data set are of the same order of magnitude. For both data sets we consider 8 price bins and 16 production levels. Note that with these running times it would be possible to use the single-hour or multi-hour 1UC as the Lagrangian relaxation subproblem of a large system model, as discussed in Section 3.1, especially if the Lagrangian subproblems were run in parallel.

3.6 Conclusions and discussion

We consider the stochastic multi-stage 1UC problem with hourly updating of intra-hour electricity price information. We present two DP formulations that both make hourly binary UC decisions whereas continuous ED decisions are made in the DP subproblem, which is a convex quadratic program with higher time resolution, but differ in their assumptions regarding non-anticipativity. In our single-hour model the hourly plans are adapted to current prices. In our multi-hour model, however, plans are made for an entire online period on the basis of current prices. We show that the multi-hour model can handle ramping in an exact manner and with little computational effort, whereas the single-hour model requires discretization of the hourly production levels. However, multi-hour planning results in a substantial reduction in profits compared to single-hour planning. Moreover, the single-hour model can be solved with a sufficiently fine discretization to obtain a satisfactory computational precision within reasonable running times.

Our results show that there is a significant difference in profits between low and high time resolution cases, especially when prices are close to marginal costs and the ramping ability is high. Furthermore, the ability of intra-hour ramping is revealed with the high time resolution with profit differences of up to 2.00% which was not detectable in the low time resolution model. However, these effects are mitigated when the intra-hour time resolution is decreased to 15 and 30 minutes, confirming the importance of the high time resolution.

Future work includes the implementation of Lagrangian relaxation of the system-wide stochastic UC problem to further investigate computational tractability of our approach.

Acknowledgements

We wish to thank Andy Philpott for valuable discussions in the preliminary work on the models and for comments at the conference presentation of the problem. T. K. Boomsma and P. Pinson are both partly supported by the Danish Strategic

Council for Strategic Research through the project 5s - Future Electricity Markets (no. 12-132636/DSF).

APPENDIX

3.A Proof of Proposition 1

Lemma 1 Denote the value functions under Assumption 1 and 2 by f_{hk_h} and $\tilde{f}_{hk_h}$, respectively. Then,

$$\tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) \leq f_{hk_h}(\mathbf{p}_{hk_h}).$$

Proof: Let $\mathbf{q}_h = (q_t)_{t=1}^{T_h}$, and let

$$\begin{aligned} \tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) = \max \ & \mathbb{E}\Big[\sum_{k=h}^{k_h}\sum_{t=1}^{T_k}(p_t q_t - c(q_t))\Big|\mathbf{p}_h\Big] \\ \text{st } & q^{min} \leq q_t \leq q^{max}, \ \ t \in T_k, k = h, \ldots, k_h \\ & \mathbf{q}_k \text{ is } \mathcal{F}_h\text{-measurable for } k = h, \ldots, k_h. \end{aligned}$$

Due to the stronger non-anticipativity constraints, the multi-hour ED problem provides a lower bound to the multi-hour ED problem. Under Assumption 2, this becomes

$$\begin{aligned} \tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) = \max \ & \sum_{k=h}^{k_h}\sum_{t=1}^{T_k}(\mathbb{E}[p_t|\mathbf{p}_h]q_t - c(q_t)) \\ \text{st } & q^{min} \leq q_t \leq q^{max}, \ \ t \in T_k, k = h, \ldots, k_h \\ & \mathbf{q}_k \text{ is } \mathcal{F}_h\text{-measurable for } k = h, \ldots, k_h, \end{aligned}$$

which is a deterministic problem.

Proposition 1 Denote the value functions under Assumption 1 and 2 by F_h^j and $\tilde{F}_h^j$, respectively. Then,

$$\tilde{F}_h^j(\mathbf{p}_h) \leq F_h^j(\mathbf{p}_h), j = 0, 1.$$

Proof: For $k = h, \ldots, k_h$, the constraint $\mathbf{u}_k$ is $\mathcal{F}_k$-measurable in the single-hour UC problem is replaced by $\mathbf{u}_k$ is $\mathcal{F}_h$-measurable in the multi-hour UC problem, which is a restriction of the problem since $\mathcal{F}_h \subseteq \mathcal{F}_k$. By definition, $u_h = u_{h+1} = \cdots = u_k \neq u_{k+1}$ if and only if $k_h = k$. Combining this with Lemma 1, we obtain the desired result.

3.B Preprocessing of the price data Supplementary material to *A dynamic programming approach to the ramp-constrained intra-hour stochastic single-unit commitment problem*

Outliers are removed from a data set on the basis of the histogram for the log transformed data. The price data is log transformed via the formula

$$\tilde{p} = \ln(\varepsilon + p), \tag{3.10}$$

in which we assume $\varepsilon = 1$.

For the special day data set the histogram can be seen in Figure 3.B.1. The histogram looks very similar for the week day data set.

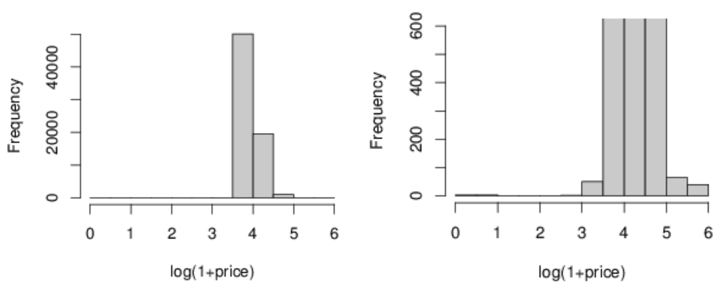


Figure 3.B.1: The histogram of the log transformed price data in New South Wales for the special day data set. The figure to the left is a closer look at the histogram, i.e. with lower values on the y-axis.

We decide to consider all values above $p = 150$, i.e. approximately 5 in the histogram, as outliers and remove them from the data set before using it to determine the Markov chain parameters.

3.C Results for the special day data set Supplementary material to *A dynamic programming approach to the ramp-constrained intra-hour stochastic single-unit commitment problem*

Clearly, the same patterns apply for the special day data set as for the week day data set, except that prices are generally lower resulting in overall lower profits.

In fact, the price is sufficiently low for the expensive units, 1c, 1d and 1e, to have negative profits in some of the deterministic special day cases for both models.

Table 3.C.1: Unit profits for the single-hour model and the special day data set. The profits for 5-, 15- and 30-minute time resolutions are listed along with their corresponding 60-minute benchmarks. Finally, the difference between high time resolution profit and 60-minute benchmark profit relative to the high time resolution profit is shown. Except for the relative difference all values are in AUD.

		Deterministic			Stochastic		
	Unit	Solution	60-min bench.	Diff.	Solution	60-min bench.	Diff.
5 min res.	1a	138161.73	137539.09	0.45%	139826.09	139175.91	0.46%
	1b	138325.60	137539.09	0.57%	139995.17	139175.91	0.59%
	1c	-400.68	-443.18	10.60%	5139.67	5101.37	0.75%
	1d	-380.23	-443.18	16.56%	5233.56	5101.37	2.53%
	1e	-113.38	-364.41	221.41%	5307.35	5175.16	2.49%
15 min res.	1a	138096.10	137509.58	0.42%	139763.23	139152.27	0.44%
	1b	138200.05	137509.58	0.50%	139870.76	139152.27	0.51%
	1c	-388.71	-430.80	10.83%	5083.76	5042.90	0.80%
	1d	-369.88	-430.80	16.47%	5177.93	5078.69	1.92%
	1e	-185.16	-406.53	119.55%	5251.68	5151.08	1.92%
30 min res.	1a	137922.80	137463.52	0.33%	139593.52	139116.92	0.34%
	1b	137922.80	137463.52	0.33%	139593.52	139116.92	0.34%
	1c	-370.34	-410.73	10.91%	4986.24	4898.00	1.77%
	1d	-370.34	-410.73	10.91%	5073.37	5041.86	0.62%
	1e	-336.77	-410.73	21.96%	5151.95	5112.73	0.76%

Table 3.C.2: Unit profits for the multi-hour model and the special day data set. The profits for 5-, 15- and 30-minute time resolutions are listed along with their corresponding 60-minute benchmarks. Finally, the difference between high time resolution profit and 60-minute benchmark profit relative to the high time resolution profit is shown. Except for the relative difference all values are in AUD.

		Deterministic			Stochastic		
	Unit	Solution min	60-min bench.	Diff.	Solution	60-min bench.	Diff.
5 min res.	1a	138161.73	137539.09	0.45%	138555.64	137908.20	0.47%
	1b	138325.60	137539.09	0.57%	138726.52	137908.20	0.59%
	1c	-445.31	-607.07	36.32%	469.49	415.92	11.41%
	1d	-445.31	-607.07	36.32%	575.45	415.92	27.72%
	1e	-445.31	-607.07	36.32%	708.10	540.49	23.67%
15 min res.	1a	138096.10	137509.58	0.42%	138494.59	137884.25	0.44%
	1b	138200.05	137509.58	0.50%	138603.14	137884.25	0.52%
	1c	-600.71	-664.84	10.68%	397.90	356.56	10.39%
	1d	-508.21	-649.83	27.87%	503.48	384.00	23.73%
	1e	-508.21	-649.83	27.87%	633.52	509.58	19.56%
30 min res.	1a	137922.80	137463.52	0.33%	138325.89	137848.14	0.35%
	1b	137922.80	137463.52	0.33%	138325.89	137848.14	0.35%
	1c	-693.80	-780.09	12.44%	280.62	219.17	21.90%
	1d	-635.94	-712.62	12.06%	372.38	341.85	8.20%
	1e	1773.80	1675.39	5.55%	502.88	471.77	6.19%

– 4 –

Open- and closed-loop equilibrium models for the day-ahead and balancing markets

Ditte M. Heide-Jørgensen, Salvador Pineda and Trine K. Boomsma,
Department of Mathematical Sciences, University of Copenhagen

Abstract The increasing penetration of inflexible and uncertain renewable energy generation is often accompanied by a sequential market setup with a day-ahead market that serves to balance supply and demand with an hourly time resolution and a balancing market in which flexible generation handles imbalances closer to real-time and with a higher time resolution. Different market characteristics, such as time resolution, the time of market offering and the information available at this time, price elasticities of demand and the number of market participants, allow producers to exercise market power to different degrees. In view of this, we consider a joint oligopolistic market setup of the day-ahead and balancing markets with Cournot competition. To represent uncertain production from renewables, we assume a linear inverse demand function with a stochastic intercept. We consider two equilibrium models for the sequential markets. The first is the open-loop model of the problem which can be solved straightforwardly via the Karush Kuhn Tucker conditions. The second is a more realistic, but computationally harder, closed-loop model, that results in an equilibrium problem with equilibrium constraints which we solve by diagonalization. We compare the open- and closed-loop models with special emphasis on the potential of exercising market power, and use the closed-loop model to further analyse the case in which access to the balancing market is limited to units with a high degree of flexibility. Contrary to similar models of other sequential electricity markets our results show that the open- and closed-loop solutions are in general not the same and, surprisingly, it is not always beneficial for a producer in a duopoly to enter the balancing market compared to the producer that can only participate in the day-ahead market. Finally, our case studies show that a higher time resolution in the balancing market reveals profit potential for the power producers.

