Orthogonal Polynomials and Special Functions

Workshop program and abstracts

November 16–17, 2016

Copenhagen, Denmark

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Aim

The purpose of the workshop is to bring together people working in the areas of orthogonal polynomials and special functions. The workshop is supported by grant DFF|181-00502 from The Danish Council for Independent Research | Natural Sciences.

Program

The talks take place in Auditorium 10 at the H.C. Ørsted Institute. The auditorium is near the department of Mathematical Sciences and is located on the first floor of the long walking area in the H.C. Ørsted Institute.

Invited talks are scheduled for 50 minutes plus questions, and contributed talks for 20 minutes plus questions. We aim for a short intermission between the talks.

Wednesday November 16

09:30 – 11:15	Coffee	
11:15 – 12.15	Clarkson	Orthogonal Polynomials and Integrable Systems
12:15 – 13:45	Lunch	
13:45 – 14:45	Neuschel	Asymptotic zero distribution of Jacobi-Piñeiro and multiple Laguerre polynomials
14:45 – 15:15	Blaschke	Asymptotic analysis via calculus of hypergeometric functions
15:15 – 15:45	Coffee	
15:45 – 16:45	Wielonsky	Large deviation principle in logarithmic potential theory
16:45 – 17:15	Askitis	On the median of the beta distribution
19:00 –	Dinner	

Thursday November 17

10:45 – 11:45	Schiefermayr	Inverse Polynomial Images
11:45 – 12.15	Vargas	Scaling Limits for Partial Sums of Power Series
12:15 – 13:45	Lunch	
13:45 – 14:45	Goncharov	Orthogonal polynomials for the generalized Julia sets
14:45 – 15:15	Alpan	Asymptotics of Jacobi matrices for a family of fractal measures
15:15 – 15:45	Coffee	
15:45 – 16:45	Kupin	On zero distribution of analytic functions from certain classes and Lieb-Thirring type inequalities
16:45 – 17:15	Szabłowski	Around Poisson–Mehler summation formula

Abstracts

Asymptotics of Jacobi matrices for a family of fractal measures

GÖKALP APLAN

BILKENT UNIVERSITY, TYRKEY

There are many results concerning asymptotics of orthogonal polynomials and Jacobi matrices associated with a measure whose essential support is a finite union of intervals on $\mathbb{R}$. In the case of totally disconnected support, spectral theory of orthogonal polynomials is less complete and numerical evaluations can be seen as a useful source of information.

In this talk, we discuss various properties and asymptotics of Jacobi matrices for a special family of fractal measures.

We focus on numerical results and conjectures. The talk is based on a joint work with Alexander Goncharov and Ahmet Nihat Şimşek.

On the median of the beta distribution

DIMITRIOS ASKITIS

UNIVERSITY OF COPENHAGEN

The median q of the beta distribution is defined implicitly by the equation

$$\int_0^q t^{a-1}(1-t)^{b-1}\mathrm{d}t = \frac{1}{2}\int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t.$$

We study the monotonicity and asymptotic properties of the median as a univariate function of the parameter a, as well as of the function $a \log q(a)$, related to its logarithm. In particular, we find asymptotic expansions for $a \to 0$ and ∞. These are related to the polygamma function and generalised Bernoulli polynomials.

Asymptotic analysis via calculus of hypergeometric functions

PETR BLASCHKE

OPAVA, CZECH REPUBLIC

The generalized hypergeometric function satisfies many identities or “transforms” which can be used to establish their asymptotic behavior for large argument and even, in some cases, for large parameters. We will show that using just three transforms alone, valid for a large class of multivariate hypergeometric functions, we can use a similar “calculus” to compute asymptotic expansions even in higher dimensions.

Orthogonal Polynomials and Integrable Systems

PETER CLARKSON

UNIVERSITY OF KENT, U.K.

In this talk I will discuss the relationship between orthogonal polynomials with respect to semi-classical weights, which are generalisations of the classical weights and arise in applications such as random matrices, and integrable systems, in particular the Painlevé equations and discrete Painlevé equations. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation. I will show that for some semi-classical weights the coefficients in the recurrence relation can be expressed in terms of Hankel determinants, which are Wronskians, that also arise in the description of special function solutions of Painlevé equations. The determinants arise as partition functions in random matrix models and the recurrence coefficients satisfy a discrete Painlevé equation. The semi-classical orthogonal polynomials discussed will include a generalization of the Freud weight and an Airy weight.

Orthogonal polynomials for the generalized Julia sets

ALEXANDER GONCHAROV

BILKENT UNIVERSITY, TURKEY

The talk deals with the orthogonal polynomials with respect to the equilibrium measure for the generalized Julia sets. The main subject of our consideration is the family of equilibrium Cantor sets. We discuss asymptotic behavior of Widom factors and the concept of the Szegő class for general (in particular for singular continuous) measures.

