QUADRATIC ALGEBRAS OF TYPE AIII; III

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Abstract. This is the last article in a series of three where we study some quadratic algebras related to quantized matrix algebras. In here we determine the full set of so-called cyclic representations of the quantized matrix algebra $M_q(3)$. Specifically, these are irreducible representations in which all generators are invertible and the assumption on q is that it is a primitive mth root of 1, with $m \geq 3$.

1. Introduction

The representation theory of $M_q(n)$ is related to the representation theory of the algebra of functions on the quantized $SL(n)$, roughly speaking by setting the quantum determinant equal to 1. Thus the investigation in [?] is of importance. Even more so are the articles by De Concini and Lyubashenko [?] and De Concini and Procesi [?], [?] which culminate in a complete classification of the irreducible representations of the quantum function algebras, corresponding to a simple complex Lie group, at a root of unity. But besides these representations, a completely new class of representations, cyclic representations, of $M_q(n)$ have been constructed and studied by different authors (e.g. [?], [?]).

In this article we study the cyclic representations of the quantum matrix algebra $M_q(3)$ and construct all the irreducible cyclic $M_q(3)$ modules explicitly.

2. Cyclic $M_q(3)$-modules

Recall that $M_q(n)$ is given by the relations

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$$\begin{aligned} Z_{i,j}Z_{i,k} &= qZ_{i,k}Z_{i,j} \text{ if } j<k, \\ Z_{i,j}Z_{k,j} &= qZ_{k,j}Z_{i,j} \text{ if } i<k, \\ Z_{i,j}Z_{s,t} &= Z_{s,t}Z_{i,j} \text{ if } i<s, t<j, \\ Z_{i,j}Z_{s,t} &= Z_{s,t}Z_{i,j} + (q-q^{-1})Z_{i,t}Z_{s,j} \text{ if } i<s, j<t, \end{aligned}$$

Let $\mathcal{I} = \{(x_1, x_2, \cdots, x_n)) \mid x_i^m = 1 \text{ for } i = 1, 2, \cdots, n\}$. Define

$$\tau_i : \mathcal{I} \longrightarrow \mathcal{I},$$

$$(x_1, x_2, \cdots, x_n) \mapsto (x_1, \cdots, x_{i-1}, qx_i, x_{i+1}, \cdots, x_n).$$

Let G be the automorphism group of $\mathcal{I}$ generated by τ_i for all $i = 1, 2, \cdots, n$. Let T be the subgroup of G generated by $\eta_i = \tau_i\tau_{i+1}$ for all $i = 1, 2, \cdots, n-1$.

Definition 2.1. *An $M_q(n)$-module V is called cyclic if every generator $Z_{i,j}$ is invertible on V.*

By using induction it is easy to prove that

Lemma 2.2. *If $i < k$ and $j < l$, then*

$$Z_{k,l}Z_{i,j}^s = Z_{i,j}^s Z_{k,l} + (q^{-1} - q^{2s-1})Z_{i,l}Z_{k,j}Z_{i,j}^{s-1}$$

and

$$Z_{i,j}Z_{k,l}^s = Z_{k,l}^s Z_{i,j} + (q - q^{1-2s})Z_{i,l}Z_{k,j}Z_{k,l}^{s-1}.$$

Corollary 2.3. *If q is an mth root of unity, then $Z_{i,j}^m$ is a central element for all $i, j = 1, 2, \cdots, n$.*

Remark 2.4. *Let V be an irreducible cyclic module. Since $Z_{i,j}^m$ is central there exists an $a_{i,j} \in \mathbb{C}^*$ such that $Z_{i,j}^m = a_{i,j}$ on V.*

For any $\chi = (\lambda_1, \lambda_2, \cdots, \lambda_n, \mu_1, \mu_2, \cdots, \mu_n) \in (\mathbb{C}^*)^{2n}$ we define an automorphism χ of $M_q(n)$ by

$$\chi(Z_{i,j}) = \lambda_i \mu_j Z_{i,j} \text{ for all } i, j = 1, 2, \cdots, n.$$

These automorphisms χ generate a group K of automorphisms of $M_q(n)$ which is isomorphic with $(\mathbb{C}^*)^{2n-1}$.