KEYWORDS *market power; stochastic programming; electricity markets; complementarity modelling; sequential market modelling*

4.1 Introduction

Electricity systems today are challenged by a growing penetration of renewables. In many countries, the inflexibility, intra-hour variability and uncertainty of renewable generation such as wind power is handled in a market setup with a day-ahead spot market and a real-time balancing market. The spot market serves to balance supply and demand with an hourly time resolution. Since market offering is a day ahead of operation, however, deviations from forecasted net demand, mainly due to renewable production, can be substantial. The purpose of the balancing market is to determine the optimal redispatch of generating units to balance unexpected mismatches between production and consumption. Offering is closer to real-time, e.g. an hour ahead of operation, and therefore the deviations from the net demand forecast made at this point in time are much smaller than for the day-ahead forecasts. Furthermore, the higher time resolution in this market allows for adjusting sufficiently flexible production when e.g. wind power varies substantially within the hour. Finally, the two markets may differ in terms of price elasticities of demand. These different market characteristics allow producers to exercise market power to different degrees.

In this paper, we study the characteristics of electricity markets with a high penetration of renewables, with special emphasis on the modelling of sequential day-ahead and balancing markets, in which producers may exercise market power. We consider a joint oligopolistic market setup with Cournot competition and assume linear inverse demand functions. The day-ahead forecast of renewable production is known, whereas uncertainty in realised production is represented by a stochastic intercept of the inverse demand function. We consider two equilibrium models for the sequential markets. The first is an open-loop model of the problem in which day-ahead and balancing decisions are made simultaneously and which can be solved straightforwardly via the Karush Kuhn Tucker conditions. The second is based on sequential decision making and more closely resembles actual practise by power producers. It is a computationally harder closed-loop model that results in an equilibrium problem with equilibrium constraints and can solved by e.g. diagonalization.

A review of modelling trends for sequential markets represented as equilibrium models is found in Ventosa et al. (2005). The authors highlight two different approaches: the Cournot based models and the supply function equilibrium (SFE) models. The SFE electricity market approach, which was introduced in Klemperer and Meyer (1989), is occasionally argued to create better models as players submit a supply function, i.e. a set of volumes and prices at which the player is willing to sell the volumes. The SFE approach, however, is computationally challenging to a degree that limits the representation of electricity system characteristics, and Cournot models are often preferred, although offers consist of quantities only, Ventosa et al. (2005). In other words, Cournot competition assumes that market participants submit the volume they wish to sell or buy and the market settles the price according to an inverse demand function, see Allaz (1992). Compared to offering prices only, this is a reasonable assumption for electricity markets as power producers have a maximum capacity, which must be respected even if the electricity price varies, and they are often producing close to this limit. Consumers may be modelled as explicit players in the market, Kazempour and Zareipour (2014), but are typically modelled implicitly via an inverse demand curve that depends on the electricity price and specifies how consumers are willing to adapt consumption to the price, see Allaz (1992); Gülpınar and Oliveira (2014) amongst others. Besides pure Cournot models, similar models include Stackelberg competition models, e.g. Hu and Ralph (2007), which likewise assume competition in quantities, but with a leader (or multiple leaders) in the market and followers, who know the decision of the leader before they determine their offers by e.g. competing in Cournot fashion between themselves. Some work has been done on more general conjectural variations models in which solutions apply to several kinds of competition determined by a specific parameter, see Song et al. (2003); Wogrin et al. (2013). These models have been argued to better reflect offering decisions, while maintaining computational tractability, as the Cournot models, Ventosa et al. (2005). In spite of this, we focus on Cournot competition and leave the extension to general conjectural variations as future research.

Several interesting electricity market problems can be formulated as two-stage stochastic market equilibrium models. Examples include the sequential spot and forward markets considered by Allaz (1992) and the capacity expansion models with capacity decisions in the first stage and subsequent spot market dispatch in the second stage by Wogrin et al. (2013). Many references account for uncertainty in net demand, assuming that the first and second stages differ by the information available for decision making, i.e. by whether uncertainty has been realized or not. If decisions in the first and second stages are made simultaneously, these are referred to as open-loop models, see Gülpınar and Oliveira (2014) for an example. These models are usually computationally tractable since they can be solved by use of the Karush Kuhn Tucker (KKT) optimality conditions. If, in contrast, decisions are made sequentially in time, the result is a so-called closed-loop model, which can be found in Allaz (1992); Hu and Ralph (2007); Zhang et al. (2010); Twomey and Neuhoff (2010); Shanbhag et al. (2011); Wogrin et al. (2013); Ito and Reguant (2015). Typically, the equilibrium of the second market is represented by the set of optimality conditions which is then used in the optimization problem for each of the players in the equilibrium of the first market. Thus, the players anticipate optimal decision making in the second market when participating in the first, but cannot take advantage of the first market when making decisions in the second. The problem of finding the Nash equilibrium of the first market results in an equilibrium model with equilibrium constraints. With Cournot competition in both markets this can be seen as a Stackelberg game with multiple leaders (the participants in the first market), and multiple followers (the participants in the second market), and the equilibrium is sometimes referred to as a subgame perfect (Nash) equilibrium, see Hu and Ralph (2007). We will consider this type of equilibrium in the sequential market setup for the day-ahead and balancing markets.

In some cases the solutions to open- and closed-loop problems are the same, and hence the computationally tractable open-loop model can be used. This is, however, not true in general, when the time resolution is not the same in the two markets or equivalently if the inverse demand function for the second market is stochastic, see Wogrin et al. (2013). For day-ahead and balancing market sequential market we find that even when time resolution is the same in the two markets, it is important whether solutions are made simultaneously or sequentially, i.e. that solutions of the open-loop and closed-loop model differ. In this case, the closed-loop problem must be solved by approximate methods, e.g. relying on diagonalization.

4.2 Previous work

One of the first two-stage stochastic sequential market models for electricity markets is found in Allaz (1992), who models the forward and day-ahead market in a duopolistic closed-loop setting. More recently, Hu and Ralph (2007) formulated a bilevel optimisation model formulated as a closed-loop model, for the forward and spot market as a multi-leader multi-follower Stackelberg game and has shown existence of the equilibrium. Moreover, a two-stage duopolistic closed-loop model for the forward and spot market is used by Twomey and Neuhoff (2010) to investigate how market power affects the profits from renewable generation. Another two-stage, but oligopolistic and stochastic closed-loop model for the forward and day-ahead market is presented in Zhang et al. (2010). The authors of Shanbhag et al. (2011) likewise propose a closed-loop model for the forward and spot market. The authors prove existence and uniqueness of an equilibrium for the resulting EPEC and present a globally convergent, scalable algorithm to find it. Furthermore, they prove that the open-loop and closed-loop solutions are the same for their problem. The comparison of open-loop and closed-loop equilibria for the capacity expansion problem is studied in Wogrin et al. (2013). The authors derive some assumptions under which the open- and closed-loop solutions are the same, but also have examples of multi-period problems in which solutions may differ.

So far, there has only been little interest in stochastic and sequential day-ahead and balancing market models. Exceptions include Kazempour and Zareipour (2014), who presents a variation of the SFE model, a step-wise offer curve model, for the day-ahead and balancing markets. Two different SFE models for the New Zealand electricity markets are studied in Khazaei et al. (2014). One of these models resembles the electricity markets of New Zealand with a day-ahead/pre-dispatch market and a subsequent balancing market. In the other model suppliers bid both a supply function and linear deviation cost at which they are willing to change production in the balancing market. They prove that the solutions are equivalent, but that the second model yields higher social welfare than the first. To derive analytical solutions, the authors assume symmetry of the producers and disregard transmission constraints. Finally, Ito and Reguant (2015) formulate a closed-loop two-stage Cournot model for the forward (or day-ahead) and real-time markets. The market setup includes a monopolist and a number of fringe suppliers and look and study their behaviour in both markets under different arbitrage assumptions and how this affects the social welfare.

In this paper we likewise consider the sequential day-ahead and balancing markets. This setup differs from the forward and spot market models and the capacity expansion models by having two demand functions and by the inverse demand function in the balancing market depending on total production and thereby variables from both markets. The other problems only include the variables from the second market. Finally, the time resolution in the day-ahead market is lower than in the balancing market. For the forward-spot models the time resolution is the same in both markets, whereas this is not necessarily the case for the capacity expansion models. Our time resolutions result in several inverse demand functions for the balancing market for a given hour, but only one for the day-ahead market. Equivalently, a stochastic setup involves several intercepts for the inverse demand function of the balancing market, but only one for the day-ahead market. The difference in time resolution and/or uncertainty excludes the results from the literature on open- and closed-loop equivalence.

The different characteristics of the day-ahead and balancing markets allow producers to exercise market power to different degrees. We investigate the impact of different time resolutions, the time of market offering and the information available at this time and price elasticities of demand on the open and closed loop solutions and the difference between them. Furthermore, we consider two special cases: In the first case, we study limited access to the balancing market, and discuss whether balancing market access is an advantage for a power producer or not. In the second case, we consider the effects of excluding arbitrage, in line with some branches of the literature, e.g. Shanbhag et al. (2011).

The paper is structured as follows. We go through the open- and closed-loop models in Section 4.3. Section 4.4 extends to the case in which arbitrage is handled explicitly. Numerical results can be found in Section 4.5 including studies of differences between open-loop and closed-loop solutions, profit gain from intra-hour time resolution and reflections on market power, and finally conclusions are drawn in Section 4.6.

4.3 Modelling the day-ahead and balancing markets

In the day-ahead market, every producer $i \in I$ solves a profit maximisation problem to determine its production, x_{ih}, in hour $h \in H$ subject to its capacity limit, $x_i^{\max}$. We assume that day-ahead demand is known but flexible and responds to the market price according to the following affine inverse demand function

$$p_h^{\text{da}} := p_h^{\text{da}}(d_h^{\text{da}}) = \beta_h^{\text{da}} - \alpha_h^{\text{da}} d_h^{\text{da}},$$

where $\alpha_h^{\text{da}} > 0$ is the demand elasticity of price, $\beta_h^{\text{da}} > 0$ is the intercept corresponding to the inelastic price and d_h^{da} is the day-ahead demand in hour $h \in H$. Note that we may, without loss of generality, consider this a relation between market price and net demand, i.e. the difference between demand and inelastic generation (e.g. intermittent renewable production), by accounting for the latter in the intercept. This could result in negative prices for some time periods, but as this only happens in less than 0.5% of the time[1], we will not go further into this here. We present the problem with the possibility to include several hours, $h \in H$, for completeness, since the day-ahead market is usually cleared for 24 hours at the time, even though we leave intertemporal constraints like ramping for future work.

A producer determines its production in the balancing market in a similar fashion as in the day-ahead market. However, the balancing market has a higher time resolution with intra-hour time intervals $t \in T$, where $|T| \geq |H|$ and every interval belongs to a given hour, i.e. $t \in T_h$ for $h \in H$. In this market producers may offer both upwards balancing power $x_{it\omega}^+$ (increase of day-ahead scheduled supply, x_{ih}) or downwards balancing power $x_{it\omega}^-$ (decrease of supply), also known as up-regulation and down-regulation, respectively. Up-regulation is subject to the remaining availability of capacity following day-ahead market clearing, $(x_i^{\max} - x_{ih})\tau_h$, where $\tau_h = \frac{1}{|T_H|}$. Down-regulation is limited by the day-ahead production corresponding to the current intra-hour time period, $x_{ih}\tau_h$, assuming that hourly day-ahead production is evenly distributed across the intra-hour time

[1]The numbers are from Denmark 2013, see the Nordic Market Report 2014: http://www.nordicenergyregulators.org/wp-content/uploads/2014/06/Nordic-Market-Report-2014.pdf

intervals. We assume that day-ahead quantities are determined facing an uncertain demand, while balancing quantities are made once the uncertainty is disclosed. More specifically, we assume that the intercept β_t^{bal} is a random variable on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Furthermore, we assume a discrete distribution of this random variable and represent its realizations by the so-called scenarios $\beta_{t\omega}^{\text{bal}}$ for $\omega \in \Omega$. A stochastic intercept could for example be justified by inelastic and intermittent renewable production, which would result in shifts of the demand function according to the scenarios. The inverse demand function in the balancing market is given by the affine function

$$p_{t\omega}^{\text{bal}} := p_{t\omega}^{\text{bal}}(\tau_h d_h^{\text{da}}, d_{t\omega}^{\text{bal}}) = \beta_{t\omega}^{\text{bal}} - \alpha_t^{\text{balx}} \tau_h d_h^{\text{da}} - \alpha_t^{\text{bal}} d_{t\omega}^{\text{bal}}$$

where $\beta_{t\omega}^{\text{bal}} > 0$ denotes intercept, i.e. the inelastic price in the balancing market, $\alpha_t^{\text{bal}} > 0$ denotes the balancing market demand elasticity of the balancing market price, $\alpha_t^{\text{balx}} > 0$ denotes the day-ahead market demand elasticity of the price and $d_{t\omega}^{\text{bal}}$ is the intra-hour net demand in time interval $t \in T$ and scenario $\omega \in \Omega$.

