The tower of K-theory of truncated polynomial algebras. A graphics illustration

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The figures on the following pages illustrate the map from [**1**, Thm. A] that is defined by the composition

$$\mathbb{W}_{m(i+1)}(\mathbb{F}_p) \xrightarrow{\text{res}} \mathbb{W}_{n(i+1)}(\mathbb{F}_p) \xrightarrow{m_\alpha} \mathbb{W}_{n(i+1)}(\mathbb{F}_p) \tag{1}$$

of the restriction map and the multiplication by an element $\alpha = \alpha_p(m,n,i)$ of $\mathbb{W}(\mathbb{F}_p)$ that is characterized, up to multiplication by a unit of $\mathbb{W}(\mathbb{F}_p)$, by the divisor

$$\operatorname{div}(\alpha_p(m,n,i)) = \sum_{0 \leqslant h < i} \big(\operatorname{div}(\mathbb{W}_{m(h+1)}(\mathbb{F}_p)) - \operatorname{div}(\mathbb{W}_{n(h+1)}(\mathbb{F}_p)) \big). \tag{2}$$

We recall from *op. cit.*, §1, that there is a canonical isomorphism

$$\eta \colon \mathbb{W}_{k(i+1)}(\mathbb{F}_p) \xrightarrow{\sim} \prod_{j \in I_p} W_s(\mathbb{F}_p),$$

where the product is indexed by the set I_p of positive integers j that are not divisible by p, and where $s = s_p(k,i,j)$ is the larger of 0 and the unique integer that satisfies the inequality $p^{s-1}j \leqslant k(i+1) < p^s j$. Let $\alpha_p(m,n,i,j)$ be the jth component of the image of $\alpha_p(m,n,i)$ by the isomorphism η, for $k=\infty$. Then the divisorial equation (2) above is equivalent to the equations

$$v_p(\alpha_p(m,n,i,j)) = \sum_{0 \leqslant h < i} \big(s_p(m,h,j) - s_p(n,h,j) \big), \tag{3}$$

for $j \in I_p$. Hence, we see that the component indexed by $j \in I_p$ of the map (1) is equal to zero if and only if

$$v_p(\alpha_p(m,n,i,j)) \geqslant s_p(n,i,j). \tag{4}$$

The figures on the following pages illustrate this inequality. The index $j \in I_p$ is depicted along the horizontal axis, and the non-negative integer i along the vertical axis. The color red indicates the inequality (4) is *not* satisfied or, equivalently, that the jth factor of the map (1) is non-zero; compare [**1**, Thm. B]. At present, the author does not understand the nature or the value of the apparent interlocking slopes that are seen in the figures below.

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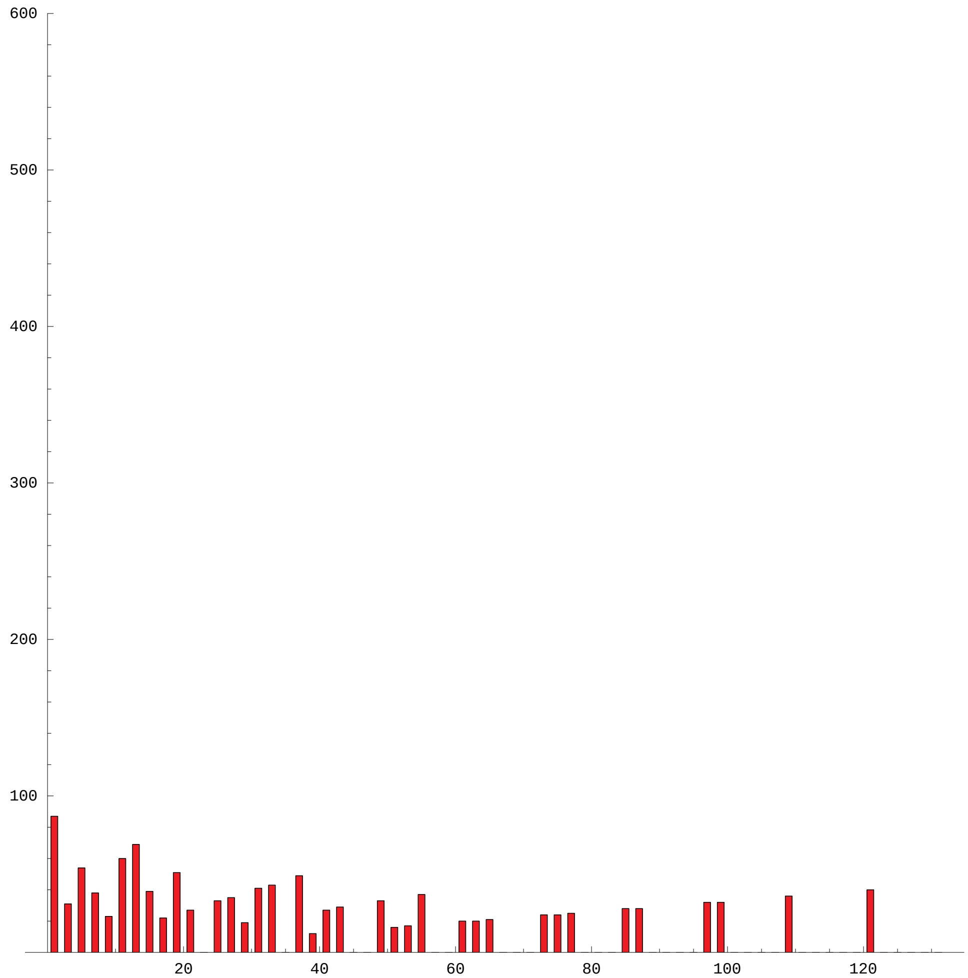

Figure 1: $m = 12$, $n = 11$, and $p = 2$.