By the action of K we can assume that

$$Z_{n+1-i,i}^m = 1 \text{ for all } i$$

on an irreducible cyclic $M_q(n)$-module V. Hence the cyclic module V admits a weight space decomposition with respect to the commutative subalgebra H generated by $Z_{n+1-i,i}$ for all i:

$$V = \oplus_{\mathcal{I}} V(x_1, x_2, \cdots, x_n),$$

where

$$V(x_1, x_2, \cdots, x_n) = \{v \in V \mid Z_{n+1-i,i}v = x_i v \text{ for all } i\}.$$

If $V(x_1, x_2, \cdots, x_n) \neq 0$, $(x_1, x_2, \cdots, x_n)$ is called a weight of V and $V(x_1, x_2, \cdots, x_n)$ is called a weight space, a non-zero element from $V(x_1, x_2, \cdots, x_n)$ is called a weight vector of weight $(x_1, x_2, \cdots, x_n)$ and $\dim V(x_1, x_2, \cdots, x_n)$ is called the multiplicity of the weight $(x_1, x_2, \cdots, x_n)$. We denote by $P(V)$ the set of weights of the module V. Clearly we can always assume that $(1, 1, \cdots, 1)$ is a weight of V, by the action of K.

Theorem 2.5. *Let V be an irreducible cyclic $M_q(n)$-module. Then $\dim V = dm^{n-1}$ for some positive integer d.*

Proof. For any weight space $V(x_1, x_2, \cdots, x_n)$ we have

$$Z_{n-i,i}V(x_1, x_2, \cdots, x_n) \subset V(\eta_i^{-1}(x_1, x_2, \cdots, x_n))$$

for $i = 1, 2, \cdots, n-1$ and

$$Z_{n-i+1,i+1}V(x_1, x_2, \cdots, x_n) \subset V(\eta_i(x_1, x_2, \cdots, x_n))$$

for $i = 1, 2, \cdots, n-1$. Since both $Z_{n-i,i}$ and $Z_{n-i+1,i+1}$ are invertible we have

$$\dim V(x_1, x_2, \cdots, x_n) = \dim V(\phi(x_1, x_2, \cdots, x_n)) \text{ for all } \phi \in T.$$

So the weight set $P(V)$ is T-invariant and the weight multiplicities are also T-invariant. Obviously, each T-orbit of $P(V)$ consists of m^{n-1} elements. This completes the proof. □

Definition 2.6. *If $d = 1$, the module V is called a minimal cyclic module.*

Clearly, if V is a minimal cyclic module, then $P(V) = T(1, 1, \cdots, 1)$ and each weight is of multiplicity one.

Obviously we have

Lemma 2.7. *Let V be a minimal cyclic module and let $v \in V(x_1, x_2, \cdots, x_n)$. Then*

$$\{\Pi_{i=1}^{n-1} Z_{n-i+1,i+1}^{s_i} v \mid s_i = 0, 1, \cdots, m-1, i = 1, 2, \cdots, n-1\}$$

is a basis of V.

Let $\sigma, D \in End(\mathbb{C}^m)$ be defined such that with respect to the standard basis $v_0, v_1, \cdots, v_{m-1}$ of $\mathbb{C}^m$,

$$\sigma(v_j) = v_{j+1}, \text{ and } D(v_j) = q^j v_j \text{ for all } j = 0, \ldots, m-1 \in \mathbb{Z}/m\cdot\mathbb{Z}.$$

We denote by σ_i and D_i, for $i = 1, 2, \cdots, r$, the operators $1 \otimes 1 \otimes \cdots \otimes \sigma \otimes 1 \cdots \otimes 1$ and $1 \otimes 1 \otimes \cdots \otimes D \otimes 1 \cdots \otimes 1$ on $(\mathbb{C}^m)^r$ with σ and D, respectively, in the ith position.

