Holomorphic Day, April 16, 2010
Department of Mathematical Sciences
University of Copenhagen

Abstract

The purpose of the holomorphic day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant 272-07-0321 from FNU.

1 Schedule

The lectures take place at the premises of the Department of Mathematical Sciences, Universitetsparken 5, Copenhagen.

Arrival, coffee, tea: 9.45-10.15 in E 419 (Fourth floor)

Christian Henriksen: 10.15-11.00 in Aud. 7
Thurston's Algorithm

Michel Zinsmeister: 11.15-12.00 in Aud. 7
From Bieberbach to Schramm, a 20th century saga

Lunch: 12.00-13.15

Alexander Rashkovskii: 13.15-14.00 in Aud. 6
Classical and new loglog-theorems

Henrik Laurberg Pedersen: 14.15-15.00 in Aud. 6
Real results by complex methods: volume of the unit ball in $\mathbb{R}^n$ *and the class of Nevanlinna-Pick functions*

Coffee break: 15.00-15.30

John H. Hubbard: 15.30-16.15 in Aud. 6
Pinched ball models for Hénon maps

Stanislav Smirnov: 16.30-17.15 in Aud. 6
Quasiconformal maps and harmonic measure

Dinner: 18.30-

2 Abstracts

Christian Henriksen, Associate professor, Technical University of Denmark: *Thurston's Algorithm.*

Thurston's has given a precise criterion for when a ramified covering map F is equivalent to a rational mapping. The proof works by showing that a certain endomorphism of a Teichmuller space has a fixed point. Using the ideas in Thurston's proof, one can construct an iterative scheme that finds a sequence of rational maps that converges towards a rational map equivalent to F, when there exists such a rational map. This scheme has been modified so that it can be implemented on a computer for two classes of degree 2 mappings. We will indicate how this is also possible for ramified coverings of degree greater than two.

Michel Zinsmeister, Professor, University of Orléans: *From Bieberbach to Schramm, a 20th century saga.*

Let $f(z) = a_0 + a_1 z + a_2 z^2 + \dots$ be a function holomorphic in the unit disk which is moreover injective. Then obviously a_1 is different from 0 and Bieberbach proved in 1916 that $|a_2| \leq 2|a_1|$. He conjectured at the same time that $|a_n| \leq n|a_1|$. This famous conjecture has interested many mathematicians and was finally proven in 1984 by de Branges. De Branges used crucially two methods that have been previously brought up to solve the Bieberbach conjecture. The case n=3 is due to Loewner and consists in an extremely elegant analysis of growth of slit domains. The proof of de Branges is a combination of use of Loewner's method, an inequality of Lebedev and Milin and an extra argument coming from combinatorics. In 1999 Oded Schramm revived Loewner's theory by introducing randomness in it and creating in this way the fascinating SLE processes. The aim of this talk is to revisit the Bieberbach conjecture story within the framework of Schramm's theory of SLE .

Alexander Rashkovskii, Professor, University of Stavanger: *Classical and new loglog-theorems.*

Celebrated loglog-theorems of Carleman, Wolf, Levinson, Sjöberg give uniform bounds on analytic functions whose majorants are double logarithmically integrable. We present a unified approach to such theorems and, moreover, obtain stronger results by replacing the original pointwise bounds with integral ones. A typical statement is that if the integral of a nonnegative subharmonic function with respect to a certain measure on the circle of radius R has the upper bound $V(R)$, then the function has the pointwise upper bound $cV(bR)$ for some constants c and b. We show that a famous theorem due to Matsaev on entire functions with special lower bound can be treated in the same way as well. The main ingredient is a simple description for radial projections of harmonic measures of bounded star-shaped domains in the plane, which, in particular, explains where the loglog-conditions come from. This can be of interest in other questions, where functions of this kind appear.

Henrik Laurberg Pedersen, Associate Professor, Faculty of Life Sciences, University of Copenhagen: *Real results by complex methods: volume of the unit ball in $\mathbb{R}^n$ and the class of Nevanlinna-Pick functions.*

The volume Ω_n of the unit ball in $\mathbb{R}^n$ can be expressed in terms of Eulers gamma function Γ in the formula $\Omega_n = \pi^{n/2}/\Gamma(1 + n/2)$. This leads to an investigation of the function

$$f(x) = \left(\frac{\pi^{x/2}}{\Gamma(1 + x/2)}\right)^{1/x \log x}.$$

We obtain that $f(x + 1)$ is a completely monotonic function and hence that

$$\{\Omega_{n+2}^{1/(n+2)\ln(n+2)}\}$$

is a Hausdorff moment sequence, i.e. of the form $\int_0^1 x^n \, d\mu(x)$ for a positive measure μ. The proofs are based on properties of the function F_2, where

$$F_a(z) = \frac{\log \Gamma(z + 1)}{z \log(az)}.$$

We show that this is a Nevanlinna-Pick function, i.e. has non-negative imaginary part in the upper half plane, when $a \geq 1$ and obtain from this the properties of $f(x + 1)$.

John H. Hubbard, Professor, Cornell and Marseille University: *Pinched ball models for Hénon maps.*

The pinched disk model for Julia sets of polynomials is our main tool for understanding their geometry. The fact that the leaves of the corresponding *laminations* must not cross puts drastic restrictions on what laminations can arise; As a consequence Julia sets are almost determined (topologically) by this requirement and the compatibility with $\theta \mapsto 2\theta$. The objective of the talk will be to try to find something analogous for Hénon mappings. The situation is of course much more complicated; it isn't clear in what ball to put the lamination, or what the lamination should be, or what non-crossing means,... In collaboration with Remus Radu and Raluca Tanase, we have come up with possible answers.

Stanislav Smirnov, Professor, University of Geneva: *Quasiconformal maps and harmonic measure.*

Many questions in complex analysis can be reduced to the multifractal properties of harmonic measures. Those are still poorly understood, and we will discuss possible approaches to them using quasiconformal maps and holomorphic motions. Motivation comes from dynamical systems and mathematical physics, but as a result we return again to questions from classical complex analysis. Partially based on joint work with Kari Astala and Istvan Prause.

Organized by **Christian Berg, Bodil Branner, Henrik Laurberg Pedersen, Carsten Lunde Petersen**