Production costs in an intra-hour time interval are given by the convex quadratic function

$$\begin{aligned} c_i\left(\tau_h x_{ih}, x_{it\omega}^+, x_{it\omega}^-\right) =& a_i\left(\tau_h x_{ih} + x_{it\omega}^+ - x_{it\omega}^-\right)^2 + b_i\left(\tau_h x_{ih} + x_{it\omega}^+ - x_{it\omega}^-\right) \\ &+ \gamma^+ x_{it\omega}^+ + \gamma^- x_{it\omega}^-, \ \ i \in I, \ \ t \in T_h, \ \ h \in H, \end{aligned}$$

for $a_i \geq 0$ and with $\gamma^+ \geq 0$ and $\gamma^- \geq 0$ representing the additional costs of stress imposed on generating units that provide balancing services.

For every producer that engages in both the day-ahead and balancing markets, the profit maximisation problem is a two-stage stochastic program, with the first stage representing the day-ahead market and the second stage representing the balancing market. In particular, a producer offers day-ahead production prior to the realization of uncertainty in net demand, whereas up-regulation and down-regulation are offered in response to it.

When producers compete, the day-ahead equilibrium problem consists of the profit maximisation problem of each producer in the market and a market clearing condition to balance supply and demand. The latter consists of the inverse demand functions and the matching of supply and demand

$$d_h^{\text{da}} = \sum_{i \in I} x_{ih}^{\text{da}}, \ \ d_{t\omega}^{\text{bal}} = \sum_{i \in I}\left(x_{it\omega}^+ - x_{it\omega}^-\right).$$

4.3.1 Open-loop

We start by formulating the problem as an open-loop problem, assuming that the producers make their day-ahead and balancing market decisions simultaneously.

The equilibrium problem for the day-ahead and balancing markets is then the two-stage stochastic problem

$$\max \left\{ \sum_{h\in H} p_h^{\text{da}} x_{ih} + \sum_{\omega\in\Omega} \pi_\omega \sum_{h\in H} \sum_{t\in T_h} \left(p_{t\omega}^{\text{bal}}(x_{it\omega}^+ - x_{it\omega}^-) - c_i\left(\tau_h x_{ih}, x_{it\omega}^+, x_{it\omega}^-\right)\right) : \right.$$

$$0 \overset{(\underline{\sigma}_{ih})}{\leq} x_{ih} \overset{(\bar{\sigma}_{ih})}{\leq} x_i^{\max},\ h\in H$$

$$0 \overset{(\underline{\sigma}_{it\omega}^+)}{\leq} x_{it\omega}^+ \overset{(\bar{\sigma}_{it\omega}^+)}{\leq} \tau_h(x_i^{\max} - x_{ih}),\ t\in T_h,\ h\in H,\ \omega\in\Omega$$

$$\left. 0 \overset{(\underline{\sigma}_{it\omega}^-)}{\leq} x_{it\omega}^- \overset{(\bar{\sigma}_{it\omega}^-)}{\leq} \tau_h x_{ih},\ t\in T_h,\ h\in H,\ \omega\in\Omega \right\},\ i\in I$$

$$p_h^{\text{da}} = \beta_h^{\text{da}} - \alpha_h^{\text{da}} d_h^{\text{da}},\ h\in H$$

$$p_{t\omega}^{\text{bal}} = \beta_{t\omega}^{\text{bal}} - \alpha_t^{\text{balx}} \tau_h d_h^{\text{da}} - \alpha_t^{\text{bal}} d_t^{\text{bal}},\ t\in T_h,\ h\in H,\ \omega\in\Omega$$

$$d_h^{\text{da}} = \sum_{i\in I} x_{ih},\ h\in H$$

$$d_{t\omega}^{\text{bal}} = \sum_{i\in I}\left(x_{it\omega}^+ - x_{it\omega}^-\right),\ t\in T_h,\ h\in H,\ \omega\in\Omega,$$

where $\underline{\sigma}_{ih}, \bar{\sigma}_{ih}$ etc. denote the dual variables of the corresponding constraints.

It should be remarked that the objective function is concave under certain (reasonable) assumptions on the inverse demand function parameters and the cost function, see Appendix 4.D. Under these circumstances we replace the problem by the necessary and sufficient KKT conditions (4.1a)-(4.1j). Along with the market clearing conditions (4.1k)-(4.1n), this results in a mixed complementarity formulation of the equilibrium problem

$$\frac{\partial p_h^{\text{da}}}{\partial x_{ih}} x_{ih} + p_h^{\text{da}} + \sum_{\omega\in\Omega}\sum_{t\in T_h} \pi_\omega \left(\frac{\partial p_{t\omega}^{\text{bal}}}{\partial x_{ih}} (x_{it\omega}^+ - x_{it\omega}^-) - \frac{\partial c_i}{\partial x_{ih}} \right) \tag{4.1a}$$

$$+\underline{\sigma}_{it} - \bar{\sigma}_{it} - \tau_h \sum_{\omega\in\Omega}\sum_{t\in T_h} (\bar{\sigma}_{it\omega}^+ - \bar{\sigma}_{it\omega}^-) = 0,\ i\in I,\ h\in H \tag{4.1b}$$

$$\pi_\omega \left(\frac{\partial p_{t\omega}^{\text{bal}}}{\partial x_{it\omega}^+} (x_{it\omega}^+ - x_{it\omega}^-) + p_{t\omega}^{\text{bal}} - \frac{\partial c_i}{\partial x_{it\omega}^+} \right) - \bar{\sigma}_{it\omega}^+ + \underline{\sigma}_{it\omega}^+ = 0,\ (*) \tag{4.1c}$$

$$\pi_\omega \left(\frac{\partial p_{t\omega}^{\text{bal}}}{\partial x_{it\omega}^-} (x_{it\omega}^+ - x_{it\omega}^-) - p_{t\omega}^{\text{bal}} - \frac{\partial c_i}{\partial x_{it\omega}^-} \right) - \bar{\sigma}_{it\omega}^- + \underline{\sigma}_{it\omega}^- = 0,\ (*) \tag{4.1d}$$

$$0 \leq x_{ih} \perp \underline{\sigma}_{ih} \geq 0,\ h\in H,\ i\in I \tag{4.1e}$$

$$0 \leq x_i^{\max} - x_{ih} \perp \bar{\sigma}_{ih} \geq 0,\ h\in H,\ i\in I \tag{4.1f}$$

$$0 \leq x_{it\omega}^+ \perp \underline{\sigma}_{it\omega}^+ \geq 0,\ (*) \tag{4.1g}$$

$$0 \leq \tau_h(x_i^{\max} - x_{ih}) - x_{it\omega}^+ \perp \bar{\sigma}_{it\omega}^+ \geq 0,\ (*) \tag{4.1h}$$

$$0 \leq x_{it\omega}^- \perp \underline{\sigma}_{it\omega}^- \geq 0,\ (*) \tag{4.1i}$$

$$0 \leq \tau_h x_{ih} - x^-_{it\omega} \perp \bar{\sigma}^-_{it\omega} \geq 0, \ \ (*) \tag{4.1j}$$

$$p^{\text{da}}_h = \beta^{\text{da}}_h - \alpha^{\text{da}}_h d^{\text{da}}_h, \ \ h \in H \tag{4.1k}$$

$$p^{\text{bal}}_{t\omega} = \beta^{\text{bal}}_{t\omega} - \alpha^{\text{balx}}_t \tau_h d^{\text{da}}_h - \alpha^{\text{bal}}_t d^{\text{bal}}_t, \ \ t \in T_h, \ \ h \in H, \ \ \omega \in \Omega \tag{4.1l}$$

$$d^{\text{da}}_h = \sum_{i \in I} x_{ih}, \ \ h \in H \tag{4.1m}$$

$$d^{\text{bal}}_{t\omega} = \sum_{i \in I} \left(x^+_{it\omega} - x^-_{it\omega} \right), \ \ t \in T_h, \ \ h \in H, \ \ \omega \in \Omega, \tag{4.1n}$$

where the $(*)$ conditions are for $i \in I, \ t \in T_h, \ h \in H, \ \omega \in \Omega$. We refer to 4.B for the partial derivatives. The mixed complementarity formulation can be solved with a standard mixed complementarity solver.

4.3.2 Closed-loop

We proceed by formulating the closed-loop problem, in which producers make their day-ahead decisions prior to making balancing market decisions. Hence, day-ahead market decisions are made in an upper-level problem, while anticipating the profit from optimal offering in the balancing market. In contrast, balancing market decisions are made in a lower-level problem in response to the offers and the corresponding profit in the day-ahead market. Whereas this increases model complexity, it is current practice of many power producers in reality. In Denmark, for example, balancing offers are accepted up to 15 minutes before each hour and therefore, power producers prefer to postpone their offers until some of the uncertainty is revealed.

