The numbers a, i, j and r are given.

Prove that

$$\sum_{m=0}^{a} \frac{(-1)^m}{\binom{a}{m}} \left[\sum_{s=0}^{a} \binom{a-i}{s} \binom{i}{m-s} \binom{a-j}{a+r-i-j-s} \binom{j}{i+j+s-r-m} \right],$$

with the obvious rearrangement

$$\sum_{s=0}^{a} \binom{a-i}{s} \binom{a-j}{a+r-i-j-s} \left[\sum_{m=0}^{a} \frac{(-1)^m}{\binom{a}{m}} \binom{i}{m-s} \binom{j}{i+j+s-r-m} \right],$$

is equal to 0 if $i+j \neq a$, and if $i+j=a$, then this expression is

$$(-1)^{i+r} \frac{\binom{a}{r}}{\binom{a}{i}} = (-1)^{i+r} \frac{i!j!}{r!(a-r)!}.$$

(For motivation, this expression is roughly the coefficient of x^r in some series in the ijth entry of an $a \times a$ matrix.)