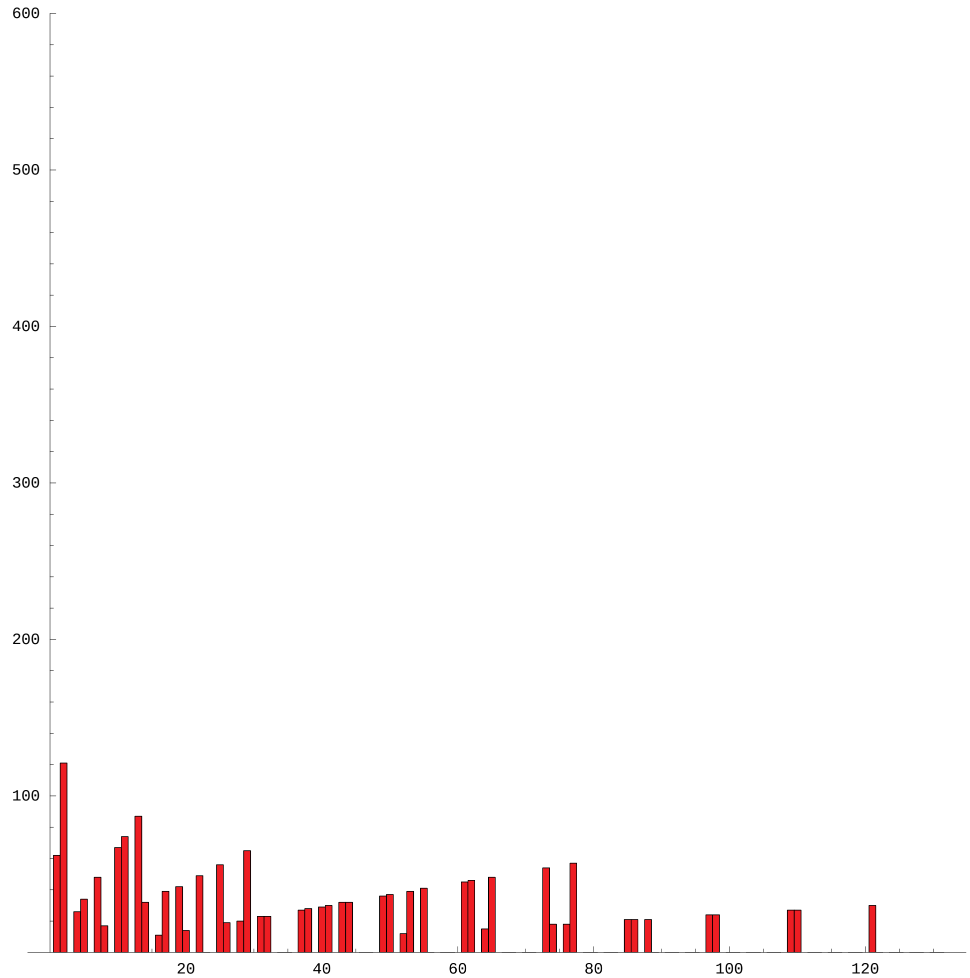

Figure 2: $m = 12$, $n = 11$, and $p = 3$.

