# A TABLE OF $A_5$-FIELDS.

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For an extension $K$ of $\mathbb{Q}$ with Galois group isomorphic to $A_5$, we shall by a **root field** of $K$ mean any extension of $\mathbb{Q}$ of degree 5 contained in $K$. Table 1 is a table containing all $A_5$-extensions of $\mathbb{Q}$ which are non-real and for which the discriminant of a root field is at the most $2083^2$. This table was used in section 4 of I, and our present purpose is to describe how the table was obtained.

Our starting point is the following theorem of Hunter.

**Theorem.** *(cf. [3]) Suppose that $F/\mathbb{Q}$ is an extension of degree 5 and discriminant $D$. Then there is an algebraic integer $\theta$ in $F$ such that*

$$F = \mathbb{Q}(\theta),$$

*and such that, denoting by $\theta_1 = \theta, \theta_2, \theta_3, \theta_4, \theta_5$ the conjugates of $\theta$,*

$$\sum_i \theta_i \in \{0, \pm 1, \pm 2\},$$

*and*

$$\sum_i |\theta_i|^2 \leq \left(\frac{8}{5}|D|\right)^{\frac{1}{4}} .$$

Let $D$ be a positive real number. If $K/\mathbb{Q}$ is a non-real Galois extension with Galois group isomorphic to $A_5$ such that the discriminant of a root field of $K$ is bounded by $D$, then we conclude that $K$ is the splitting field of a polynomial

$$f(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 \in \mathbb{Z}[x],$$

such that

$$a_1 \in \{0, \pm 1, \pm 2\},$$

and, denoting by $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ the roots of $f$,

$$\sum_i |\theta_i|^2 \leq \left(\frac{8}{5}D\right)^{\frac{1}{4}} .$$

We want to bound the coefficients of $f$.

Suppose first that $a_1 = 0$. Now, $f(x)$ has 1 real root and 2 pairs of complex conjugate roots, so we may write:

$$\theta_1 = r \ , \quad \theta_2 = \sigma_1 + it_1 \ , \quad \theta_3 = \sigma_1 - it_1 \ , \quad \theta_4 = \sigma_2 + it_2 \ , \quad \theta_5 = \sigma_2 - it_2 \ ,$$

with $r, \sigma_1, \sigma_2, t_1, t_2 \in \mathbb{R}$. We have:

$$r + 2\sigma_1 + 2\sigma_2 = \sum_i \theta_i = 0,$$

i.e:

$$r = -2(\sigma_1 + \sigma_2).$$

We then compute:

$$\begin{aligned}
\sum |\theta_i|^2 &= 6\sigma_1^2 + 8\sigma_1\sigma_2 + 6\sigma_2^2 + 2t_1^2 + 2t_2^2 \ , \\
a_2 &= -3\sigma_1^2 - 4\sigma_1\sigma_2 - 3\sigma_2^2 + t_1^2 + t_2^2 \ , \\
a_3 &= 2\sigma_1^3 + 8\sigma_1^2\sigma_2 + 8\sigma_1\sigma_2^2 + 2\sigma_2^3 + 2\sigma_1 t_1^2 + 2\sigma_2 t_2^2 \ , \\
a_4 &= -4\sigma_1^3\sigma_2 - 7\sigma_1^2\sigma_2^2 - 4\sigma_1\sigma_2^3 - 3\sigma_1^2 t_2^2 - 3\sigma_2^2 t_1^2 - 4\sigma_1\sigma_2 t_1^2 \\
&\quad -4\sigma_1\sigma_2 t_2^2 + t_1^2 t_2^2 \ .
\end{aligned}$$

From this it follows immediately that

$$|a_2| \le \frac{1}{2}\sum |\theta_i|^2 \ .$$

Put $\alpha_1 = \sigma_1^2 + t_1^2$ and $\alpha_2 = \sigma_2^2 + t_2^2$. Then:

$$\begin{aligned}
\sum |\theta_i|^2 &= 4\sigma_1^2 + 8\sigma_1\sigma_2 + 4\sigma_2^2 + 2\alpha_1 + 2\alpha_2 \ , \\
a_3 &= 8\sigma_1^2\sigma_2 + 8\sigma_1\sigma_2^2 + 2\sigma_1\alpha_1 + 2\sigma_2\alpha_2 \ , \\
a_4 &= -4\sigma_1^2\alpha_2 - 4\sigma_1\sigma_2\alpha_1 - 4\sigma_1\sigma_2\alpha_2 - 4\sigma_2^2\alpha_1 + \alpha_1\alpha_2 \ .
\end{aligned}$$