Now let us focus on the classification of the cyclic $M_q(3)$-modules. Let V be a minimal cyclic module over $M_q(3)$. Then we have

$$Z_{31} = D_1, Z_{22} = D_1 D_2, Z_{13} = D_2;$$

$$Z_{32} = \lambda_1 \sigma_1, Z_{23} = \lambda_2 \sigma_2.$$

Lemma 2.8.

$$Z_{33} = q\lambda_1\lambda_2 D_1^{-1} D_2^{-1} \sigma_1 \sigma_2 = q^{-1} \lambda_1 \lambda_2 \sigma_1 \sigma_2 D_1^{-1} D_2^{-1}.$$

Proof. By Lemma **??** we need only compute the action of $Z_{3,3}$ on each basis element $Z_{3,2}^a Z_{2,3}^b v$. Clearly we have

$$Z_{3,3} Z_{3,2}^a Z_{2,3}^b v = q^{-a-b} Z_{3,2}^a Z_{2,3}^b Z_{3,3} v.$$

Obviously $Z_{3,3}v$ is also a weight vector of the same weight as $Z_{3,2}Z_{2,3}v$ with respect to D_1 and D_2. Hence there exists a $c \in \mathbb{C}^*$ such that

$$Z_{3,3} v = c Z_{3,2} Z_{2,3} v.$$

Therefore

$$Z_{3,3} Z_{3,2}^a Z_{2,3}^b v = c\lambda_1\lambda_2\sigma_1\sigma_2 D_1^{-1} D_2^{-1} (Z_{3,2}^a Z_{2,3}^b v).$$

By

$$Z_{2,2} Z_{3,3} = Z_{3,3} Z_{2,2} + (q - q^{-1}) Z_{2,3} Z_{3,2}$$

we have $c = q^{-1}$. This completes the proof. ☐

Lemma 2.9.

$$Z_{21} = \sigma_1' = \beta D_2 \sigma_1^{-1} + \lambda_1^{-1} q D_1^2 D_2 \sigma_1^{-1},$$

$$Z_{12} = \sigma_2' = \eta D_1 \sigma_2^{-1} + \lambda_2^{-1} q D_1 D_2^2 \sigma_2^{-1}$$

for some $\beta, \eta \in \mathbb{C}$.

Proof. By computing the weight of $Z_{2,1}v$ we know that there exists a non-zero complex number d such that

$$Z_{2,1} v = d Z_{3,2}^{-1} v.$$

Let us compute the action of $Z_{2,1}$ on $Z_{3,2}^a Z_{2,3}^b v$. We have

$$Z_{2,1} Z_{3,2}^a Z_{2,3}^b v = (Z_{3,2}^a Z_{2,1} + (q - q^{1-2a}) Z_{2,2} Z_{3,1} Z_{3,2}^{a-1}) Z_{2,3}^b v$$

$$= dq^b Z_{3,2}^{a-1} Z_{2,3}^b v + q D_1^2 D_2 Z_{3,2}^{a-1} Z_{2,3}^b v - q^{1-2a} D_1^2 D_2 Z_{3,2}^{a-1} Z_{2,3}^b v.$$

So $Z_{2,1} = \beta D_2\sigma_1^{-1} + \beta' D_1^2 D_2\sigma_1^{-1}$ for some $\beta, \beta' \in \mathbb{C}$. By

$$Z_{2,1}Z_{3,2} = Z_{3,2}Z_{2,1} + (q - q^{-1})Z_{2,2}Z_{3,1}$$

we have

$$Z_{2,1} = \beta D_2\sigma_1^{-1} + \lambda_1^{-1} q D_1^2 D_2\sigma_1^{-1}.$$

Similarly we can determine $Z_{1,2}$. This completes the proof. □

Analogously to the computation of Z_{33} we get

$$Z_{11} = (\beta + q\lambda_1^{-1}D_1^2)(\eta + q\lambda_2^{-1}D_2^2)\sigma_1^{-1}\sigma_2^{-1}.$$

By

$$Z_{11}Z_{33} = Z_{33}Z_{11} + (q - q^{-1})Z_{13}Z_{31}$$

we obtain

$$\beta\eta = 0.$$

This concludes our analysis. It is not hard to see that the imposed conditions also are sufficient to guarantee that we have a module.