Lower-level

At the time of making balancing market decisions, the day-ahead production, x_{ih}, is known. Now, the lower-level problem consists of the equilibrium of the balancing market

$$\max_{x^+_{it\omega} x^-_{it\omega}} \left\{ \sum_{h \in H} \sum_{t \in T_h} \left(p^{\text{bal}}_{t\omega} (x^+_{it\omega} - x^-_{it\omega}) - c_i \left(\tau_h x_{ih}, x^+_{it\omega}, x^-_{it\omega} \right) \right) : \right. \tag{4.2a}$$

$$\left. \begin{aligned} 0 \overset{(\underline{\sigma}^+_{it\omega})}{\leq} x^+_{it\omega} \overset{(\bar{\sigma}^+_{it\omega})}{\leq} \tau_h (x^{\max}_i - x_{ih}), \ \ t \in T_h, \ \ h \in H, \ \ \omega \in \Omega \quad (4.2b) \\ 0 \overset{(\underline{\sigma}^-_{it\omega})}{\leq} x^-_{it\omega} \overset{(\bar{\sigma}^-_{it\omega})}{\leq} \tau_h x_{ih}, \ \ t \in T_h, \ \ h \in H, \ \ \omega \in \Omega \end{aligned} \right\}, \ \ i \in I, \tag{4.2c}$$

$$p^{\text{bal}}_{t\omega} = \beta^{\text{bal}}_{t\omega} - \alpha^{\text{balx}}_t \tau_h d^{\text{da}}_h - \alpha^{\text{bal}}_t d^{\text{bal}}_{t\omega}, \ \ t \in T_h, \ \ h \in H, \ \ \omega \in \Omega \tag{4.2d}$$

$$d^{\text{da}}_h = \sum_{i \in I} x_{ih}, \ \ h \in H \tag{4.2e}$$

$$d^{\text{bal}}_{t\omega} = \sum_{i \in I} \left(x^+_{it\omega} - x^-_{it\omega} \right), \ \ t \in T_h, \ \ h \in H, \ \ \omega \in \Omega. \tag{4.2f}$$

The nested profit maximisation problem of each producer is concave, which can be proved similarly as for the open-loop problem above. Thus, (4.2a)-(4.2c) can be replaced by the necessary and sufficient KKT conditions (4.3a)-(4.3f). Along with the market clearing conditions (4.3g)-(4.3i) these make up the lower-level balancing market equilibrium problem as follows

$$\frac{\partial p^{\text{bal}}_{t\omega}}{\partial x^{+}_{it\omega}}(x^{+}_{it\omega} - x^{-}_{it\omega}) + p^{\text{bal}}_{t\omega} - \frac{\partial c_i}{\partial x^{+}_{it\omega}} - \bar{\sigma}^{+}_{it\omega} + \underline{\sigma}^{+}_{it\omega} = 0, \quad (*) \tag{4.3a}$$

$$\frac{\partial p^{\text{bal}}_{t\omega}}{\partial x^{-}_{it\omega}}(x^{+}_{it\omega} - x^{-}_{it\omega}) - p^{\text{bal}}_{t\omega} - \frac{\partial c_i}{\partial x^{-}_{it\omega}} - \bar{\sigma}^{-}_{it\omega} + \underline{\sigma}^{-}_{it\omega} = 0, \quad (*) \tag{4.3b}$$

$$0 \leq x^{+}_{it\omega} \perp \underline{\sigma}^{+}_{it\omega} \geq 0, \quad (*) \tag{4.3c}$$

$$0 \leq \tau_h(x^{\max}_i - x_{ih}) - x^{+}_{it\omega} \perp \bar{\sigma}^{+}_{it\omega} \geq 0, \quad (*) \tag{4.3d}$$

$$0 \leq x^{-}_{it\omega} \perp \underline{\sigma}^{-}_{it\omega} \geq 0, \quad (*) \tag{4.3e}$$

$$0 \leq \tau_h x_{ih} - x^{-}_{it\omega} \perp \bar{\sigma}^{-}_{it\omega} \geq 0, \quad (*) \tag{4.3f}$$

$$p^{\text{bal}}_{t\omega} = \beta^{\text{bal}}_{t\omega} - \alpha^{\text{balx}}_{t}\tau_h d^{\text{da}}_h - \alpha^{\text{bal}}_t d^{\text{bal}}_{t\omega}, \quad t \in T_h, \quad h \in H, \quad \omega \in \Omega \tag{4.3g}$$

$$d^{\text{da}}_h = \sum_{i \in I} x_{ih}, \quad h \in H \tag{4.3h}$$

$$d^{\text{bal}}_{t\omega} = \sum_{i \in I} \left(x^{+}_{it\omega} - x^{-}_{it\omega}\right), \quad t \in T_h, \quad h \in H, \quad \omega \in \Omega, \tag{4.3i}$$

where again the $(*)$ conditions are for $i \in I, \ t \in T_h, \ h \in H, \ \omega \in \Omega$.

Upper-level

In the upper-level, the equilibrium of the producers committing to a day-ahead production schedule is found, while taking into account the equilibrium of the lower-level balancing market. The upper-level day-ahead market equilibrium problem is therefore

$$\max \left\{ \sum_{h \in H} p^{\text{da}}_h x_{ih} \right. \tag{4.4a}$$

$$+ \sum_{\omega \in \Omega} \pi_\omega \sum_{h \in H} \sum_{t \in T_h} \left(p^{\text{bal}}_{t\omega}(x^{+}_{it\omega} - x^{-}_{it\omega}) - c_i\left(\tau_h x_{ih}, x^{+}_{it\omega}, x^{-}_{it\omega}\right)\right) : \tag{4.4b}$$

$$\left. 0 \leq x_{ih} \leq x^{\max}_i, \ h \in H, \ (4.3) \right\}, \ i \in I \tag{4.4c}$$

$$p^{\text{da}}_h = \beta^{\text{da}}_h - \alpha^{\text{da}}_h \sum_{i \in I} x_{ih}, \quad h \in H. \tag{4.4d}$$

This problem is an equilibrium problem with equilibrium constraints, which is hard to solve. One approach is to use Fortuny-Amat and McCarl linearization Gabriel et al. (2013); Fortuny-Amat and McCarl (1981) of the complementarity constraints and diagonalization of the upper-level problem Gabriel et al. (2013); Hu and Ralph (2007), which is a similar as the Gauss-Seidel iteration used in Ito and Reguant (2015). The pseudo-code for the algorithm can be found in Appendix 4.A.

4.4 Excluding arbitrage

In the above formulations of sequential market clearing, a producer can profit from offering production to the day-ahead market and subsequently offer down-regulation to the balancing market in all scenarios, if the day-ahead price exceeds the intra-day price and the additional balancing costs. Such arbitrage, however, may be prohibited by the market operator. For this reason, we extend the formulation to include an arbitrageur who participates in both markets with the objective to maximise profits from buying (resp. selling) q_h in hour $h \in H$ in the day-ahead market and selling (resp. buying) $q_{t\omega}$ in the balancing market for every intra-hour time period $t \in T_h$ and every scenario, ω, in hour $h \in H$. Since the arbitrageur does not produce anything we must have that the amount bought or sold in the day-ahead market for a given hour is the same as the amount sold or bought in the balancing market for all intra-hour time intervals of the hour and for all scenarios.

The problem of the arbitrageur is

$$\max_{q_h, q_{t\omega}} \left\{ \sum_{h \in H} p_h q_h + \sum_{\omega \in \Omega} \pi_\omega \sum_{h \in H} \sum_{t \in T_h} p_{t\omega} q_{t\omega} : \right.$$
$$\left. \tau_h q_h + q_{t\omega} = 0, \;\; t \in T_h, \;\; h \in H, \;\; \omega \in \Omega \right\}$$

By isolation of $q_{t\omega}$ and substitution into the objective function we obtain

$$\max_{q_h, q_{t\omega}} \left\{ \sum_{h \in H} p_h q_h - \sum_{\omega \in \Omega} \pi_\omega \sum_{h \in H} \tau_h q_h \sum_{t \in T_h} p_{t\omega} \right\}$$

Assuming the arbitrageur is a price-taker, the KKT optimality conditions for this problem are

$$-p_h + \tau_h \sum_{\omega \in \Omega} \pi_\omega \sum_{t \in T_h} p_{t\omega} = 0, \;\; h \in H.$$

This constraint is included in the open-loop problem and in the upper-level of the closed-loop problem. Note that the constraint implies that

$$\beta_h^{\text{da}} - \alpha_h^{\text{da}} d_h^{\text{da}} = \tau_h \sum_{\omega \in \Omega} \pi_\omega \sum_{t \in T_h} \left(\beta_{t\omega}^{\text{bal}} - \alpha_t^{\text{balx}} \tau_h d_h^{\text{da}} - \alpha_t^{\text{bal}} d_{t\omega}^{\text{bal}} \right).$$

The nested profit maximising problems of the producers in the above open- and closed-loop equilibrium model formulations are always concave, when including this equation.

4.5 Results

For clarity, we consider a duopoly with two identical units with capacity $x_i^{\max} = 150$ and cost function parameters $a_i = 0, b_i = 10, \gamma^+ = 5, \gamma^- = 1$. Note, the difference in cost of up- and down-regulation eliminates the possibility of multiple solutions. The cost function parameters are chosen in accordance with the inverse demand function parameters below such that the units produce less than at their full capacity to be able to observe if market power is exercised. Since the two units are identical, results are reported for one unit only, except in the case of limited balancing market access. For model analysing purposes, the introduction of a number of scenarios (with equal probability) is equivalent to having the same number of intra-hour time periods in a deterministic model. Therefore, we assume in the following that $|\Omega| = 1$ (and omit the scenario index). Further, we assume $|H| = 1$, since we do not have any intertemporal constraints.

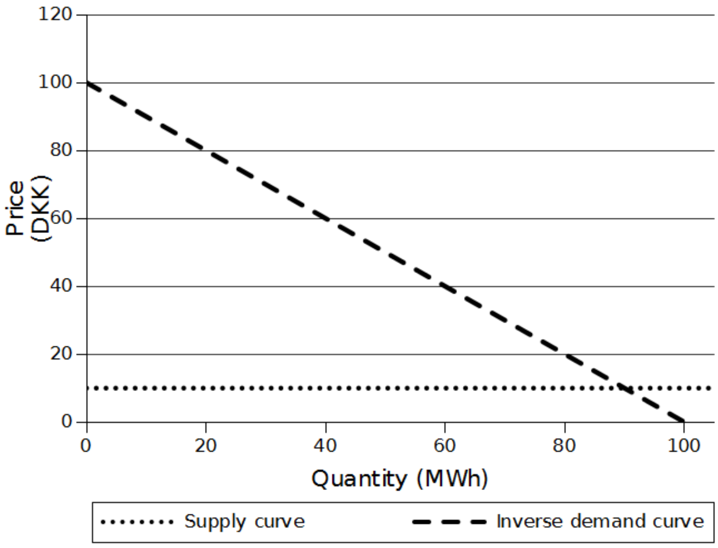
The next sections shed light on the following issues. First, in Section 4.5.1 we determine that there may be a significant profit difference between open-loop and closed-loop solutions and we determine some of the factors influencing the size of the difference. In Section 4.5.2 we establish that there is a profit gain from considering a higher time resolution in the balancing market than hourly and that differences between open-loop and closed-loop solutions also are present in a setup with high time resolution. Surprisingly, we find in Section 4.5.3 that for two producers participating in only the day-ahead market, the inclusion of one of them to the balancing market may benefit the other producer more than the one in both markets, depending on the inverse demand function parameters. Finally, for the special case in Section 4.5.4 in which there is no arbitrage, we find that open- and closed-loop solutions differ to a very high degree.

4.5.1 Hourly balancing market time resolution

In our most simple case we assume $|T| = 1$ and compare the three models: Open-loop perfect competition, open-loop Cournot and closed-loop Cournot. Note, that closed-loop perfect competition is not considered, since for perfect competition producers offer production at marginal costs obviating the need for a closed-loop formulation. For all three models we consider 5 cases. For our base case we assume that demand in the two markets is the same and reacts in the same way to price, i.e. the inverse demand function parameters for the two markets are the same. Specifically we assume $\alpha_1^{\text{da}} = \alpha_1^{\text{balx}} = \alpha_1^{\text{bal}} = 1$ and $\beta_1^{\text{bal}} = \beta_1^{\text{da}} = 100$.

