Probabilistic Modeling of Bed-load Composition

Ian McEwan[1], Michael Sørensen[2], John Heald[3],
Simon Tait[4], Gavin Cunningham[5], Derek Goring[6] and Brian Willetts[7]

1. Reader, Department of Engineering, University of Aberdeen, UK, AB24 3UE
2. Professor, Department of Statistics and Operations Research, University of Copenhagen, Denmark
3. Graduate Student, Department of Engineering, University of Aberdeen, UK, AB24 3UE
4. Lecturer, Department of Civil and Structural Engineering, University of Sheffield, UK
5. Engineer, Cundall Johnston & Partners, Horsely House, Regent Centre, Newcastle-upon-Tyne, UK, NE3 3LU
6. Principal Scientist, National Institute of Water and Atmospheric Research, Christchurch, New Zealand
7. Professor, Department of Engineering, University of Aberdeen, UK, AB24 3UE

Corresponding Author

Ian McEwan
Department of Engineering
University of Aberdeen
Aberdeen
AB24 3UE
United Kingdom

Email: i.mcewan@eng.abdn.ac.uk
Tel: 00 44 1224 272510
Fax: 00 44 1224 272497

ABSTRACT

This paper proposes that the changes which occur in composition of the bed-load during the transport of mixed-grain-size sediments are largely controlled by the distributions of critical entrainment shear stress for the various size fractions. This hypothesis is examined for a uni-modal sediment mixture by calculating these distributions with a discrete particle model and using them in a probabilistic calculation of bed-load composition. The estimates of bed-load composition compare favorably with observations of fractional transport rates made in a laboratory flume for the same sediment suggesting that the hypothesis is reasonable. The analysis provides additional insight, in terms of grain mechanics, into the processes which determine bed-load composition. These insights strongly suggest that better prediction methods will result from taking account of the variation of threshold with size fractions something that most previous studies have generally neglected.

KEYWORDS *uniform sediment, mixed grain size sediment, entrainment, threshold, discrete element modeling*

INTRODUCTION

Background

The prediction of bed-load composition is notoriously difficult. Early calculation of transport rates in graded sediments used a single representative grain diameter in formulae which had been developed for uniform grain size data (Meyer-Peter and Muller, 1948; Einstein, 1942). Later it was recognized that the interaction of the size fractions was complex and so fraction by fraction calculations were introduced. This enabled empirical hiding functions to be derived which adjusted the relative mobility of size fractions to account for various mixture effects. A large number of such hiding functions, each associated with a particular transport equation, have been put forward in the literature (Sutherland, 1992). Generally the performance of a particular hiding function deteriorates when it is applied to data other than that from which it was derived; this may imply a lack of generality in the concept.

More recently, Parker *et al*. (1982) introduced the equal mobility hypothesis which rests on two empirical results both derived from data obtained from Oak Creek by Milhous (1973). These are that the threshold for each size fraction is independent of grain size and that perfect similarity exists for the fractional transport rate relations. Parker *et al*. (1982) demonstrated that these conditions held approximately for the conditions of Oak Creek and then used them to construct a general model for predicting fractional transport rates. However Diplas (1987), in a re-analysis of the Oak Creek data, showed that important departures from equal mobility were present in the data so that perfect similarity of the fractional transport relations did not hold. In recognition of this Diplas (1987) used the Oak Creek data to derive, for the first time, a hiding function which depended on shear stress, acknowledging that bed-load composition varied strongly with shear stress. However, in a telling contribution, Wilcock (1992b) synthesized fractional transport rate data for a wide range of mixtures and demonstrated that there appears to be no systematic pattern linking the fractional transport relations for different mixtures. This strongly suggests that the derivation of a generalized empirical hiding function will be problematic. Given the difficulty that this goal has presented hitherto, this conclusion is not surprising.

Therefore fresh approaches are required. In this paper a probabilistic method for calculating bed-load composition is introduced and its predictions compared with bed-load data taken from Wilcock (1992). The approach is distinguished by two important characteristics. First, it takes greater account of the physical processes than previous calculations because fluid turbulence and the heterogeneous behavior within size fractions are explicitly accounted for. Second, the method does not rely on the same fractional transport rate data for both its derivation and evaluation.