Theorem 2.10. *Let V be a minimal cyclic $M_q(3)$ module. Then we can identify V with $(\mathbb{C}^m)^{\otimes 2}$ and choose the basis of V properly such that the action of the generators of the algebra $M_q(3)$ are given by the following formulas:*

$$Z_{31} = D_1, Z_{22} = D_1 D_2, Z_{13} = D_2;$$

$$Z_{32} = \lambda_1\sigma_1, Z_{23} = \lambda_2\sigma_2;$$

$$Z_{21} = \beta D_2\sigma_1^{-1} + \lambda_1^{-1} q D_1^2 D_2\sigma_1^{-1};$$

$$Z_{12} = \eta D_1\sigma_2^{-1} + \lambda_2^{-1} q D_1 D_2^2\sigma_2^{-1};$$

$$Z_{11} = (\beta + q\lambda_1^{-1}D_1^2)(\eta + q\lambda_2^{-1}D_2^2)\sigma_1^{-1}\sigma_2^{-1}$$

$$Z_{33} = q\lambda_1\lambda_2 D_1^{-1}D_2^{-1}\sigma_1\sigma_2;$$

where λ_1, λ_2 are free non-zero parameters and $\beta, \eta \in \mathbb{C}$ satisfy $\beta\eta = 0$, $\beta^m + \lambda_1^{-m} \neq 0$ and $\eta^m + \lambda_2^{-m} \neq 0$.

In the following we can assume that $\dim V > m^2$ for an irreducible cyclic $M_q(3)$ module.

Proposition 2.11. *Let q be a primitive mth root of unity for some odd integer m. Let V be an irreducible cyclic $M_q(3)$-module which is not a minimal cyclic module. Then*

$$\dim V = m^3,$$

and

$$\{Z_{3,2}^a Z_{2,3}^b Z_{3,3}^c v \mid a, b, c = 0, 1, \cdots, m-1\}$$

is a basis of V for any weight vector $v \in V$.

Proof. If $Z_{3,3}v$ is also a weight vector, then a similar computation as in the minimal cyclic case shows that $Z_{3,3} = q\lambda_1\lambda_2\sigma_1\sigma_2$ and furthermore the action of the $Z_{1,2}, Z_{2,1}$ and $Z_{1,1}$ are the same as the minimal cyclic case which is a contradiction! Hence $Z_{3,3}v$ is not a weight vector if V is an irreducible cyclic $M_q(3)$ module and $\dim V > m^2$. Clearly we can assume that

$$Z_{33}v = \sum_{x_2'} v_{qx_1, x_2', qx_3},$$

where $v_{qx_1,x_2',qx_3} \in V(qx_1, x_2', qx_3)$. By

$$Z_{22}Z_{33} = Z_{33}Z_{22} + (q - q^{-1})Z_{2,3}Z_{3,2}$$

we get

$$Z_{33}v = v_{qx_1,x_2,qx_3} + v_{qx_1,q^2x_2,qx_3}$$

and $v_{qx_1,x_2,qx_3} \neq 0, v_{qx_1,q^2x_2,qx_3} \neq 0$. Hence the weight set $P(V)$ of V is $< T, \tau_2^2 >$ invariant. Since m is odd we get $P(V) = \mathcal{I}$. This proves that $\dim V = m^3$ and each weight space is of dimension one. Hence

$$\{Z_{3,2}^a Z_{2,3}^b Z_{3,3}^c v \mid a, b, c = 0, 1, \cdots, m-1\}$$

is a basis of V for any weight $v \in V$. □

Now we fix a weight vector $v \in V(1,1,1)$ and identify V with $(\mathbb{C}^m)^{\otimes 3}$ by the following linear map:

$$(\nabla) \qquad Z_{3,2}^a Z_{2,3}^b Z_{3,3}^c v \mapsto \lambda_1^a \lambda_2^b \lambda_3^c v_a \otimes v_b \otimes v_c$$