Figure 4.1 shows the resulting inverse demand function for the day-ahead and balancing markets and the aggregated supply curve for the base case. To

Figure 4.1: The inverse demand function (the same for both markets) and the aggregated supply curve.



understand the parameter setting in the remaining four cases we first consider the inverse demand functions and isolate the demand in each market

$$d_h^{\mathrm{da}} = \frac{\beta_h^{\mathrm{da}}}{\alpha_h^{\mathrm{da}}} - \frac{1}{\alpha_h^{\mathrm{da}}} p_h^{\mathrm{da}}, \tag{4.5}$$

$$d_t^{\mathrm{bal}} = \frac{\beta_t^{\mathrm{bal}}}{\alpha_t^{\mathrm{bal}}} - \frac{\alpha_t^{\mathrm{balx}}}{\alpha_t^{\mathrm{bal}}} d_h^{\mathrm{da}} - \frac{1}{\alpha_h^{\mathrm{bal}}} p_t^{\mathrm{bal}}. \tag{4.6}$$

Clearly, varying the day-ahead demand elasticity of price, α_h^{da}, corresponds to varying both the inflexible day-ahead demand, $\frac{\beta_h^{\mathrm{da}}}{\alpha_h^{\mathrm{da}}}$, and the price elasticity of demand, $\frac{1}{\alpha_h^{\mathrm{da}}}$. We assume $\alpha_t^{\mathrm{balx}} = \alpha_t^{\mathrm{bal}}$ in all 5 cases. This means that day-ahead demand should be subtracted from the balancing market demand function intercept. Now, we investigate how the profit of each power producing unit is affected by

I) Inelastic demand being lower in the day-ahead market than in the balancing market, letting $\beta_h^{\mathrm{da}} = 60$. This may be the case if renewable energy production is lower than expected. Further, outages of power production units and transmission lines can also be translated to higher balancing market demand.

II) Inelastic demand being higher in the day-ahead market than in the balancing market, in two cases by letting a) $\beta_h^{\mathrm{da}} = 140$ and b) $\beta_h^{\mathrm{da}} = 180$. This

Table 4.1: The cost and demand function parameters for the single intra-hour time period case.

Case	a	b	β_1^{da}	α_1^{da}	β_1^{bal}	α_1^{balx}	α_1^{bal}
Base	0	10	100	1	100	1	1
I	0	10	60	1	100	1	1
IIa	0	10	140	1	100	1	1
IIb	0	10	180	1	100	1	1
III	0	10	100	0.8	100	1	1
IV	0	10	100	1.25	100	1	1

may be the case in a stochastic setting if renewable generation is higher than expected or if transmission from external areas is higher than planned.

III) The willingness of demand to adjust to the price being higher in the day-ahead market than in the balancing market, by letting $\alpha_h^{\text{da}} = 0.8$ and $\beta_h^{\text{da}} = 80$ corresponding to an inelastic demand of 100 (as for the base case) and price elasticity of the day-ahead demand of 1.25, see (4.5), whereas the price elasticity of demand for the balancing market is 1 (as in the base case). This is reasonable if we assume that the demand in the balancing market is unwilling to adjust to the price as would be the case with a high share of inflexible renewables.

IV) The willingness of demand to adjust to the price being lower in the day-ahead market than in the balancing market, by letting $\alpha_h^{\text{da}} = 1.25$ and $\beta_h^{\text{da}} = 125$, corresponding to an inelastic demand of 100 (as in the base case) and price elasticity of day-ahead demand of 0.8, again see 4.5, whereas it is 1 (as in the base case) for the balancing market demand/price. This is interesting in future electricity systems where flexible demand controlled by smart solutions may enter the balancing market.

Open-loop perfect competition

Table 4.2 shows the solution to the open-loop perfect competition model presented above in (4.1). For the base case up- and down-regulation is zero, confirming that the producers offer their production to the day-ahead market only, when the inverse demand functions are the same, due to additional balancing market costs.

Up- respectively down-regulation occur when the intercept of the inverse demand function for the day-ahead market is lower respectively higher than that of the balancing market, see I respectively IIa and IIb in Table 4.2. Further, there is down-regulation for case III with demand elasticity of price being lower in the day-ahead market than in the balancing market, since this will induce higher prices in the day-ahead market, unless production is increased as is the case in Table 4.2. Likewise there is up-regulation for case IV in which the day-ahead market has a lower demand elasticity of price than the balancing market. For all cases, the market price equals the marginal costs since the players are perfectly competitive: 10 DKK/MWh for the day-ahead market and 15 and 9 DKK/MWh for the balancing market when up- respectively down-regulation occurs. As expected, profits are therefore zero.

Table 4.2: Open-loop perfect competition. Varying inverse demand function parameters. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

Case	${x_{i1}}^*$	x_{i1}^{+*}	x_{i1}^{-*}	Total	p_1^{da}	p_1^{bal}	Profit
Base	45.00	0.00	0.00	45.00	10.00	10.00	0.00
I	25.00	17.50	0.00	42.50	10.00	15.00	0.00
IIa	65.00	0.00	19.50	45.50	10.00	9.00	0.00
IIb	85.00	0.00	39.50	45.50	10.00	9.00	0.00
III	56.25	0.00	10.75	45.50	10.00	9.00	0.00
IV	37.50	5.00	0.00	42.50	10.00	15.00	0.00

Open-loop Cournot

Table 4.3 shows the open-loop Cournot results. When comparing to the perfect competition case in Table 4.2, we observe that in general, when competition is imperfect, producers hold back production from the day-ahead market to increase the market price in both markets and thereby their profit, see the base case in the tables. The same behaviour is not observed in the balancing market, since production in the balancing market does not affect the price in the day-ahead market. With this increase in balancing market price, it becomes profitable to increase up- and down-regulation even though this reduces the balancing market price. The higher inelastic demand in the day-ahead market or the higher the demand elasticity of price, the higher total production and profits. In other words the higher production at equilibrium, the higher benefits of being able to exercise market power.

Closed-loop Cournot

Table 4.4 shows the solution to the closed-loop Cournot formulation in (4.4) for hourly balancing market time resolution. In the closed-loop solution, producers hold back less production in the day-ahead market than in the open-loop solution in Table 4.3. The total production of a unit is higher, and the day-ahead market price, the balancing market price and the profit are therefore lower. Thus, when the day-ahead and balancing market offers are made sequentially, the ability to exercise market power is lower. In the open-loop model, the producers can

Table 4.3: Open-loop Cournot. Varying inverse demand function parameters. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

Case	${x_{i1}}^*$	x_{i1}^{+*}	x_{i1}^{-*}	Total	p_1^{da}	p_1^{bal}	Profit
Base	26.43	10.71	0.00	37.14	47.14	25.71	1096.43
I	9.29	22.14	0.00	31.43	41.43	37.14	782.14
IIa	43.33	0.00	0.00	43.33	53.33	13.33	1877.78
IIb	59.86	0.00	9.57	50.29	60.29	-0.57	3101.57
III	24.04	12.31	0.00	36.35	41.54	27.31	909.62
IV	28.11	9.59	0.00	37.70	54.73	24.59	1349.32

benefit from deciding on production in both markets simultaneously. In other applications of sequential market models, like the forward-spot models, this may not advantageous, but in the day-ahead and balancing market setup, the demand (and thereby the production of a given unit) in the day-ahead market influences the price in the balancing market and makes it profitable to decide on the day-ahead and balancing market production simultaneously compared to the sequential decisions in the closed-loop model.

Table 4.4: Closed-loop Cournot. Varying inverse demand function parameters. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

	${x_{i1}}^*$	x_{i1}^{+*}	x_{i1}^{-*}	Total	p_1^{da}	p_1^{bal}	Profit
Base	27.83	9.78	0.00	37.61	44.35	24.78	1051.47
I	12.17	20.22	0.00	32.39	35.65	35.22	721.03
IIa	43.33	0.00	0.00	43.33	53.33	13.33	1877.78
IIb	58.61	0.00	8.74	49.87	62.78	0.26	3169.89
III	26.14	10.91	0.00	37.05	38.18	25.91	855.58
IV	29.08	8.95	0.00	38.03	52.31	23.95	1310.31

Now, we investigate when the solutions of the two models are different and how the inverse demand function parameters influence this difference. Table 4.5 shows that the relative profit difference between the open-loop and closed-loop Cournot solutions is in most cases significant, with differences above 4%. Since the difference between the open- and closed-loop models is the interaction between the day-ahead and balancing markets, the solutions are exactly the same for case IIa with $\beta_h^{\text{da}} = 140$, in which production occurs only in the day-ahead market. For the same reason, the larger the production in the balancing market (the larger the up-regulation), the larger the difference in profit. Larger balancing market production tend to occur in case I and III when the day-ahead inverse demand function intercept is lower than that of the balancing market or the day-ahead demand elasticity of price is lower (corresponding to a higher price elasticity of demand) than that of the balancing market. Finally, when increasing the day-ahead inverse demand function intercept to $\beta_h^{\text{da}} = 180$ in case IIb, down-regulation is non-zero enlarging again the differences between the profits. In a

Table 4.5: Differences between open- and closed-loop Cournot solutions compared to the open-loop solution while varying the inverse demand function parameters.

	$x_{i1}{}^*$	x_{i1}^{+*}	x_{i1}^{-*}	Total prod	p_1^{da}	p_1^{bal}	Profit
Base	-5.30%	8.68%	-	-1.27%	5.92%	3.62%	4.10%
I	31.00%	8.67%	-	3.05%	13.95%	5.17%	7.81%
IIa	0.00%	-	-	0.00%	0.00%	0.00%	0.00%
IIb	2.09%	-	8.67%	0.84%	-4.13%	-145.61%	-2.20%
III	-8.74%	11.37%	-	-1.93%	8.09%	5.13%	5.94%
IV	-3.45%	6.67%	-	-0.88%	4.42%	2.60%	2.89%

market with a lot of renewable generation, the intercepts of the inverse demand function in the day-ahead and balance market will differ and the closed-loop model is in this case clearly the most appropriate to use to analyse the market equilibrium.

4.5.2 30-minute balancing market time resolution

In the previous case studies, both the day-ahead and the balancing market are cleared once for each hour. In the results presented in this section, the day-ahead market is still cleared on an hourly basis, but the balancing market time resolution is 30 minutes. We define the parameters for the 30-minute time resolution base case such that demand corresponds to the demand from the single intra hour base case above being divided equally over the two intra-hour time periods. Hence, $\beta_t^{\text{bal}} = \beta_1^{\text{da}} = 100$, $\alpha_t^{\text{bal}} = \alpha_t^{\text{balx}} = T_1 = 2$ and $\alpha_t^{\text{da}} = 1$. To explore what happens when the market is cleared every 30 minutes we vary the balancing market parameters inverse demand function intercept β_t^{bal}, see Table 4.6.

We will concentrate on the differences between solutions with hourly and 30-minute market clearing in the balancing market and differences between open-loop and closed-loop Cournot solutions since the perfect competition case is similar to the case for hourly market clearing in the balancing market.

Table 4.6: The cost and demand function parameters for the 30 minute market clearing.

Case	a	b	β_1^{da}	α_1^{da}	β_1^{bal}	α_1^{balx}	α_1^{bal}	β_2^{bal}	α_2^{balx}	α_2^{bal}
Base	0	10	100	1	100	2	2	100	2	2
	0	10	100	1	110	2	2	90	2	2
	0	10	100	1	130	2	2	70	2	2

Open-loop Cournot

Table 4.7 shows the open-loop Cournot results for the 30-minute market clearing in the balance market. At the top the two time period base case is depicted. Here, the solution is the same as for the single time period base case in Table 4.3, only the intra-hour values are with 30-minute resolution. When intra-hour demand varies across the two time periods, however, the profit increases due to additional intra-hour flexibility (the average of the two demand functions in the 30-minute resolution case is equivalent to the hourly balancing market inverse demand function, whereas the production can adjust to the demand in each of the two time periods). When the inverse demand function intercept varies with ±10% from the average, the profit difference between the two intra-hour and the single intra-hour case differs with 1.06%, but when the intercept varies with ±30% from the average the profit differs with almost 10%. Hence, with intra-hourly variations in renewable generation, the value of flexibility clearly increases. Power producers participating in the day-ahead and balancing markets without perfect competition thus benefit from a higher time resolution in the balancing market than hourly.