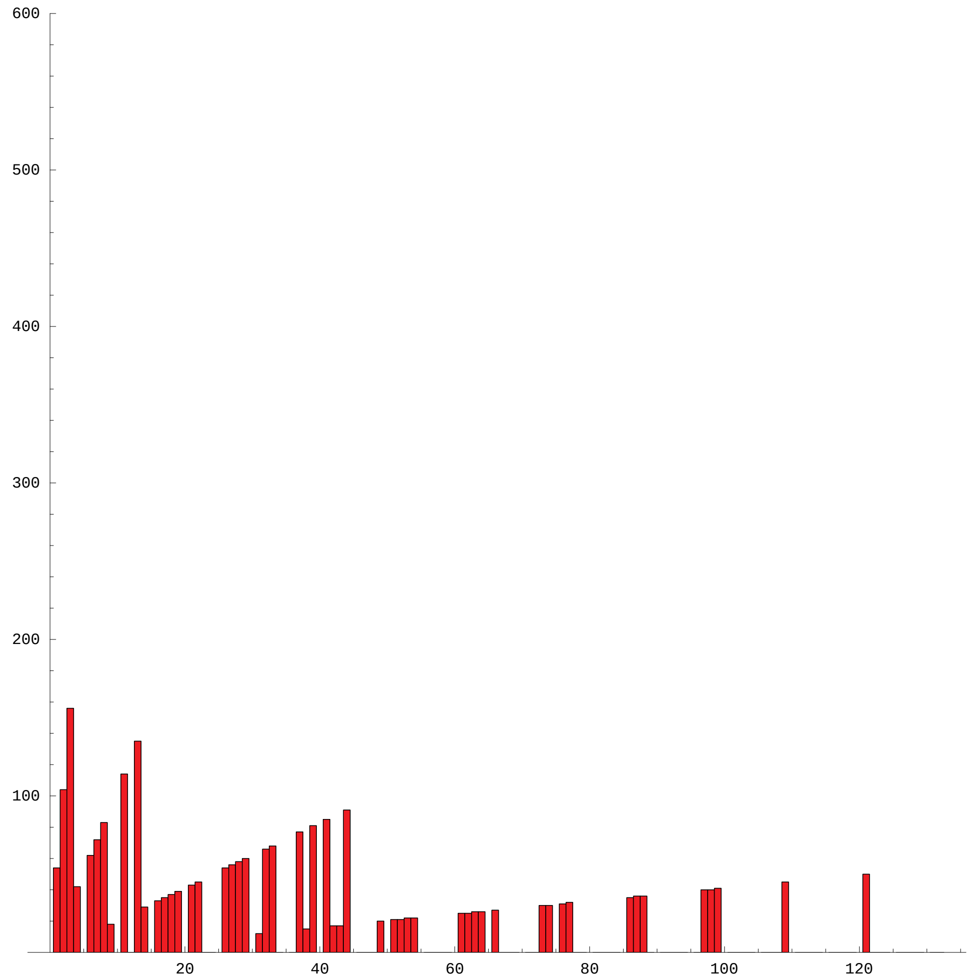

Figure 3: $m = 12$, $n = 11$, and $p = 5$.

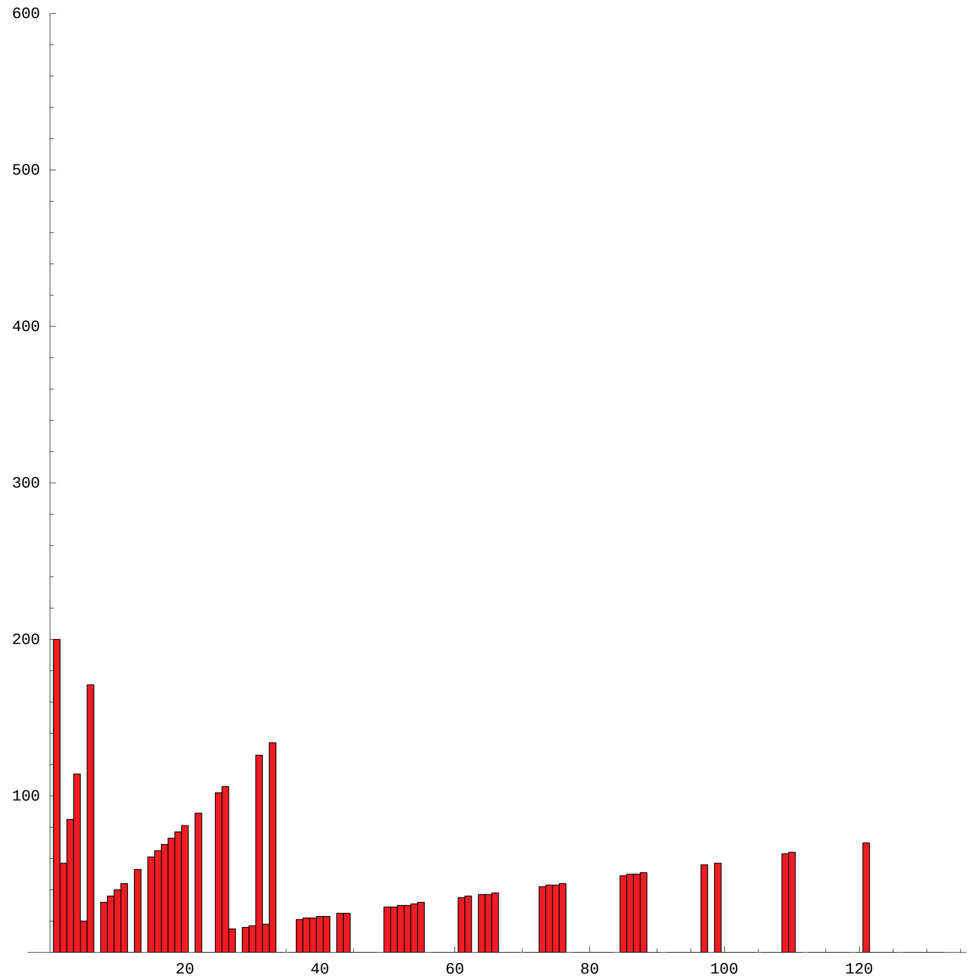

Figure 4: $m = 12$, $n = 11$, and $p = 7$.

