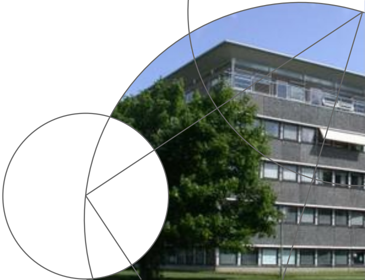
Faculty of Science

Do phantom Cuntz-Krieger algebras exist?

Sara Arklint
Department of Mathematical Sciences



What is a phantom Cuntz-Krieger algebra?

Definition

A C^*-algebra A *looks like a purely infinite Cuntz-Krieger algebra* if

- A is unital, purely infinite, nuclear, separable, and of real rank zero,
- A has finitely many ideals,
- for all $I \trianglelefteq J \trianglelefteq A$, the group $K_*(J/I)$ is finitely generated, the group $K_1(J/I)$ is free, and $\operatorname{rank} K_0(J/I) = \operatorname{rank} K_1(J/I)$,
- the simple subquotients of A are in the bootstrap class.

All purely infinite Cuntz-Krieger algebras look like purely infinite Cuntz-Krieger algebras.

Definition

A *phantom Cuntz-Krieger algebra* is a C^*-algebra that looks like a purely infinite Cuntz-Krieger algebra but is not isomorphic to one.

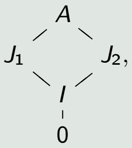
Using K-theory on phantom Cuntz-Krieger algebras

Theorem (Restorff)

Let A and B be purely infinite Cuntz-Krieger algebras with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$. Then $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$ implies $A \otimes \mathbb{K} \cong B \otimes \mathbb{K}$.

Example (Reduced filtered K-theory $\mathrm{FK}_{\mathcal{R}}$)

For a C^*-algebra A with ideal lattice



its $\mathrm{FK}_{\mathcal{R}}(A)$ consists of

$$\begin{array}{ccc} K_0(I) & \longrightarrow & K_0(J_n) \\ \uparrow & & \\ K_0(J_n/I) & & \qquad K_1(I), \end{array}$$

$$n \in \{1,2\}.$$

Theorem (A-Bentmann-Katsura)

Let A be a C^*-algebra that looks like a purely infinite Cuntz-Krieger algebra. Then there exists a purely infinite Cuntz-Krieger algebra B with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$ and $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$.

Using K-theory on phantom Cuntz-Krieger algebras

Theorem (A-Bentmann-Katsura)

Let A be a C^*-algebra that looks like a purely infinite Cuntz-Krieger algebra. Then there exists a purely infinite Cuntz-Krieger algebra B with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$ *and* $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$.

Theorem (Kirchberg, Meyer-Nest, Bentmann-Köhler)

Let A and B be Kirchberg X-algebras with X an accordion space. Then $\mathrm{FK}(A) \cong \mathrm{FK}(B)$ *implies* $A \otimes \mathbb{K} \cong B \otimes \mathbb{K}$.

Theorem (A-Bentmann-Katsura)

Let A and B be C^*-algebras that looks like purely infinite Cuntz-Krieger algebras and assume that $\mathrm{Prim}(A)$ *and* $\mathrm{Prim}(B)$ *are homeomorphic accordion spaces. Then* $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$ *implies* $\mathrm{FK}(A) \cong \mathrm{FK}(B)$.

Corollary

A phantom Cuntz-Krieger algebra A with $\mathrm{Prim}(A)$ *an accordion space cannot exist.*