If we fix the value of $\sum |\theta_i|^2$ to be $\beta$, we may then seek the extremal points of $a_3$ and $a_4$ considered as functions of $\sigma_1, \sigma_2, \alpha_1, \alpha_2$ under the restrictions

$$\alpha_1 \ge \sigma_1^2 \quad \text{and} \quad \alpha_2 \ge \sigma_2^2 \ .$$

It turns out to be unproblematic to find these extremal points by applying the Langrangian method. We shall not repeat the computations here, but merely note that one finds the following possible extremal values for $a_3$ and $a_4$:

$$\pm\frac{1}{3\sqrt{30}}\beta^{\frac{3}{2}} \ , \quad \pm\frac{1}{3\sqrt{3}}\beta^{\frac{3}{2}} \ , \quad \pm\frac{1}{5}\beta^{\frac{3}{2}} \ , \quad \pm\frac{1}{2\sqrt{5}}\beta^{\frac{3}{2}} \ , \quad \text{for} \quad a_3 \ ,$$

and

$$-\frac{4}{61}\beta^2 \ , \quad -\frac{1}{20}\beta^2 \ , \quad -\frac{3}{80}\beta^2 \ , \quad 0 \ , \quad \frac{1}{16}\beta^2 \ , \quad \frac{1}{15}\beta^2 \ , \quad \frac{3}{40}\beta^2 \ , \quad \text{for} \quad a_4 \ .$$

For $a_5$ we may use the inequality between arithmetic and geometric means:

$$\left(|a_5|^2\right)^{\frac{1}{5}} = \left(\prod |\theta_i|^2\right)^{\frac{1}{5}} \le \frac{1}{5}\sum |\theta_i|^2 \ ,$$

i.e.:

$$|a_5| \le \left(\frac{1}{5}\sum |\theta_i|^2\right)^{\frac{5}{2}} \ .$$

Noting that we may well assume that $a_5$ is positive, we conclude that (in the case $a_1 = 0$):

$$\begin{array}{rcccl} -\frac{1}{2}\beta(D) & \leq & a_2 & \leq & \frac{1}{2}\beta(D) \ , \\ -\frac{1}{2\sqrt{5}}\beta(D)^{\frac{3}{2}} & \leq & a_3 & \leq & \frac{1}{2\sqrt{5}}\beta(D)^{\frac{3}{2}} \ , \\ -\frac{4}{61}\beta(D)^2 & \leq & a_4 & \leq & \frac{3}{40}\beta(D)^2 \ , \\ 1 & \leq & a_5 & \leq & \left(\frac{1}{5}\beta(D)\right)^{\frac{5}{2}} \ , \end{array}$$

with

$$\beta(D) = \left(\frac{8}{5}D\right)^{\frac{1}{4}} .$$

If $a_1 \neq 0$, we first note that the bounds for $a_5$ obviously still hold. Secondly, we consider the polynomial

$$\tilde{f}(x) = f\left(x - \frac{1}{5}a_1\right) = x^5 + \tilde{a}_2 x^3 + \tilde{a}_3 x^2 + \tilde{a}_4 x + \tilde{a}_5 \ ,$$

where:

$$\begin{array}{rcl} a_2 & = & \tilde{a}_2 + \frac{2}{5}a_1^2 \ , \\ a_3 & = & \tilde{a}_3 + \frac{3}{5}a_1\tilde{a}_2 + \frac{2}{25}a_1^3 \ , \\ a_4 & = & \tilde{a}_4 + \frac{2}{5}a_1\tilde{a}_3 + \frac{3}{25}a_1^2\tilde{a}_2 + \frac{1}{125}a_1^4 \ . \end{array}$$

Now, $\tilde{f}(x)$ has the roots $\theta_i + \frac{1}{5}a_1$, $i = 1, 2, 3, 4, 5$, and we find (using $\sum \theta_i = -a_1$):

$$\sum_i |\theta_i + \frac{1}{5}a_1|^2 = -\frac{1}{5}a_1^2 + \sum_i |\theta_i|^2.$$

The above considerations then give us bounds for $\tilde{a}_2, \tilde{a}_3, \tilde{a}_4$ in terms of $\tilde{\beta}(D) = -\frac{1}{5}a_1^2 + \beta(D)$, and this of course gives us immediately bounds for $a_2, a_3, a_4$.