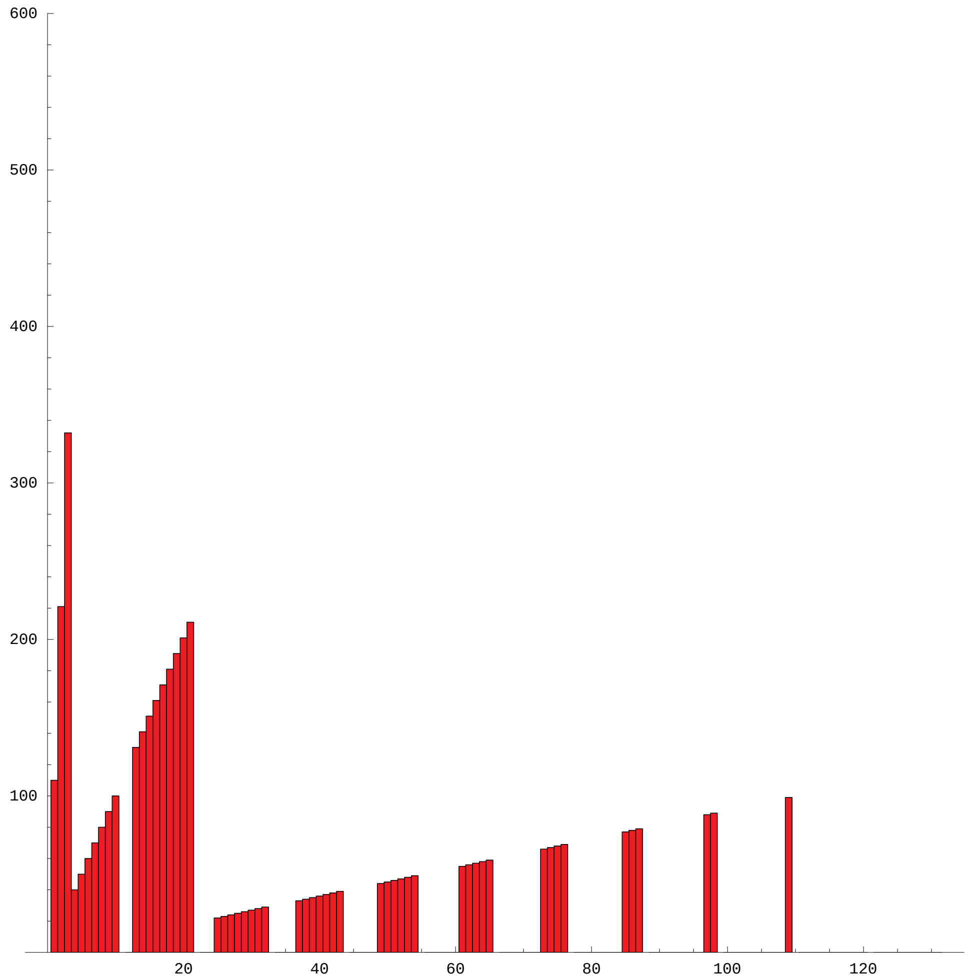

Figure 5: $m = 12$, $n = 11$, and $p = 11$.