Then a simple computation shows that

$$Z_{3,2} = \lambda_1\sigma_1, Z_{2,3} = \lambda_2\sigma_2, Z_{3,3} = \lambda_3 D_1^{-1} D_2^{-1}\sigma_3,$$

$$Z_{3,1} = D_1 D_3, Z_{1,3} = D_2 D_3,$$

Lemma 2.12.

$$Z_{2,2} = Z := q^{-1}\lambda_1\lambda_2\lambda_3^{-1}D_1D_2\sigma_1\sigma_2\sigma_3^{-1} + \sum_{i=0}^{m-1} a_i D_1D_2D_3^{-2i}\sigma_1^i\sigma_2^i\sigma_3^{-i},$$

where $a_i \in \mathbb{C}$*.*

Proof. At first we only consider the relations among the generators $Z_{i,j}, i+j \geq 4$. By direct verification we see that the stated $Z_{2,2} = Z$ satisfies the relations for arbitrary $a_i \in \mathbb{C}$ for $i = 0, 1, \cdots, m-1$. If $Z_{2,2} = X$ satisfies the same relations among the generators $Z_{i,j}, i+j \geq 4$, then we write $X - q^{-1}\lambda_1\lambda_2\lambda_3^{-1}D_1D_2\sigma_1\sigma_2\sigma_3^{-1}$ into a sum of different monomials of D_1, D_2, D_3 and $\sigma_1, \sigma_2, \sigma_3$. Then each of the monomials Y commute with $Z_{3,1}, Z_{3,3}$ and $Z_{1,3}$ and satisfy:

$$YZ_{3,2} = qZ_{3,2}Y, \; YZ_{2,3} = qZ_{2,3}Y.$$

Hence Y must be a multiple of $D_1D_2D_3^{-2i}\sigma_1^i\sigma_2^i\sigma_3^{-i}$ for some i. This completes the proof. □

Lemma 2.13.

$$Z_{2,1} = \lambda_1^{-1}qD_1D_3Z\sigma_1^{-1} + cD_2D_3\sigma_1^{-1},$$

$$Z_{1,2} = \lambda_2^{-1}qD_2D_3Z\sigma_2^{-1} + bD_1D_3\sigma_2^{-1}$$

for some $b, c \in \mathbb{C}$*.*

Proof. By the results in [?] we know that $(Z_{2,1}Z_{3,2} - qZ_{3,1}Z_{2,2})Z_{1,3}^{m-1}$ and $(Z_{1,2}Z_{2,3} - qZ_{1,3}Z_{2,2})Z_{3,1}^{m-1}$ are central elements of the algebra $M_q(3)$. Therefore they are scalars when acting on an irreducible $M_q(3)$ module. Hence the most general form of the elements are the given. It is straightforward to verify that these elements do satisfy all the relations with the elements $Z_{i,j}$ for $i + j \geq 4$. □

Lemma 2.14.

$$Z_{1,1} = \lambda_1^{-1}\lambda_2^{-1}D_1D_2D_3^2\sigma_1^{-1}\sigma_2^{-1}Z$$
$$+q\lambda_1^{-1}bD_1^2D_3^2\sigma_1^{-1}\sigma_2^{-1} + cD_2^2D_3^2\sigma_1^{-1}\sigma_2^{-1} + q^2cdD_1D_2D_3^2\sigma_1^{-1}\sigma_2^{-1}Z^{-1},$$

where b, c *are complex constants.*

Proof. Let $Z_{1,1} = \sum a_{ijkrst}D_1^iD_2^jD_3^k\sigma_1^r\sigma_2^s\sigma_3^t$. By

$$Z_{1,1}Z_{1,3} = qZ_{1,3}Z_{1,1}, \; Z_{1,1}Z_{3,1} = qZ_{3,1}Z_{1,1}$$

we have

$$Z_{1,1} = \sum a_{ijks}D_1^iD_2^jD_3^k\sigma_1^s\sigma_2^s\sigma_3^{m-s-1}.$$

By

$$Z_{1,1}Z_{3,2} = Z_{3,2}Z_{1,1} + (q - q^{-1})Z_{1,2}Z_{3,1}$$

we have

$$Z_{1,1} =$$
$$q\lambda_1^{-1}bD_1^2D_3^2\sigma_1^{-1}\sigma_2^{-1} + \lambda_1^{-1}\lambda_2^{-1}D_1D_2D_3^2\sigma_1^{-1}\sigma_2^{-1}Z + (\star_1),$$

where $(\star_1)$ does not contain D_1.