For the case:

$$D = 2083^2,$$

we obtain the following bounds:

$$\begin{array}{lrl} \text{For} \quad a_1 = 0 \ : & -25 & \leq a_2 \leq 25 \ , \\ & -82 & \leq a_3 \leq 82 \ , \\ & -172 & \leq a_4 \leq 197 \ , \\ & 1 & \leq a_5 \leq 337 \ . \end{array}$$

$$\begin{array}{lrl} \text{For} \quad a_1 = \pm 1 \ : & -25 & \leq a_2 \leq 25 \ , \\ & -97 & \leq a_3 \leq 97 \ , \\ & -207 & \leq a_4 \leq 231 \ , \\ & 1 & \leq a_5 \leq 337 \ . \end{array}$$

$$\begin{aligned} \text{For} \quad a_1 = \pm 2 \ : \quad -23 &\le a_2 \le 27 \ , \\ -108 &\le a_3 \le 108 \ , \\ -236 &\le a_4 \le 260 \ , \\ 1 &\le a_5 \le 337 \ . \end{aligned}$$

This gives us about $7.7 \cdot 10^9$ polynomials, and these polynomials are now investigated computationally through the following steps:

**(A)** The polynomial is eliminated, if its discriminant is not a square modulo some prime in $\{3, 5, 7, 11, 13, 17, 19, 23, 29\}$.

**(B)** For the remaining polynomials the polynomial discriminant is calculated, and the polynomial is eliminated, if this discriminant is not a square in $\mathbb{Z}$.

**(C)** The reducible survivors from **(B)** are eliminated.

**(D)** For the remaining polynomials it is tested whether the splitting field has Galois group isomorphic to $A_5$ or not. If not, the polynomial is eliminated.

**(E)** Among the remaining polynomials, those are selected for which the field discriminant of a field obtained by adjoining 1 root of the polynomial to $\mathbb{Q}$ is $\le 2083^2$.

**(F)** These selected polynomials are put in classes such that 2 polynomials are in the same class, if and only if their splitting fields are identical. From each class, 1 polynomial is chosen.

We shall briefly describe the methods used to perform these steps.

**(A)**, **(B)** : The discriminant (mod $p$ or over $\mathbb{Z}$) of a polynomial

$$f(x) = x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5 \in \mathbb{Z}[x],$$

is the resultant $R(f, f')$, where

$$f'(x) = 5x^4 + 4a_1 x^3 + 3a_2 x^2 + 2a_3 x + a_4,$$

i.e., it is the determinant of the matrix:

$$\begin{pmatrix} 1 & a_1 & a_2 & a_3 & a_4 & a_5 & 0 & 0 & 0 \\ 0 & 1 & a_1 & a_2 & a_3 & a_4 & a_5 & 0 & 0 \\ 0 & 0 & 1 & a_1 & a_2 & a_3 & a_4 & a_5 & 0 \\ 0 & 0 & 0 & 1 & a_1 & a_2 & a_3 & a_4 & a_5 \\ 5 & 4a_1 & 3a_2 & 2a_3 & a_4 & 0 & 0 & 0 & 0 \\ 0 & 5 & 4a_1 & 3a_2 & 2a_3 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 5 & 4a_1 & 3a_2 & 2a_3 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 5 & 4a_1 & 3a_2 & 2a_3 & a_4 & 0 \\ 0 & 0 & 0 & 0 & 5 & 4a_1 & 3a_2 & 2a_3 & a_4 \end{pmatrix} .$$