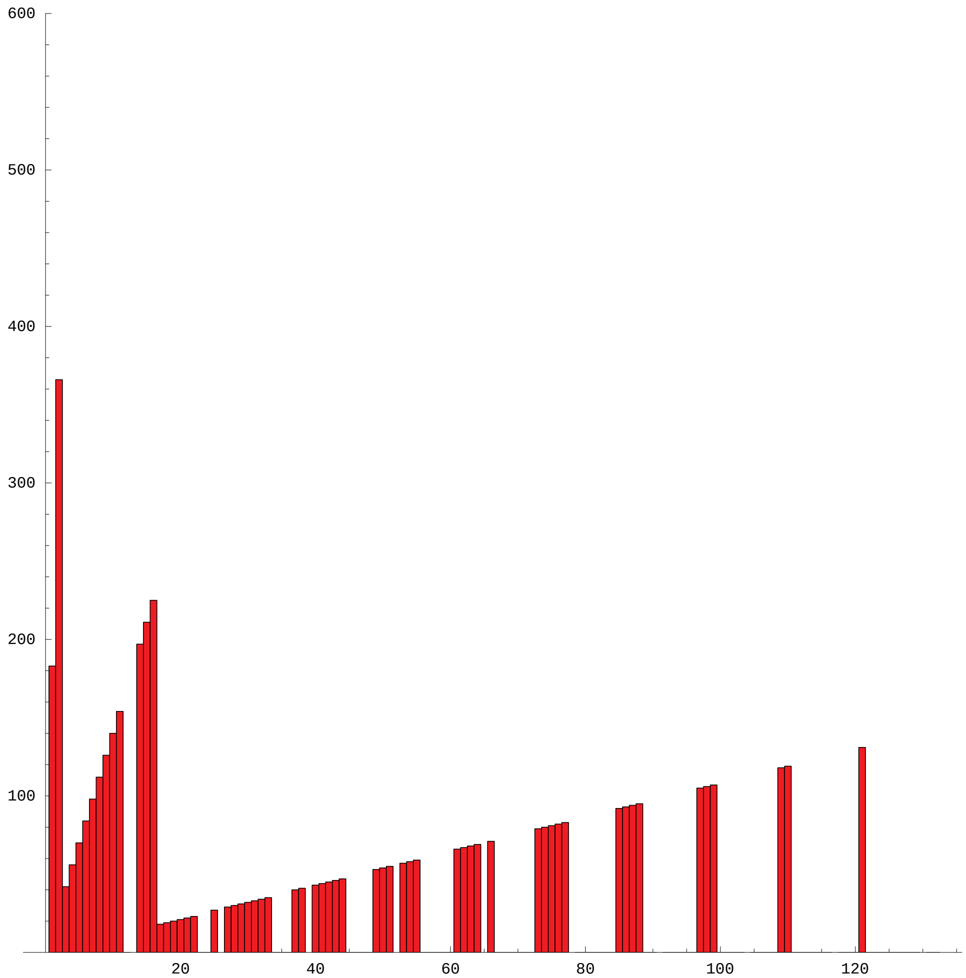

Figure 6: $m = 12$, $n = 11$, and $p = 13$.

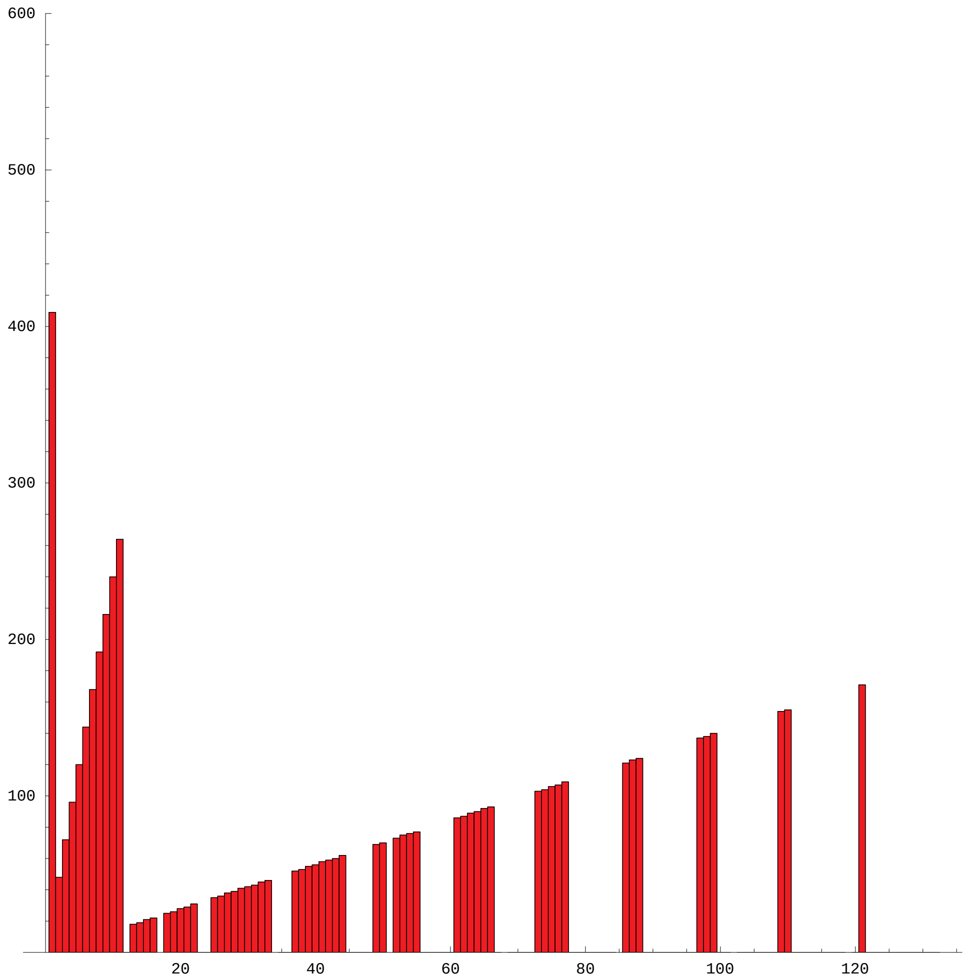

Figure 7: $m = 12$, $n = 11$, and $p = 17$.