The Grain Mechanics Perspective

Commonly, the sediment transport of graded sediments has been predicted by dividing the mixture into its constituent size fractions. It has been usual to assume that all grains of like-size within a mixture behave in a similar way so that the behavior of a size fraction may be specified in terms of its threshold shear stress and the mean flow parameters (e.g. Egiazoroff, 1965, Parker *et al.,* 1982). The equal mobility hypothesis of Parker *et al*. (1982) may be expressed as $p_i = f_i$ where p_i and f_i represent, respectively, the proportions by weight of the i^{th} size fraction in the bed-load and bulk mix. A consequence if equal mobility holds is that the total bed-load transport rate can be calculated by a single representative grain diameter; Parker *et al*. (1982) recommended that the d_{50} should be used. However, more recent work has shown that for low and moderate shear stresses this is not the case (Wilcock, 1992). More specifically Wilcock showed that in this crucial range of shear stresses (which prevail in many gravel bed streams) the changes in relative mobility depend on both grain size and shear stress in a complex way. One of the key findings has been that a condition of partial transport exists (Wilcock and McArdell, 1993) in which a proportion of some size fractions remain immobile on the surface at a given shear stress (i.e. $p_i \neq f_i$). Clearly this means that there is variation in mobility *within size fractions* and that this variation is significant in influencing the bulk behavior of the sediment.

Thus the assumption, implicit in most previous treatments of the problem, that behavior within a size fraction is homogeneous is not valid. It seems likely then that there is scope for increasing the accuracy of predictions of bed-load composition if methods are developed which relax this assumption. Wilcock (1997) has taken the first investigative steps into this by calculating variation in critical entrainment shear stress within size fractions from fractional transport data and observations of size distribution of the surface, including the proportion of immobile grains in a particular size fraction.

In this paper Wilcock's treatment of the problem is extended by developing a probabilistic method to calculate bed-load composition. Many previous studies have also attempted to describe bed-load transport stochastically, explicitly accounting for the turbulent velocity fluctuations and/or the possibility of variable behavior within size fractions, (e.g. Gessler, 1967; Paintal 1971b; Sutherland and Irvine, 1973; Shen and Cheong, 1981; Nakagawa and Tsujimoto, 1981; and Hsu and Holly, 1992). Despite the promise of these approaches, they have not been generally adopted as a means to predict sediment transport rates. It is suggested that this is because they are generally more complex to implement than deterministic formulae without significantly improving the accuracy of the prediction. However, it is considered that particular facets of the sediment transport problem will prove to be amenable to probabilistic solution. It is considered that one of these is mixed-grain-size sediment transport.

Fundamentally, the probabilistic approach examines sediment grain mechanics. Recent work by Kirchner *et al.* (1990) and Buffington *et al.* (1992) has addressed the problem of describing the entrainment of mixed-grain-size sediments in probabilistic terms. They measured friction angles of particles placed on the surface of a mixed-grain-size sediment bed by tilting the bed to an angle at which a particle of interest moved from its position. Johnston *et al.* (1998) obtained similar results from in-situ field measurements of particle stability. These studies demonstrate that the heterogeneity of mixed-grain-size sediment beds leads to a distribution of critical entrainment shear stresses within size fractions.

This paper proposes that the changes which occur in composition of the bed-load are largely controlled by the distributions of critical entrainment shear stress for the various size fractions. This hypothesis is examined and tested by calculating these distributions for the 1ϕ sediment mixture (Wilcock, 1992) with a discrete particle model (McEwan and Heald, 2001) and then use them to estimate the bed-load composition as a function of shear stress, comparing the results with the original flume data. The motivation for the work is two-fold. Firstly, it is to provide additional physical insight into a number of key observations concerning the transport dynamics of mixed-grain sediments, in particular addressing compositional changes in bed-load with increasing shear stress. Secondly, it will assess the potential of a quantitative method to predict these changes, the success of which is rooted in grain mechanics.