Introducing

$$c_i = -2a_{i+1} + a_1 a_i \ , \text{for} \quad i = 1, 2, 3, 4,$$

and

$$c_5 = a_1 a_5 \ ,$$

we see that the discriminant of $f(x)$ is the determinant of:

$$\begin{pmatrix} 1 & a_1 & a_2 & a_3 & a_4 & a_5 \\ c_4 - a_1c_1 & c_3 - a_2c_1 & c_2 - a_3c_1 & c_1 - a_4c_1 & -a_5c_1 & 0 \\ 0 & c_4 - a_1c_1 & c_3 - a_2c_1 & c_2 - a_3c_1 & c_1 - a_4c_1 & -a_5c_1 \\ 0 & c_1 & c_2 & c_3 & c_4 & c_5 \\ 0 & -a_1 & -2a_2 & -3a_3 & -4a_4 & -5a_5 \\ 0 & 5 & 4a_1 & 3a_2 & 2a_3 & a_4 \end{pmatrix}$$

This determinant is computed using Gauß elimination to triangulize this matrix.

Of course one could also use explicit formulas expressing the polynomial discriminant as a polynomial in the $a_i$'s, but the above method is considerably faster.

In step **(A)** it is convenient first to generate tables of all polynomials over $\mathbb{Z}/\mathbb{Z}p$ for which the discriminant is a square in $\mathbb{Z}/\mathbb{Z}p$, for $p = 3, 5, 7, 11, 13, 17, 19, 23, 29$. A polynomial $f(x) \in \mathbb{Z}[x]$ is then tested by reducing its coefficients modulo $p$ and checking whether this reduction occurs in the table belonging to $p$. If this is not the case for one of the above primes, the polynomial is eliminated.

**(C)**, **(F)** : Here the problem is to factorize a polynomial $f(x) \in \mathbb{Z}[x]$ either over $\mathbb{Q}$ (for step **(C)**) or over an extension of degree 5 over $\mathbb{Q}$ (for step **(F)**), where of course the latter case is the most complicated. The algorithm given in [6] was used.

**(D)** : The Galois group of the splitting field of any polynomial which is tested in step **(D)** is isomorphic to a transitive subgroup of $A_5$. The question is whether this Galois group is solvable or not. Let us recall how this question can be resolved by use of the 'Cayley resolvent'.

Consider the symmetric group $S_5$ on the symbols $1, 2, 3, 4, 5$, and in it the permutations:

$$\sigma = (1\ 2\ 3\ 4\ 5)\ , \quad \tau = (1\ 2\ 4\ 3).$$

Then $\sigma$ and $\tau^2$ generate a subgroup of $A_5$ isomorphic to the dihedral group of order 10. The elements

$$1, (1\ 2\ 5), (1\ 5\ 2), (1\ 4\ 5), (1\ 5\ 4), (2\ 3\ 5)$$

form a complete set of (right) coset representatives of $\langle \sigma, \tau^2 \rangle$ in $A_5$. Of course, $S_5 = A_5 \cup \tau A_5$. Consider the field $\mathbb{Q}(x_1, x_2, x_3, x_4, x_5)$ with the action of $S_5$ through permutation of the $x_i$'s. Let $a_1, a_2, a_3, a_4, a_5$ be the elementary symmetric polynomials in the $x_i$, i.e.:

$$\prod_i (t - x_i) = t^5 + a_1t^4 + a_2t^3 + a_3t^2 + a_4t + a_5\ .$$

Let $v = x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + x_1x_5$. Then both $v$ and $\tau v$ are invariant under $\langle \sigma, \tau^2 \rangle$. Hence the element

$$u_1 = v - \tau v = x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + x_1x_5 - x_2x_4 - x_1x_4 - x_1x_3 - x_3x_5 - x_2x_5$$

is also invariant under $\langle \sigma, \tau^2 \rangle$. It has 6 translates under $A_5$:

$$\begin{aligned}
u_2 &= (1\ 2\ 5)u_1 &= \quad & x_2x_5 + x_3x_5 + x_3x_4 + x_1x_4 + x_1x_2 \\
&&& -x_4x_5 - x_2x_4 - x_2x_3 - x_1x_3 - x_1x_5 \ , \\
u_3 &= (1\ 5\ 2)u_1 &= \quad & x_1x_5 + x_1x_3 + x_3x_4 + x_2x_4 + x_2x_5 \\
&&& -x_1x_4 - x_4x_5 - x_3x_5 - x_2x_3 - x_1x_2 \ , \\
u_4 &= (1\ 4\ 5)u_1 &= \quad & x_2x_4 + x_2x_3 + x_3x_5 + x_1x_5 + x_1x_4 \\
&&& -x_2x_5 - x_4x_5 - x_3x_4 - x_1x_3 - x_1x_2 \ , \\
u_5 &= (1\ 5\ 4)u_1 &= \quad & x_2x_5 + x_2x_3 + x_1x_3 + x_1x_4 + x_4x_5 \\
&&& -x_1x_2 - x_1x_5 - x_3x_5 - x_3x_4 - x_2x_4 \ , \\
u_6 &= (2\ 3\ 5)u_1 &= \quad & x_1x_3 + x_3x_5 + x_4x_5 + x_2x_4 + x_1x_2 \\
&&& -x_3x_4 - x_1x_4 - x_1x_5 - x_2x_5 - x_2x_3 \ .
\end{aligned}$$

Furthermore,

$$\tau u_1 = -u_1 \ , \quad \tau u_2 = -u_2 \ , \quad \tau u_3 = -u_4 \ ,$$

$$\tau u_4 = -u_6 \ , \quad \tau u_5 = -u_3 \ , \quad \tau u_6 = -u_5 \ .$$

Define $b_1, b_2, b_3, b_4, b_5, b_6$ by:

$$\prod_i (y - u_i) = y^6 + b_1 y^5 + b_2 y^4 + b_3 y^3 + b_4 y^2 + b_5 y + b_6 \ .$$

Now, $b_1, b_3, b_5$ are skew-symmetric in the $x_i$'s, and are therefore multiples of

$$\Delta = \prod_{i<j} (x_j - x_i)$$

in $\mathbb{Q}(x_1, \ldots, x_5)$. Since $b_1, b_3, b_5, \Delta$ are homogeneous in the $x_i$'s of degrees $2, 6, 10$ and 10 respectively, we conclude that:

$$b_1 = b_3 = 0 \ , \quad \frac{b_5}{\Delta} \in \mathbb{Q} \ .$$

Substituting (for example) $0, 1, -1, 2, -2$ for $x_1, \ldots, x_5$, one easily finds:

$$b_5 = -32\Delta \ .$$

The elements $b_2, b_4, b_6$ are symmetric in the $x_i$'s, and they are therefore polynomials in the $a_i$'s. Using the computer-algebra system MAPLE, we found:

$$
\begin{aligned}
b_2 &= -3a_2^2 + 8a_1a_3 - 20a_4 \ , \\
b_4 &= 3a_2^4 - 16a_1a_2^2a_3 + 16a_1^2a_3^2 + 16a_1^2a_2a_4 - 64a_1^3a_5 + 16a_2a_3^2 - 8a_2^2a_4 \\
&\quad -112a_1a_3a_4 + 240a_1a_2a_5 + 240a_4^2 - 400a_3a_5 \ , \\
b_6 &= -a_2^6 + 8a_1a_2^4a_3 - 16a_1^2a_2^3a_4 - 16a_1^2a_2^2a_3^2 + 64a_1^3a_2a_3a_4 - 64a_1^4a_4^2 \\
&\quad -16a_2^3a_3^2 + 28a_2^4a_4 + 64a_1a_2a_3^3 - 112a_1a_2^2a_3a_4 + 48a_1a_2^3a_5 \\
&\quad -128a_1^2a_3^2a_4 + 224a_1^2a_2a_4^2 - 192a_1^2a_2a_3a_5 + 384a_1^3a_4a_5 + 224a_2a_3^2a_4 \\
&\quad -64a_3^4 - 176a_2^2a_4^2 - 80a_2^2a_3a_5 - 64a_1a_3a_4^2 + 640a_1a_3^2a_5 \\
&\quad -640a_1a_2a_4a_5 - 1600a_1^2a_5^2 + 320a_4^3 - 1600a_3a_4a_5 + 4000a_2a_5^2 \ .
\end{aligned}
$$

The conclusion is, that if a polynomial

$$f(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 \in \mathbb{Q}[x]$$

is irreducible and has discriminant $d^2$ with $d \in \mathbb{Q}$, then its Galois group is isomorphic to $A_5$, if and only if the polynomial:

$$y^6 + b_2y^4 + b_4y^2 - 32dy + b_6 \ ,$$

where $b_2, b_4, b_6$ are defined in terms of the $a_i$'s by the above formulas, does not have a rational root.