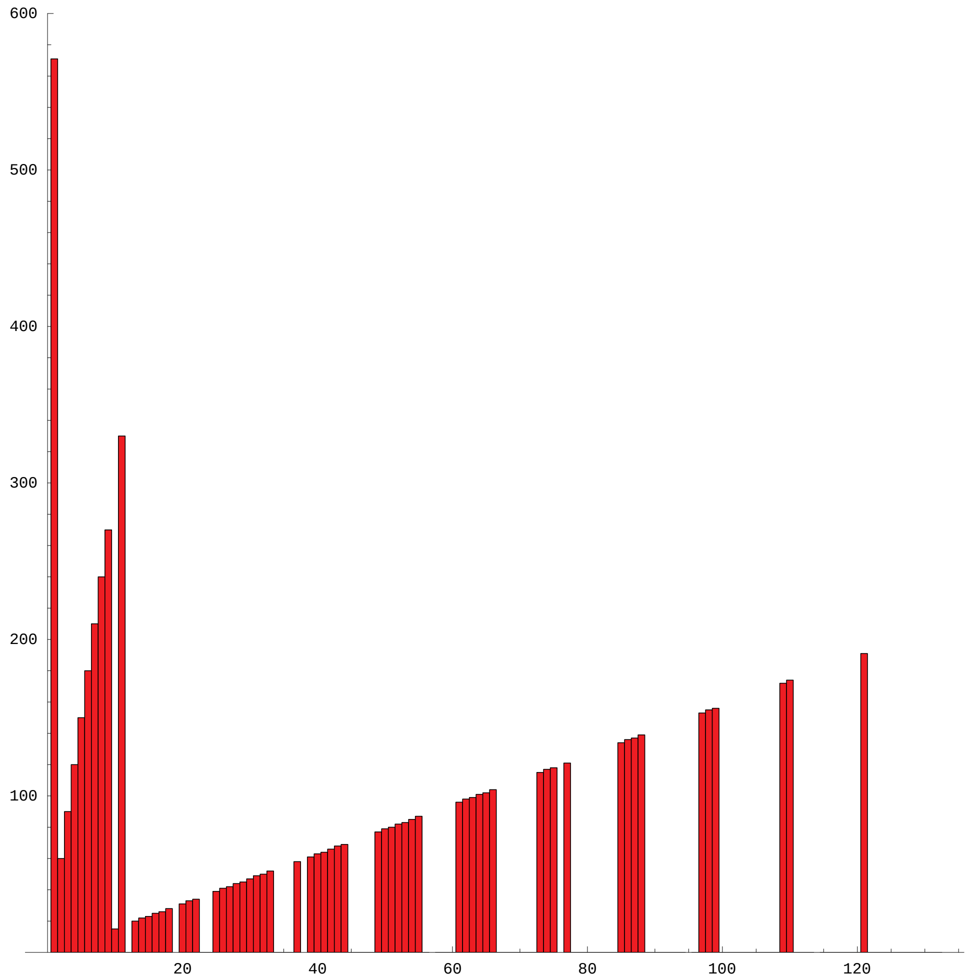

Figure 8: $m = 12$, $n = 11$, and $p = 19$.

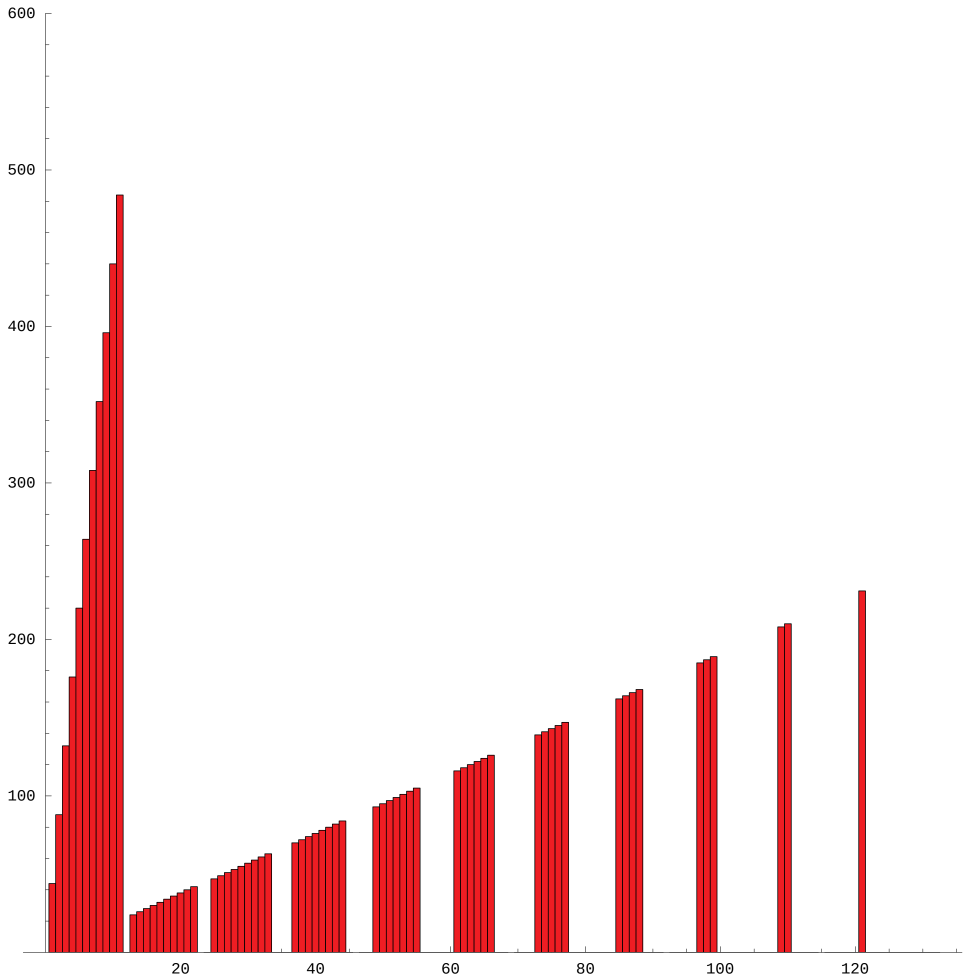

Figure 9: $m = 12$, $n = 11$, and $p = 23$.

