Holomorphic Day, April 17, 2009
Department of Mathematical Sciences
University of Copenhagen

Abstract

The purpose of the holomorphic day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant 272-07-0321 from FNU.

1 Schedule

The lectures take place at the premises of the Department of Mathematical Sciences, Universitetsparken 5, Copenhagen.

Arrival, coffee, tea: 9.00-9.30 in E 419 (Fourth floor)

Jacob Stordal Christiansen: 9.30-10.20 in Aud. 10
The isospectral torus.

Magnus Aspenberger: 10.30-11.20 in Aud. 10
On Semi-hyperbolic rational maps and Misiurewicz maps.

Kealey Dias: 11.30-12.20 in Aud. 10
Classification of complex polynomial vector fields in $\mathbb{C}$.

Lunch: 12.30-14.00
Alexandru Aleman: 14.00-14.50 in Aud. 5
Some applications of near invariance.

Sebastien Godillon: 15.00-15.50 in Aud. 5
Examples of dynamics on components of disconnected Julia sets.

Coffee break: 15.50-16.15

Jan-Fredrik Olsen: 16.15-17.05 in Aud. 5
The boundary behaviour of a class of modified zeta functions.

Adam Epstein: 17.15-18.05 in Aud. 5
Matings of Quadratic Polynomials.

Dinner: 19.00-

2 Abstracts

Jacob Stordal Christiansen, Assistant professor, University of Copenhagen: The isospectral torus.

When studying orthogonal polynomials on several intervals instead of a single interval, the notion of convergence of the recurrence coefficients has to be reconsidered. The coefficients no longer have limits in the usual sense but approach a torus of certain (almost) periodic sequences. In the talk, I'll use a covering map formalism to define and study this isospectral torus. In particular, I'll explain how the Abel map "linearizes" coefficient stripping so that coefficient stripping simply corresponds to multiplication (by a certain character). To illustrate different approaches to the isospectral torus, I'll present a theorem of Remling and a result on Szegő asymptotics from joint work with Simon and Zinchenko. The limiting behaviour of the orthogonal polynomials can be described in terms of Jost functions.

Magnus Aspenberger, Postdoc, University of Kiel: On Semi-hyperbolic rational maps and Misiurewicz maps.

The talk will be focused on recent progress and some conjectures connected to rational Misiurewicz maps in Complex Dynamics. These maps are non-hyperbolic maps without parabolic periodic points and such that the critical set on the Julia set is non-recurrent, i.e. if $Crit(f)$ is the set of critical points for f and $J(f)$ is the Julia set for f, then for every $c \in Crit(f) \cap J(f)$ we have $\omega(c) \cap Crit(f) = \emptyset$. In the space of rational functions of a given degree, the set of Misiurewicz maps have Lebesgue measure zero. On the other hand, they have full Hausdorff dimension, i.e. dimension equal to the dimension of the parameter space (the last statement proved together with J. Graczyk). This seems to be a quite universal statement also in other parameter spaces. A slightly more general type of maps are the semi-hyperbolic maps, introduced by Carleson, Jones and Yoccoz. For these maps every critical point is non-recurrent instead of the whole critical set, i.e. $c \notin \omega(c)$ for every $c \in Crit(f) \cap J(f)$. Together with J. Graczyk, we show that these maps also has Lebesgue measure zero in degree 2.

Kealey Dias, Ph.D.-student, Technical University of Denmark: Classification of complex polynomial vector fields in $\mathbb{C}$.

We consider the family of polynomial vector fields in the complex plane, that is, the space whose elements are the vector fields in $\mathbb{C}$ that in a global chart take the form $P(z)d/dz$, where P is a complex polynomial of degree d. The project consists of two parts: a characterization of the global topological and analytical structure of a given vector field; and the decomposition of parameter space for complex polynomial vector fields in $\mathbb{C}$ with a description of the possible bifurcations. An introduction and outline of the main results will be presented.

Alexandru Aleman, Professor, University of Lund: Some applications of near invariance.