**Remark 1.** *The above formula for $b_4$ does not quite coincide with the formula given on p. 99 in* [2].

**(E)** : To compute this field discriminant we used the computer-algebra system SIMATH. The algorithm involved is described in [4], chap. 4. See also [1].

In all, 238 fields emerged from this process. Only one of them, namely the field no. 176 in table 1, is real; it is marked with a star for this reason. To test whether a given field was real, we used the following well-known method (cf. [5], p. 259 and p. 284):

The irreducible polynomial:

$$f(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 \in \mathbb{Z}[x]$$

has 5 real roots if and only if the matrix

$$\begin{pmatrix} 4a_4^2-10a_3a_5 & 3a_3a_4-15a_2a_5 & 2a_2a_4-20a_1a_5 & a_1a_4-25a_5 \\ 3a_3a_4-15a_2a_5 & 6a_3^2-10a_2a_4-20a_1a_5 & 4a_2a_3-15a_1a_4-25a_5 & 2a_1a_3-20a_4 \\ 2a_2a_4-20a_1a_5 & 4a_2a_3-15a_1a_4-25a_5 & 6a_2^2-10a_1a_3-20a_4 & 3a_1a_2-15a_3 \\ a_1a_4-25a_5 & 2a_1a_3-20a_4 & 3a_1a_2-15a_3 & 4a_1^2-10a_2 \end{pmatrix}$$

and its principal minors all have positive determinant.

The 238 fields are listed in table 1. For each field the smallest possible conductor of a lifting to a 2-dimensional Galois representation over $\mathbb{Q}$ of (one, hence any of) the 2-dimensional projective representation(s) associated with the field, is displayed.

This minimal conductor can be computed from the knowledge of the structure of the associated local projective representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$, where $p$ runs over the ramified prime numbers, by using the theory given in I. But, as has been shown by Buhler (cf. [2]), it may also in most cases (and in fact in all cases of table 1) be determined alone from the knowledge of the type of factorization of the ramified prime numbers in a root field of the field in question.

The notation of table 1 is as follows: $\sqrt{d}$ denotes the square root of the discriminant of a root field of the given $A_5$-extension. 'polynomial' denotes a generating equation of the field given by its coefficients $(a_1, a_2, a_3, a_4, a_5)$. 'conductor' denotes the above-mentioned minimal conductor of liftings of the associated projective representations. Under 'ramified primes' the type of factorization of each of the ramified prime numbers in the root field is given. The notation:

$$p = f_1^{e_1} \dots f_s^{e_s}$$

means that $(p)$ factorizes as:

$$(p) = p_1^{e_1} \dots p_s^{e_s},$$

where the residue degree of the prime $p_i$ is $f_i$.

Table 2 lists the 238 fields ordered by the size of the minimal conductor mentioned above. The notation in table 2 is the same as for table 1.

It should be noted that the book [2] of J. P. Buhler contains also a table of $A_5$-fields with small discriminants of the corresponding root fields. In fact, one of the examples occurring in his table served as a motivation for fixing the bound $D \leq 2083^2$ above. The main difference between his table and ours is that the above analysis of the bounds of the polynomial coefficients is lacking in [2]; this means that one does not know for what bound on $D$ his table is complete (though it is estimated in [2] that the table in [2] is complete for the bound $D \leq 200^2$, cf. pp. 42–43 in [2]). But for use in section 5 of I (and in VI) we must have completeness for the bound $D \leq 2083^2$, and for this bound the table in [2] is certainly not complete as a comparison with our table 1 shows. Thus, for the purposes in section 5 of I ( and in VI) the table of Buhler, though it served as a starting point, would have been insufficient.

## References

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