Table 4.7: Open-loop Cournot. Varying β_t^{bal}. The table shows the variable values as well as the profit difference for each case compared to the single intra-hour base case. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

t	β_t^{bal}	$x_{i1}{}^*$	x_{it}^{+*}	x_{it}^{-*}	Total	p_1^{da}	p_t^{bal}	Profit	Diff
1	100.00	26.43	5.36	0.00	18.57	47.14	25.71	1096.43	0.00%
2	100.00		5.36	0.00	18.57		25.71		
1	110.00	26.43	7.02	0.00	20.24	47.14	29.05	1107.54	1.01%
2	90.00		3.69	0.00	16.90		22.38		
1	130.00	26.43	10.36	0.00	23.57	47.14	35.71	1196.43	9.12%
2	70.00		0.36	0.00	13.57		15.71		

Closed-loop Cournot

Table 4.8 shows the closed-loop Cournot results for the two time period case with varying balancing market demand throughout the hour as described in Table 4.6. When the inverse demand function intercept varies with ±10% from the average, the profit difference between the two intra-hour and the single intra-hour case differs with 1.06%, but when the intercept varies with ±30% from the average the profit differs with almost 10%. Thus also for the closed-loop model the value of flexibility increases with intra-hourly variations in renewable generation.

In Table 4.9 the open-loop Cournot and the closed-loop Cournot solutions for the two intra-hour time periods case are compared. For all the sets of β_t^{bal} values, the profit difference is about 4%, which corresponds to the differences found for

Table 4.8: Closed-loop Cournot. Varying β_t^{bal}. The table shows the variable values as well as the profit difference for each case compared to the single intra-hour base case. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

t	β_t^{bal}	$x_{i1}{}^*$	x_{it}^{+*}	x_{it}^{-*}	Total	p_1^{da}	p_t^{bal}	Profit	Diff
1	100.00	27.83	4.89	0.00	18.80	44.35	24.78	1051.47	0.00%
2	100.00		4.89	0.00	18.80		24.78		
1	110.00	27.83	6.56	0.00	20.47	44.35	28.12	1062.58	1.06%
2	90.00		3.22	0.00	17.14		21.45		
1	130.00	27.80	9.90	0.00	23.80	44.40	34.80	1152.34	9.59%
2	70.00		0.00	0.00	13.90		14.40		

the single intra-hour time period cases. Hence, the difference between the open-loop and closed-loop solutions is significant regardless of the value of the inverse demand intercept in the balancing market for the high time resolution market clearing in the balancing market. Thus the closed-loop model is also the appropriate choice in a setting with a high time resolution in the balancing market.

Table 4.9: Differences between open- and closed-loop solutions varying β_t^{bal} for 30-minute balancing market time resolution.

t	β_t^{bal}	$x_{i1}{}^*$	x_{it}^{+*}	x_{it}^{-*}	Total prod	p_1^{da}	p_t^{bal}	Profit
1	100	-5.30%	8.77%	-	-1.24%	5.92%	3.62%	4.10%
2	100		8.77%	-	-1.24%		3.62%	
1	110	-5.30%	6.55%	-	-1.14%	5.92%	3.20%	4.06%
2	90		12.74%	-	-1.42%		4.16%	
1	130	-5.18%	4.44%	-	-0.98%	5.81%	2.55%	3.69%
2	70		100.00%	-	-2.43%	-	8.34%	

4.5.3 Limited access to the balancing market

Some power producing units do not have sufficient ramping ability to participate in the balancing market, and hence, the balancing market typically consists of fewer market players. This obviously has an effect on the market equilibrium and not least the ability to exercise market power, and we therefore consider a case in which unit 1 has access only to the day-ahead market, whereas unit 2 has access to both markets. Having established that the closed-loop model is appropriate for solving these kinds of problems, we consider only the closed-loop formulation and a single time period in both markets.

Our base case is the same as above. Further we vary α_t^{balx} and β_t^{bal} in two other cases, see Table 4.10.

Table 4.10: The cost and demand function parameters for the single intra-hour time period cases with limited access to the balancing market.

Case	a	b	β_1^{da}	α_1^{da}	β_1^{bal}	α_1^{balx}	α_1^{bal}
Base	0	10	100	1	100	1	1
V	0	10	100	1	100	0.6	1
VI	0	10	60	1	100	1	1
VII	0	10	140	1	100	1	1
VIII	0	10	180	1	100	1	1

Table 4.11 shows for the base case that the total production of the two producers are the same. This is due to the symmetry of the producers and would not necessarily have been the case for asymmetric producers. The producer with access to the balancing market holds back production in the day-ahead market to exploit the monopoly-like conditions in the second market. For comparison, we found the solution to the day-ahead market equilibrium without the balancing market to be ${x_{i1}}^* = 30$, $p_1^{\text{da}} = 40$ and profit of each unit to be 900 DKK for the base case. Hence, both units gain from one of them being able to participate in the balancing market. In this case, the unit without access to the balancing market actually experience a larger increase in profits than the unit with access to both markets. Moreover, by comparing to the base case of Table 4.4, the profit for unit 1 is higher when both units have access to the balancing market, whereas unit 2 reduces its profits from entering the balancing market along with unit 1. Surprisingly, a producer would prefer only the other market players participate in the balancing market. This counter-intuitive result can be explained as follows. The two markets are linked via the inverse demand function for the balancing market in which the day-ahead demand affects the price in the balancing market. Hence, the unit without access to the balancing market still influences the balancing price. In contrast to the counter-intuitive result of the base case, we find that when the parameter α^{balx} is small, i.e. when the influence of the day-ahead market production on the balancing market price is small, the unit with access to both markets has an advantage compared to the other unit and obtains higher profits, see case V in Table 4.11. However, it is unlikely that the parameter will differ significantly from the other α parameters, so more interestingly, the same effect is seen when β_1^{da} is lower than the intercept in balancing market and induces up-regulation or higher than the intercept in the balancing market, leading to down-regulation, case VI and VIII in Table 4.11. Then the balancing market participant has higher profits than the other unit. For these cases the balancing market participant also has higher profits than when both producers participate in both markets, see Table 4.4 case I and IIb. For $\beta_1^{\text{da}} = 140$ both units only participate in the day-ahead market, and hence the profit is the same as for the corresponding case IIa in Table 4.4. Thus, when differences in day-ahead and

Table 4.11: Closed-loop Cournot, hourly time resolution in both markets. Note that the units no longer have equal production and hence the values for both units are depicted. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

i	Case	$x_{i1}{}^*$	x_{i1}^{+*}	x_{i1}^{-*}	Total prod	p_1^{da}	p_1^{bal}	Profit
1	Base	20.00	15.00	0.00	35.00	45.00	30.00	925.00
2		35.00	-	-	35.00			1225.00
1	V	19.57	26.06	0.00	45.64	45.21	41.06	1368.59
2		35.21	-	-	35.21			1239.94
1	VI	0.00	30.00	0.00	30.00	35.00	45.00	900.00
2		25.00	-	-	25.00			625.00
1	VII	43.33	0.00	0.00	43.33	53.33	13.33	1877.78
2		43.33	-	-	43.33			1877.78
1	VIII	65.60	0.00	13.40	52.20	62.20	-4.40	3603.88
2		52.20	-	-	52.20			2724.84

balancing market demand lead to down-regulation or up-regulation, e.g. with intra-hour variations and/or uncertainty in demand, or low balancing demand elasticity of balancing market price, limited market access increases the exercise of market power.

4.5.4 Excluding arbitrage

In spite of differences between the above open- and closed-loop models, these are mostly moderate. If we exclude arbitrage, however, we observe from Table 4.12 that already for the base case with hourly time resolution of the balancing market equal inverse demand functions in the two markets, the open- and closed-loop solution differ substantially. For our case, the inability to make decisions simultaneously reduces profits by 68%. As evidenced from the above results, this inflexibility is largely offset by the possibility for producers to arbitrage.

Table 4.12: Solution for the hourly time resolution in both markets with the no arbitrage constraint. Open-loop Cournot at the top, then closed-loop Cournot and the closed-loop compared to the open-loop at the bottom. Production volumes are measured in MWh, prices in DKK/MWh and profit in DKK.

$x_{i1}{}^*$	x_{i1}^{+*}	x_{i1}^{-*}	Total prod	p_1^{da}	p_1^{bal}	Profit
30.00	0.00	0.00	30.00	40.00	40.00	900.00
26.00	11.00	0.00	37.00	26.00	26.00	537.00
15.38%	-	-	-18.92%	53.85%	53.85%	67.60%

4.5.5 Convergence

The diagonalization algorithm for solving the closed loop problem was implemented in GAMS. It utilises the SBB mixed integer non-linear programming problem (MINLP) solver and the CONOPT non-linear program (NLP) solver. It converges for all the instances and typically within 12 iterations with a tolerance of 10^{-5}. For selected cases we varied the starting condition curVal (see Appendix 4.A) between 0 and the maximum capacity of the unit to search for multiple solutions, but none was found.

4.6 Conclusion

We have presented an open-loop and a closed-loop model for the day-ahead and balancing markets. Unlike similar models in the literature we find that solutions of the open- and closed-loop models for the day-ahead and balancing markets are in general not the same and may differ substantially, advocating for the computationally harder closed-loop model.

When handling arbitrage explicitly the differences between the open- and closed-loop solutions are even larger. Hence, closed-loop models, though computationally harder than open-loop models, should receive increased attention within the field of sequential market modelling, especially when modelling stochastics with demand function parameters that varies a lot between the scenarios.

Finally, when the influence of day-ahead production on the balancing price is low or when down-regulation occur, a power producer with access to the balancing market can exercise market power by adjusting production in the day-ahead market to take advantage of the balancing market and thereby increase profits above the level of a power producer with access only to the day-ahead market.

Future research includes relating the results to changes in social welfare and extending the models to general conjectural variations, including stochastic (wind power) producers as well as introducing further technical constraints in the problem such as ramp rates.

Acknowledgements

D. M. Heide-Jørgensen and T. K. Boomsma are both partly funded by the Research Council of Norway through project 228811: RISKY-RES, Investment under uncertainty: EU renewable energy support beyond 2020 project.

APPENDIX

4.A Solving the closed-loop model with diagonalization

We use diagonalization Hu and Ralph (2007), also known as Gauss-Seidel iteration Ito and Reguant (2015), to solve the closed-loop model. The idea is to solve the MPEC from Section 4.3.2 for each unit while fixing the upper-level solutions of the other units and then iterating between the units taking into account their found solutions until no changes in the upper-level decisions are seen. The procedure is as follows

prevVal$(i,h) \leftarrow 0$
curVal$(i,h) \leftarrow$ initial value $\neq 0$

while $\|\text{prevVal}(i,h) - \text{curVal}(i,h)\| >$ tolerance

  for $i = 1,\ldots,|I|$:

    prevVal$(i,h) \leftarrow$ curVal(i,h)
    for $j \neq i$ fix
      $x_{jh} \leftarrow$ curVal(j,h)
    end for
    solve the MPEC for unit i
    curVal$(i,h) \leftarrow x_{ih}$

  end for

end while

4.B Partial derivatives

Assuming Cournot competition in the market we have the partial derivatives of the inverse demand functions

$$\frac{\partial p_h^{\text{da}}}{\partial x_{ih}} = -\alpha_h^{\text{da}},\ \frac{\partial p_h^{\text{da}}}{\partial x_{it\omega}^{+}} = \frac{\partial p_h^{\text{da}}}{\partial x_{it\omega}^{-}} = 0$$

$$\frac{\partial p_{t\omega}^{\text{bal}}}{\partial x_{ih}} = -\alpha_t^{\text{balx}}\tau_h,\ \frac{\partial p_{t\omega}^{\text{bal}}}{\partial x_{it\omega}^{+}} = -\frac{\partial p_h^{\text{da}}}{\partial x_{it\omega}^{-}} = -\alpha_t^{\text{bal}}.$$

For perfect competition these partial derivatives are zero.