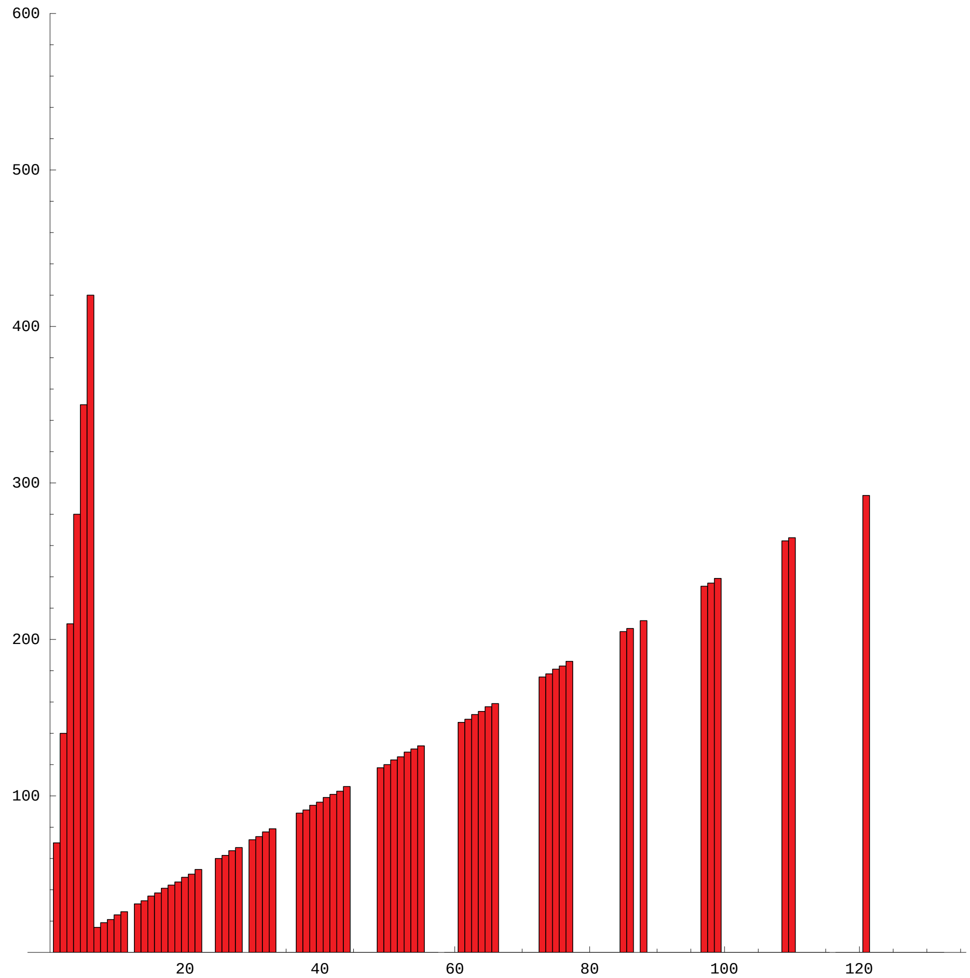

Figure 10: $m = 12$, $n = 11$, and $p = 29$.