On zero distribution of analytic functions from certain classes and Lieb-Thirring type inequalities

STANISLAS KUPIN

UNIVERSITY OF BORDEAUX, FRANCE

In the first part of this talk I shall give an overview of recent results on distribution of zeros of analytic functions satisfying some growth conditions. I shall discuss the connections of these results to Lieb-Thirring type inequalities for certain operators of mathematical physics in the second part of the presentation. The talk is mainly based on results of Borichev-Golinskii-Kupin [2009, 2016].

Asymptotic zero distribution of Jacobi-Piñeiro and multiple Laguerre polynomials

THORSTEN NEUSCHEL

UNIVERSITÉ CATHOLIQUE DE LOUVAIN, BELGIUM

Multiple orthogonal polynomials are a natural generalization of classical orthogonal polynomials, and their theory originates from questions in approximation theory and analytic number theory. After introducing basic notions, in this talk we will turn our attention to the classes of Jacobi-Piñeiro and multiple Laguerre polynomials, which generalize the classical Jacobi polynomials and the Laguerre polynomials, respectively. We show how to derive explicit representations for the asymptotic zero distributions for indices on the diagonal. The proofs rely on the computation of ratio asymptotics from the nearest neighbor recurrence relations for these polynomials. Moreover, connections to random matrix theory can be found, for instance, by relating the limiting distributions of zeros to the so-called Fuss-Catalan distributions. This is joint work with Walter Van Assche.

Inverse Polynomial Images

KLAUS SCHIEFERMAYR

UNIVERSITY OF APPLIED SCIENCES UPPER AUSTRIA, CAMPUS WELS, AUSTRIA

Let P_n be a polynomial of degree n and consider the inverse image of the interval $[-1,1]$ under this polynomial mapping, that is,

$$P_n^{-1}([-1,1]) := \{z \in \mathbb{C} : P_n(z) \in [-1,1]\}.$$

We will call such sets inverse polynomial images (IPI). In this talk, an overview of the work of the author on IPI is given with several aspects on

- geometric properties of IPI
- the Green function of IPI and improvements of the Bernstein-Walsh inequality
- proving inequalities for the norm of Chebyshev polynomials on a compact set with the help of IPI
- representation of IPI consisting of two analytic Jordan arcs with the help of Jacobian elliptic and theta functions
- sets of minimal logarithmic capacity and the Chebotarev problem

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Around Poisson–Mehler summation formula

PAWEŁ J. SZABŁOWSKI

WARSAW UNIVERSITY OF TECHNOLOGY, POLAND

We study polynomials in x and y of degree $n+m : \{Q_{m,n}(x,y|t,q)\}_{n,m\geq 0}$ that are related to the generalization of Poisson–Mehler formula i.e. to the expansion

$$\sum_{i\geq 0} \frac{t^i}{[i]_q!} H_{i+n}(x|q)\, H_{m+i}(y|q),$$

where $\{H_n(x|q)\}_{n\geq -1}$ are the so-called $q-$Hermite polynomials (qH). In particular we show that the spaces $span\{Q_{i,n-i}(x,y|t,q) : i=0,\ldots,n\}_{n\geq 0}$ are orthogonal with respect to a certain measure (two-dimensional $(t,q)-$Normal distribution) on the square

$$\left\{(x,y) : |x|,|y| \leq 2/\sqrt{1-q}\right\}$$

being a generalization of two-dimensional Gaussian measure. We study structure of these polynomials showing in particular that they are rational functions of parameters t and q. We use them in various infinite expansions that can be viewed as simple generalization of the Poisson-Mehler summation formula. Further we use them in the expansion of the reciprocal of the right hand side of the Poisson-Mehler formula.

Scaling Limits for Partial Sums of Power Series

ANTONIO R. VARGAS

KU LEUVEN, BELGIUM

This talk will describe the universal asymptotic behavior of the partial sums of Maclaurin series for entire functions which have a certain asymptotic behavior at infinity. The scaling limits we obtain give direct information about the zeros of the partial sums. With this information we partially verify for this class of functions a conjecture concerning the angular distribution of these zeros known as the Saff-Varga Width Conjecture.

Large deviation principle in logarithmic potential theory

FRANCK WIELONSKY

UNIVERSITÉ AIX-MARSEILLE, FRANCE

After recalling a few basic facts about large deviations in probability and random matrix theory, we will describe how a general large deviation principle can be proved in the framework of logarithmic potential theory on the complex plane. This involves a L^2-type discretization of weighted logarithmic energy with respect to a measure that satisfies a Bernstein-Markov property. The derived large deviation principle holds in a scalar or vector setting, and in other situations as well.

This is a joint work with Thomas Bloom (Toronto) and Norman Levenberg (Bloomington).