The partial derivatives of the cost function are

$$\frac{\partial c_i}{\partial x_{ih}} = 2a_i\tau_h\left(\tau_h x_{ih} + x^+_{it\omega} - x^-_{it\omega}\right) + b_i\tau_h$$

$$\frac{\partial c_i}{\partial x^+_{it\omega}} = 2a_i\left(\tau_h x_{ih} + x^+_{it\omega} - x^-_{it\omega}\right) + b_i + \gamma^+.$$

$$\frac{\partial c_i}{\partial x^-_{it\omega}} = -2a_i\left(\tau_h x_{ih} + x^+_{it\omega} - x^-_{it\omega}\right) - b_i + \gamma^-.$$

4.C Nomenclature

Table 4.C.1: Sets and indices

H	The set of hours in the time horizon
h	Hour in the time horizon
T	The set of all intra-hour time intervals
t	Intra-hour time interval, $t \in T$
T_h	The subset of intra-hour time intervals in hour h, $T_h \subseteq T$
I	The set of all players, i.e. the production units
i	A player or production unit, $i \in I$
Ω	The set of scenarios for the inverse demand function intercept
ω	A scenario for the inverse demand function intercept, $\omega \in \Omega$

Table 4.C.2: Parameters

τ_h	Constant for adjusting low time-resolution values to high time-resolution values. $\tau_h = 1/\lvert T_h \rvert$
$x_i^{\max}$	Maximum capacity of producer i (MWh)
α_h^{da}	Day-ahead market demand elasticity of the day-ahead market price at time h (DKK/MWh)
β_h^{da}	Intercept of the inverse demand function at time h in the day-ahead market (DKK)
α_t^{bal}	Balancing market demand elasticity of the balancing price at time t (DKK/MWh)
α_t^{balx}	Day-ahead demand elasticity of the balancing price at time t (DKK/MWh)
$\beta_{t\omega}^{\text{bal}}$	Intercept of the inverse demand function in the balancing market at time t in scenario ω (DKK)
c_i	Cost function for unit i
b_i	Coefficient of linear term of cost function, c_i, of producer i (DKK/MWh)
a_i	Non-negative coefficient of quadratic term of cost function, c_i, of producer i (DKK/MWh^2)
γ^+	Non-negative cost factor for up-regulation (DKK/MWh)
γ^-	Non-negative cost factor for down-regulation (DKK/MWh)

Table 4.C.3: Variables

x_{ih}	Production planned in the day-ahead market by producer i in hour h (MWh)
$x^{+}_{it\omega}$	Up-regulation balancing power provided by producer i at time t in scenario ω (MWh)
$x^{-}_{it\omega}$	Down-regulation balancing power provided by producer i at time t in scenario ω (MWh)
p^{da}_{h}	Day-ahead market price at time h (DKK/MWh)
d^{da}_{h}	Demand in the day-ahead market at time h (MWh)
$p^{\text{bal}}_{t\omega}$	Balancing market price at time t in scenario ω (DKK/MWh)
$d^{\text{bal}}_{t\omega}$	Demand in the balancing market at time t in scenario ω (MWh)
$\underline{\sigma}_{ih}$	Shadow cost of minimum capacity constraint of producer i at time h in the day-ahead market
$\bar{\sigma}_{ih}$	Shadow cost of maximum capacity constraint of producer i at time h in the day-ahead market
$\underline{\sigma}^{+}_{it\omega}$	Shadow cost of minimum capacity constraint of the up-regulation by producer i at time t in scenario ω in the balancing market
$\bar{\sigma}^{+}_{it\omega}$	Shadow cost of maximum capacity constraint of the up-regulation by producer i at time t in scenario ω in the balancing market
$\underline{\sigma}^{-}_{it\omega}$	Shadow cost of minimum capacity constraint of the down-regulation by producer i at time t in scenario ω in the balancing market
$\bar{\sigma}^{-}_{it\omega}$	Shadow cost of maximum capacity constraint of the down-regulation by producer i at time t in scenario ω in the balancing market

4.D Concavity of the problem

The profit maximising objective function of a single producer in the equilibrium model in Section 4.3.1 is concave for certain parameter values. To see this, observe that for $t \in T_h,\ h \in H$ and $\omega \in \Omega$, the Hessian of the profit function is

$$\begin{bmatrix} -2\tau_h(\alpha^{\text{da}}_h + \tau_h a_i) & -\tau_h(\alpha^{\text{balx}}_t + 2a_i) & \tau_h(\alpha^{\text{balx}}_t + 2a_i) \\ -\tau_h(\alpha^{\text{balx}}_t + 2a_i) & -2(\alpha^{\text{bal}}_t + a_i) & 2(\alpha^{\text{bal}}_t + a_i) \\ \tau_h(\alpha^{\text{balx}}_t + 2a_i) & 2(\alpha^{\text{bal}}_t + a_i) & -2(\alpha^{\text{bal}}_t + a_i) \end{bmatrix}$$

for which the determinant of the first, second and third leading principal minors are $-2\tau_h(\alpha^{\text{da}}_h + a_i\tau_h)$, $4\tau_h(\alpha^{\text{da}}_h + a_i\tau_h)(\alpha^{\text{bal}}_t + a_i) - \tau_h^2(\alpha^{\text{balx}}_t + 2a_i)^2$ and 0, showing that the Hessian is negatively semi-definite for $4(\alpha^{\text{da}}_h + a_i\tau_h)(\alpha^{\text{bal}}_t + a_i) \geq \tau_h^2(\alpha^{\text{balx}}_t + 2a_i)^2$ and, hence, the function is concave when this inequality is satisfied. Examples of such parameter settings are $\tau_h = 1$, and $\alpha^{\text{bal}_t} = \alpha^{\text{da}_h} = \alpha^{\text{balx}_t}$, $a = 0$ and $2\alpha^{\text{bal}_t} \geq \alpha^{\text{balx}_t}$ and $2\alpha^{\text{da}_h} \geq \alpha^{\text{balx}_t}$ (which we consider in the case studies). Hence, the maximisation problems of the producers are concave programming problems, the KKT conditions are both necessary and sufficient for optimality.

– A –

Abbreviations

Table A.1: Abbreviations

1UC	Single-unit commitment problem
AUD	Australian dollars
CP	Convex programming problem
DKK	Danish crowns
DP	Dynamic programming
ED	Economic dispatch problem
EPEC	Equilibrium problem with equilibrium constraints
KKT	Karush Kuhn Tucker
LP	Linear Programming problem
MCP	Mixed complementarity problem
MC	Markov chain
MIP	Mixed integer programming problem
MW	Mega watt
MWh	Mega watt hours
SFE	Supply function equilibrium
SO	System operator
SP	Stochastic programming problem
SUC	Stochastic unit commitment
TSO	Transmission system operator
UC	Unit commitment

Bibliography

B. Allaz. Oligopoly, uncertainty and strategic forward transactions. *International Journal of Industrial Organization*, 10(2):297 – 308, 1992. ISSN 0167-7187. doi: http://dx.doi.org/10.1016/0167-7187(92)90020-Y.

J. Andersen. *Modelling and Optimisation of Renewable Energy Systems*. PhD thesis, Aarhus University, Aarhus, Denmark, 2015.

M. F. Anjos. *Recent Progress in Modeling Unit Commitment Problems*, pages 1–29. Springer, 2013. ISBN 978-1-4614-8987-0. doi:10.1007/978-1-4614-8987-0_1.

J. M. Arroyo and A. J. Conejo. Optimal response of a thermal unit to an electricity spot market. *Power Systems, IEEE Transactions on*, 15(3):1098–1104, Aug 2000. ISSN 0885-8950. doi:10.1109/59.871739.

D. P. Bertsekas. *Dynamic programming: deterministic and stochastic models*. Prentice-Hall, Inc., 1987. ISBN 0132215810.

J. R Birge and F. Louveaux. *Introduction to Stochastic Programming*. Springer Series in Operations Reseach and Financial Engineering. Springer, 2 edition, 2011.

F. Bouffard and F. D. Galiana. Stochastic security for operations planning with significant wind power generation. *Power Systems, IEEE Transactions on*, 23(2): 306–316, 2008. ISSN 0885-8950. doi:10.1109/TPWRS.2008.919318.

D. W. Bunn and S. N. Paschentis. Development of a stochastic model for the economic dispatch of electric power. *European Journal of Operational Research*, 27(2):179–191, 1986. doi:10.1016/0377-2217(86)90059-7.

C. C. Caröe and R. Schultz. A two-stage stochastic program for unit commitment under uncertainty in a hydro-thermal power system. Technical Report SC-98-11, ZIB, Takustr.7, 14195 Berlin, 1998.

M. Carrión and J. M. Arroyo. A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. *Power Systems, IEEE Transactions on*, 21(3):1371–1378, 2006. ISSN 0885-8950. doi:10.1109/TPWRS.2006.876672.

A. J. Conejo, M. Carrión, and J. M. Morales. *Decision Making Under Uncertainty in Electricity Markets*, volume 153 of *International Series in Operations Research & Management Science*. Springer, 2010.

T. S. Dillon, K. W. Edwin, H.-D. Kochs, and R. J. Taud. Integer programming approach to the problem of optimal unit commitment with probabilistic reserve determination. *Power Apparatus and Systems, IEEE Transactions on*, PAS-97(6): 2154–2166, 1978. ISSN 0018-9510. doi:10.1109/TPAS.1978.354719.

G. Doorman and B. Nygreen. Market price calculations in restructured electricity markets. *Annals of Operations Research*, 124(1):49–67, 2003. ISSN 1572-9338. doi: 10.1023/B:ANOR.0000004762.31449.33.

E. Ela and M. O'Malley. Studying the variability and uncertainty impacts of variable generation at multiple timescales. *Power Systems, IEEE Transactions on*, 27 (3):1324–1333, 2012. ISSN 0885-8950. doi:10.1109/TPWRS.2012.2185816.

H. Farahmand and G. L. Doorman. Balancing market integration in the northern european continent. *Applied Energy*, 96(0):316 – 326, 2012. ISSN 0306-2619. doi: 10.1016/j.apenergy.2011.11.041.

J. Fortuny-Amat and B. McCarl. A representation and economic interpretation of a two-level programming problem. *The Journal of the Operational Research Society*, 32(9):783–792, 1981. ISSN 01605682, 14769360. doi:10.2307/2581394.

A. Frangioni and C. Gentile. Solving nonlinear single-unit commitment problems with ramping constraints. *Operations Research*, 54(4):767–775, 2006. doi: 10.1287/opre.1060.0309.

A. Frangioni, C. Gentile, and F Lacalandra. Solving unit commitment problems with general ramp constraints. *International Journal of Electrical Power & Energy Systems*, 30(5):316 – 326, 2008. ISSN 0142-0615. doi:10.1016/j.ijepes.2007.10.003.

S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, and C. Ruiz. *Complementarity Modeling in Energy Markets*, volume 180 of *International Series in Operations Research & Management Science*. Springer-Verlag New York, 1 edition, 2013. ISBN 978-1-4419-6123-5. doi:10.1007/978-1-4419-6123-5.