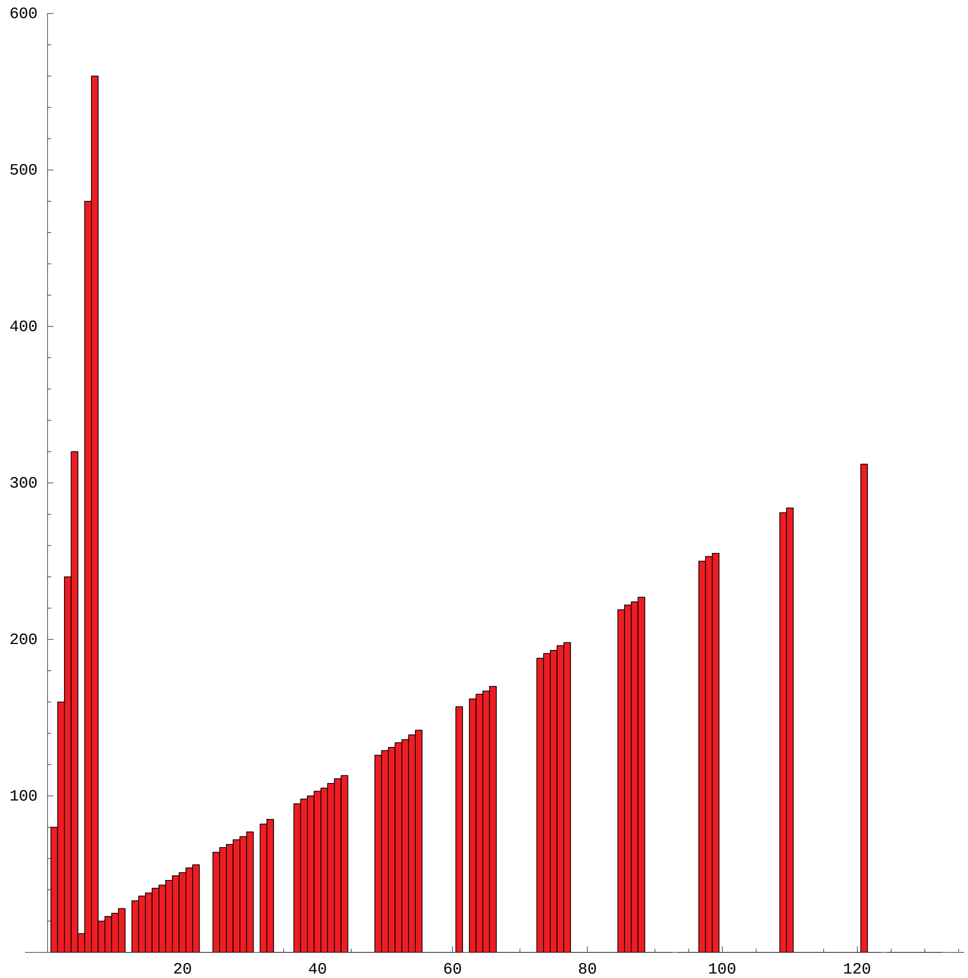

Figure 11: $m = 12$, $n = 11$, and $p = 31$.

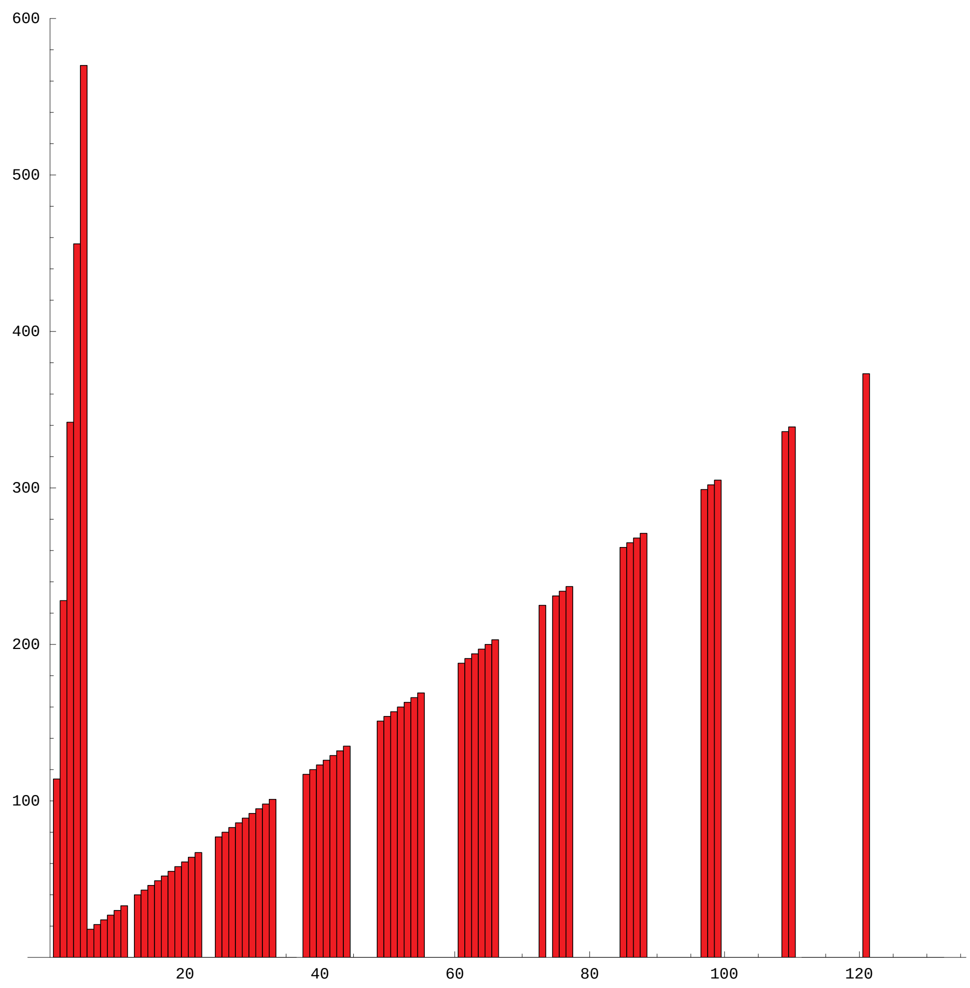

Figure 12: $m = 12$, $n = 11$, and $p = 37$.

References

[1] L. Hesselholt, *The tower of K-theory of truncated polynomial algebras*, Preprint 2007, available as `math.NT/0702877`.

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