Similarly we also get

$$Z_{1,1} =$$
$$q\lambda_2^{-1}cD_2^2D_3^2\sigma_1^{-1}\sigma_2^{-1} + \lambda_1^{-1}\lambda_2^{-1}D_1D_2D_3^2\sigma_1^{-1}\sigma_2^{-1}Z + (\star_2),$$

where $(\star_2)$ does not contain D_2.

Hence we have

$$Z_{1,1} = \lambda_1^{-1}\lambda_2^{-1}D_1D_2D_3^2\sigma_1^{-1}\sigma_2^{-1}Z$$
$$+q\lambda_1^{-1}bD_1^2D_3^2\sigma_1^{-1}\sigma_2^{-1} + cD_2^2D_3^2\sigma_1^{-1}\sigma_2^{-1} + (\star_3),$$

where $(\star_3)$ does not contain D_1, D_2, Finally, by checking the identity $Z_{1,1}Z_{2,2} - Z_{2,2}Z_{1,1} = (q-q^{-1})Z_{2,1}Z_{1,2}$ it follows that $(\star_3)$ must have the indicated form. □

That the above generators do in fact define a module is elementary to verify. In order that it be cyclic, we need to impose the further requirements that $b^m \neq -\lambda_2^{-m}$, $c^m \neq -\lambda_1^{-m}$, and $\sum_{i=0}^{m-1} a_i^m \neq -\lambda_1^m\lambda_2^m\lambda_3^{-m}$. Thus we can state

Theorem 2.15. *Let V be an irreducible cyclic $M_q(3)$ module of dimension m^3. Then we can identify V with $(\mathbb{C}^m)^3$ as given by (∇). By choosing the basis of V appropriately, the action of the generators of $M_q(3)$ are given by*

$$Z_{3,2} = \lambda_1\sigma_1, Z_{2,3} = \lambda_2\sigma_2, Z_{3,3} = \lambda_3D_1^{-1}D_2^{-1}\sigma_3,$$

$$Z_{3,1} = D_1D_3, Z_{1,3} = D_2D_3,$$

$$Z_{2,2} = Z := q^{-1}\lambda_1\lambda_2\lambda_3^{-1}D_1D_2\sigma_1\sigma_2\sigma_3^{-1} + \sum_{i=0}^{m-1} a_iD_1D_2D_3^{-2i}\sigma_1^i\sigma_2^i\sigma_3^{-i},$$

$$Z_{2,1} = \lambda_1^{-1}qD_1D_3Z\sigma_1^{-1} + cD_2D_3\sigma_1^{-1},$$

$$Z_{1,2} = \lambda_2^{-1}qD_2D_3Z\sigma_2^{-1} + bD_1D_3\sigma_2^{-1},$$

and

$$Z_{1,1} = \lambda_1^{-1}\lambda_2^{-1} D_1 D_2 D_3^2 \sigma_1^{-1}\sigma_2^{-1} Z$$
$$+q\lambda_1^{-1} b D_1^2 D_3^2 \sigma_1^{-1}\sigma_2^{-1} + c D_2^2 D_3^2 \sigma_1^{-1}\sigma_2^{-1} + q^2 cd D_1 D_2 D_3^2 \sigma_1^{-1}\sigma_2^{-1} Z^{-1},$$

where $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}^*$*,* $a_i \in \mathbb{C}$ *for* $i = 0, 1, \cdots, m-1$*, and* $b, c \in \mathbb{C}$*,* $b^m \neq -\lambda_2^{-m}$*,* $c^m \neq -\lambda_1^{-m}$*, and* $\sum_{i=0}^{m-1} a_i^m \neq -\lambda_1^m \lambda_2^m \lambda_3^{-m}$*.*

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