A. García-Alcalde, M. Ventosa, M. Rivier, A. Ramos, and G. Relano. Fitting electricity market models: A conjectural variations approach. *Proc. 14th PSCC 2002*, 2002. URL `http://www.iit.comillas.edu/aramos/papers/PSCCSeville_s12p03.pdf`.

L. L. Garver. Power generation scheduling by integer programming-development of theory. *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, 81(3):730–734, 1962. ISSN 0097-2460. doi:10.1109/AIEEPAS.1962.4501405.

R. Gollmer, M. P. Nowak, W. Römisch, and R. Schultz. Unit commitment in power generation – a basic model and some extensions. *Annals of Operations Research*, 96(1-4):167–189, 2000. ISSN 0254-5330. doi:10.1023/A:1018947401538.

A. M. González, A. M. S. Roque, and J. Garcia-González. Modeling and forecasting electricity prices with input/output hidden markov models. *Power Systems, IEEE Transactions on*, 20(1):13–24, Feb 2005. ISSN 0885-8950. doi:10.1109/TPWRS.2004.840412.

N. Gülpınar and F. S. Oliveira. Analysis of relationship between forward and spot markets in oligopolies under demand and cost uncertainties. *Computational Management Science*, 11(3):267–283, 2014. ISSN 1619-6988. doi:10.1007/s10287-014-0217-7.

F.-J. Heredia, M. J. Rider, and C. Corchero. A stochastic programming model for the optimal electricity market bid problem with bilateral contracts for thermal and combined cycle units. *Annals of Operations Research*, 193(1):107–127, 2012. doi:10.1007/s10479-011-0847-x.

B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H.-P. Chao. *The next generation of electric power unit commitment models*, volume 36. Springer Science & Business Media, 2001. doi:10.1007/b108628.

X. Hu and D. Ralph. Using epecs to model bilevel games in restructured electricity markets with locational prices. *Operations Research*, 55(5):809–827, 2007. doi:10.1287/opre.1070.0431.

K. Ito and M. Reguant. Sequential Markets, Market Power and Arbitrage. Discussion papers 15015, Research Institute of Economy, Trade and Industry (RIETI), 2015. URL `https://ideas.repec.org/p/eti/dpaper/15015.html`.

E. B. Iversen, J. M. Morales, and H. Madsen. Optimal charging of an electric vehicle using a markov decision process. *Applied Energy*, 123:1 – 12, 2014. ISSN 0306-2619. doi:10.1016/j.apenergy.2014.02.003.

S. Jaehnert and G. L. Doorman. Assessing the benefits of regulating power market integration in northern europe. *International Journal of Electrical Power & Energy Systems*, 43(1):70 – 79, 2012. ISSN 0142-0615. doi:10.1016/j.ijepes.2012.05.010.

S. J. Kazempour and H. Zareipour. Equilibria in an oligopolistic market with wind power production. *Power Systems, IEEE Transactions on*, 29(2):686–697, March 2014. ISSN 0885-8950. doi:10.1109/TPWRS.2013.2286094.

J. Khazaei, G. Zakeri, and S. S. Oren. Market clearing mechanisms under uncertainty. *Submitted to Operations Research*, 2014. URL `http://www.ieor.berkeley.edu/~oren/pubs/Javad%20and%20Gorbon+appendix.pdf`.

P. D. Klemperer and M. A. Meyer. Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):1243–1277, 1989. ISSN 00129682, 14680262. doi:10.2307/1913707.

E. Lindgren and L Söder. Minimizing regulation costs in multi-area systems with uncertain wind power forecasts. *Wind Energy*, 11(1):97–108, 2008.

H. Louie. Evaluation of probabilistic models of wind plant power output characteristics. In *Probabilistic Methods Applied to Power Systems (PMAPS), 2010 IEEE 11th International Conference on*, pages 442–447, June 2010. doi:10.1109/PMAPS.2010.5528963.

H. Lund. Renewable energy strategies for sustainable development. *Energy*, 32 (6):912–919, 2007.

J. M. Morales, A. J. Conejo, and J. Perez-Ruiz. Economic valuation of reserves in power systems with high penetration of wind power. *Power Systems, IEEE Transactions on*, 24(2):900–910, 2009a. ISSN 0885-8950. doi:10.1109/TPWRS.2009.2016598.

J. M. Morales, R Mínguez, and A. J. Conejo. A methodology to generate statistically dependent wind speed scenarios. *Applied Energy*, 87(3):843–855, 2009b. doi:10.1016/j.apenergy.2009.09.022.

G. Morales-Espana, J. M. Latorre, and A. Ramos. Tight and compact milp formulation of start-up and shut-down ramping in unit commitment. *Power Systems, IEEE Transactions on*, 28(2):1288–1296, 2013. ISSN 0885-8950. doi:10.1109/TPWRS.2012.2222938.

J. A. Muckstadt and S. A. Koenig. An application of lagrangian relaxation to scheduling in power-generation systems. *Operations Research*, 25(3):387–403, 1977. doi:10.1287/opre.25.3.387.

F. H. Murphy and Y. Smeers. Generation capacity expansion in imperfectly competitive restructured electricity markets. *Operations Research*, 53(4):646 – 661, 2005. ISSN 0030364X.

M. P. Nowak and W. Römisch. Stochastic lagrangian relaxation applied to power scheduling in a hydro-thermal system under uncertainty. *Annals of Operations Research*, 100(1-4):251–272, 2000. ISSN 0254-5330. doi:10.1023/A:1019248506301.

J. Ostrowski, M. F. Anjos, and A. Vannelli. Tight mixed integer linear programming formulations for the unit commitment problem. *Power Systems, IEEE Transactions on*, 27(1):39–46, 2012. ISSN 0885-8950. doi:10.1109/TPWRS.2011.2162008.

A. Papavasiliou and S. S. Oren. Multiarea stochastic unit commitment for high wind penetration in a transmission constrained network. *Operations Research*, 61(3):578–592, 2013. doi:10.1287/opre.2013.1174.

A. Papavasiliou, S. S. Oren, and B. Rountree. Applying high performance computing to transmission-constrained stochastic unit commitment for renewable energy integration. *Power Systems, IEEE Transactions on*, 30(3):1109–1120, May 2015. ISSN 0885-8950. doi:10.1109/TPWRS.2014.2341354.

J. A. Petersen, D. M. Heide-Jørgensen, N. K. Detlefsen, and T. K. Boomsma. Short-term balancing of supply and demand in an electricity system: forecasting and scheduling. *Annals of Operations Research*, 238(1):449–473, 2016. ISSN 1572-9338. doi:10.1007/s10479-015-2092-1.

P. Pinson. *Estimation of the uncertainty in wind power forecasting*. PhD thesis, Ecole des Mines de Paris, Paris, France, 2006.

P. Pinson and R. Girard. Evaluating the quality of scenarios of short-term wind power generation. *Applied Energy*, 96(0):12 – 20, 2012. ISSN 0306-2619. doi:10.1016/j.apenergy.2011.11.004.

P. Pinson and H. Madsen. Ensemble-based probabilistic forecasting at horns rev. *Wind Energy*, 12(2):137–155, 2009. ISSN 1099-1824. doi:10.1002/we.309.

G. Pritchard, G. Zakeri, and A. Philpott. A single-settlement, energy-only electric power market for unpredictable and intermittent participants. *Operations Research*, 58(4-Part-2):1210–1219, 2010. doi:10.1287/opre.1090.0800.

A. Rong, H. Hakonen, and R. Lahdelma. A variant of the dynamic programming algorithm for unit commitment of combined heat and power systems. *European Journal of Operational Research*, 190(3):741 – 755, 2008. ISSN 0377-2217. doi:10.1016/j.ejor.2007.06.035.

U. V. Shanbhag, G. Infanger, and P. W. Glynn. A complementarity framework for forward contracting under uncertainty. *Operations Research*, 59(4):810–834, 2011. doi:10.1287/opre.1110.0947.

G. B. Sheble and G. N. Fahd. Unit commitment literature synopsis. *Power Systems, IEEE Transactions on*, 9(1):128–135, 1994. ISSN 0885-8950. doi:10.1109/59.317549.

Y. Song, Y. Ni, F. Wen, Z. Hou, and F. F. Wu. Conjectural variation based bidding strategy in spot markets: fundamentals and comparison with classical game theoretical bidding strategies. *Electric Power Systems Research*, 67(1):45 – 51, 2003. ISSN 0378-7796. doi:http://dx.doi.org/10.1016/S0378-7796(03)00042-7.

S. Takriti, J. R. Birge, and E. Long. A stochastic model for the unit commitment problem. *Power Systems, IEEE Transactions on*, 11(3):1497–1508, 1996. ISSN 0885-8950. doi:10.1109/59.535691.

C. Tseng and G. Barz. Short-term generation asset valuation: a real options approach. *Operations Research*, 50:297–310, 2002. doi:10.1287/opre.50.2.297.429.

P. Twomey and K. Neuhoff. Wind power and market power in competitive markets. *Energy Policy*, 38(7):3198 – 3210, 2010. ISSN 0301-4215. doi:http://dx.doi.org/10.1016/j.enpol.2009.07.031. Large-scale wind power in electricity markets with Regular Papers.

M. Ventosa, Á. Baíllo, A. Ramos, and M. Rivier. Electricity market modeling trends. *Energy Policy*, 33(7):897 – 913, 2005. ISSN 0301-4215. doi:http://dx.doi.org/10.1016/j.enpol.2003.10.013.

H. von Stackelberg. *Marktform und Gleichgewicht*. J. Springer, Vienna and Berlin, 1934.

J. Wang, J. Wang, C. Liu, and J. P. Ruiz. Stochastic unit commitment with subhourly dispatch constraints. *Applied Energy*, 105:418 – 422, 2013. ISSN 0306-2619. doi:10.1016/j.apenergy.2013.01.008.

C. Weber. Adequate intraday market design to enable the integration of wind energy into the european power systems. *Energy Policy*, 38(7):3155 – 3163, 2010. ISSN 0301-4215. doi:http://dx.doi.org/10.1016/j.enpol.2009.07.040. Large-scale wind power in electricity markets with Regular Papers.

C. Weber, P. Meibom, R. Barth, and H. Brand. *WILMAR: A Stochastic Programming Tool to Analyze the Large-Scale Integration of Wind Energy*, chapter 19, pages 437–458. Energy Systems. Springer Berlin Heidelberg, 2009.

R. Weron. Electricity price forecasting: A review of the state-of-the-art with a look into the future. *International Journal of Forecasting*, 30(4):1030 – 1081, 2014. ISSN 0169-2070. doi:10.1016/j.ijforecast.2014.08.008.

S. Wogrin, B. F. Hobbs, D. Ralph, E. Centeno, and J. Barquín. Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition. *Mathematical Programming*, 140(2):295–322, 2013. ISSN 1436-4646. doi:10.1007/s10107-013-0696-2.

D. Zhang, H. Xu, and Y. Wu. A two stage stochastic equilibrium model for electricity markets with two way contracts. *Mathematical Methods of Operations Research*, 71(1):1–45, 2010. doi:10.1007/s00186-009-0283-8.

Q. P. Zheng, J. Wang, P. M. Pardalos, and Y. Guan. A decomposition approach to the two-stage stochastic unit commitment problem. *Annals of Operations Research*, pages 1–24, 2012. ISSN 0254-5330. doi:10.1007/s10479-012-1092-7.

F. Zhuang and F. D. Galiana. Towards a more rigorous and practical unit commitment by lagrangian relaxation. *Power Systems, IEEE Transactions on*, 3(2): 763–773, May 1988. ISSN 0885-8950. doi:10.1109/59.192933.