We consider Hilbert spaces H which consist of analytic functions in a domain $\Omega \subset \mathbb{C}$ and have the property that any zero of an element of H which is not a common zero of the whole space, can be divided out without leaving H. This property is called *near invariance* and is related to a number of interesting problems that connect complex analysis and operator theory. The concept probably appeared first in L. de Branges' work on Hilbert spaces of entire functions and played later a decisive role in the description of invariant subspaces of the shift operator on Hardy spaces over multiply connected domains. There are a number of structure theorems for nearly invariant spaces obtained by de Branges, Hitt and Sarason, and more recently by Feldman, Ross and myself, but the emphasis of the talk will be on some applications. We shall have a look at differentiation invariant subspaces of $C^\infty(\mathbb{R})$, and invariant subspaces of Volterra operators on spaces of power series on the unit disc. Finally, we use near invariance in the vector-valued case to study kernels of products of Toeplitz operators. More precisely, I will present in more detail the recent solution of the following problem: If a finite product of Toeplitz operators is the zero operator then one of the factors is zero.

Sebastian Godillon, Ph.D.-student, Roskilde University: Examples of dynamics on components of disconnected Julia sets.

The most interesting questions for a given holomorphic dynamical system is to understand the geometry of its associated Julia set and the dynamic of its restriction on this set. If moreover the Julia set is disconnected, the dynamic exchanges components among themselves. Since such a Julia set has uncountably many components and each of its points is an accumulation point of infinitely many disctinct components, this dynamic could be very sophisticated. In the polynomial case, we can show that if all critical points escape to infinity, the Julia set is a Cantor set and the dynamic corresponds to the associated shift map. But more complicated situations may happen in the rational case. McMullen gave an example of a Julia set homeomorphic to the product of a Cantor set with a quasi-circle and the dynamic on the components given by the shift map on the projection. In this talk I will present other examples of rational maps with disconnected Julia sets whose dynamics are coded by some weighted trees. Then I will explain how to construct such maps starting from a weighted tree. That will provide us with a well understood family of rational maps with disconnected Julia sets.

Jan-Fredrik Olsen, Ph.D.-student, NTNU Trondheim: The boundary behaviour of a class of modified zeta functions.

Given a subset $K \subset \mathbb{N}$ we define the K-zeta function

$$\zeta_K(s) = \sum_{n \in K} n^{-s}, \quad s = \sigma + \mathrm{i}t.$$

The series converges absolutely to the right of the abscissa $\sigma = 1$. For $K = \mathbb{N}$ we get Riemann's zeta function which is known to have a meromorphic continuation to the entire complex plane with a single pole at $s = 1$. In general the abscissa $\sigma = 1$ may be a natural boundary for the K-zeta function, as was shown by Kahane and Queffelec. We investigate the boundary behaviour of the K-zeta functions on finite intervals on the abscissa $\sigma = 1$ by looking at the operator defined on $L^2(-T, T)$, for $T > 0$, by

$$\mathcal{Z}_{K,T} : g \longmapsto \lim_{\delta\to 0} \frac{\chi_{(-T,T)}}{\pi} \int_{-T}^{T} g(\tau)\Re\zeta_K(1+\delta+\mathrm{i}(\tau-t))\mathrm{d}\tau.$$

Here $\chi_{(-T,T)}$ denotes the characteristic function of the interval $(-T,T)$.

Using the meromorphic continuation for Riemann's zeta function, we observe that for $K = \mathbb{N}$ this operator is a small perturbation of the identity operator in such a way that it is bounded below in norm. For general $K \subset \mathbb{N}$ we give a complete characterisation of this behaviour in terms of the asymptotic behaviour of the counting function of the set K.

[Que80] Hervé Queffélec, *Propriétés presque sûres et quasi-sûres des séries de Dirichlet et des produits d'Euler*, Can. J. Math. **XXXII** (1980), no. 3, 531–558.

Adam Epstein, Associate Professor, University of Warwick: Matings of Quadratic Polynomials.

Over a quarter century ago, Douady and Hubbard proposed a construction for *mating* appropriately compatible pairs of polynomials. Under favorable circumstances, the output is a rational map whose Julia set may be understood as the result of gluing the two polynomial Julia sets back-to-back. The existence of such matings is guaranteed by results of Tan Lei and others, all resting on fundamental and highly abstract work of Thurston. More recently, work of Bartholdi-Nekrashevych rekindled interest in the computational implementation of this procedure. This is achieved with the aid of an associated skew-product dynamical system, and we will give a concrete description (joint work with Buff and Koch) of what is arguably the simplest example. The talk will be liberally illustrated with animations by Cheritat.