The probabilistic method is based on the analysis of discrete particle simulations which create sediment beds consisting of $\sim 10^6$ spherical grains (Jefcoate, 1995; Jefcoate and McEwan, 1997; McEwan and Heald, 2001). The model is a development of the 'multi-particle simulation method' first applied to fluvial sediment transport by Jiang and Haff (1993). Preliminary results from this method suggest that it will be a fruitful technique in deriving insight into the grain mechanics. In this paper the technique is brought to bear on the problem of quantifying the distributions of critical entrainment shear stress in a sediment mixture. Sediment beds are formed numerically by dropping particles in sequence, each from a random position above the surface, the grain then undergoing a series of collisions with previously deposited particles until it settles in a stable position. The geometry of numerically deposited beds is analyzed to reveal the critical entrainment shear stresses of the particles exposed on the surface.

ESTIMATION OF BED-LOAD COMPOSITION

For a mixed-grain-size sediment consisting of *n* size fractions the total mass transport rate in bed-load can be written as,

$$q_b \equiv \sum_{j=1}^{n} q_{bi} = \langle\phi\rangle\langle\lambda\rangle \qquad (1)$$

in which q_b is the mass transport rate of the sediment [$ML^{-1}T^{-1}$], q_{bi} is the mass transport rate of the i^{th} size fraction [$ML^{-1}T^{-1}$], $\langle\phi\rangle$ is the total mass entrainment rate ($ML^{-2}T^{-1}$) and $\langle\lambda\rangle$ is a step length associated with the total mass transport rate such that $\langle\lambda\rangle = q_b/\langle\phi\rangle$.

For the i^{th} size fraction the mass transport rate is given by,

$$q_{bi} \equiv p_i q_b \equiv \phi_i \lambda_i \qquad (2)$$

in which ϕ_i is the mass entrainment rate or pick-up rate of the i^{th} size fraction [$ML^{-2}T^{-1}$] and λ_i [L] is the mean trajectory length of the i^{th} size fraction (i.e. the stream-wise displacement between rest periods).

Thus p_i may be described in terms of ϕ_i and λ_i as follows,

$$p_i = \frac{q_{bi}}{q_b} = \frac{\phi_i \lambda_i}{\sum_{j=1}^{n} \phi_j \lambda_j} \qquad (3)$$

In order to predict bed-load composition the product $\phi_i\lambda_i$ must be estimated. Recalling the hypothesis that bed-load composition is largely controlled by the distributions of critical entrainment shear stress for the various size fractions, p_i is estimated from distributions calculated by discrete particle simulations of the 1ϕ mixture and with the assumption that λ_i (henceforth λ) is constant at a given shear stress.

From this simplification it follows that the proportion of mass entrained within a size fraction is,

$$p_i = \frac{\phi_i}{\sum_{j=1}^{n} \phi_j}. \qquad (4)$$

The assumption that λ is constant at a particular shear stress is both justifiable and expedient. Consider the transport of the i^{th} size fraction of a mixed-grain-size sediment. Two processes govern its motion over the surface, namely bed friction (indicated by relative roughness ~ d_{50}/d_i) and the fluid effects (indicated by the ratio of drag force/particle weight). McEwan *et al*. (2001) showed that these two effects have a strong tendency to counter-act each other implying that, at a given shear stress, λ depends only weakly on grain size for all size fractions present. The assumption that λ is constant is useful in the present circumstances because any variation in predicted bed-load composition may be attributed solely to variation in fractional entrainment rates at a given shear stress. Therefore the degree of agreement between the predictions and the data is a measure of the validity of the hypothesis.

Following Grass (1970) the instantaneous fluid shear stress, τ_f, is considered to be represented by a distribution F with probability density function $f_F(\tau_f)$ and the critical entrainment shear stress τ_g for the individual grains in the i^{th} size fraction to be represented by a distribution G^i with probability density function $f_G^i(\tau_g)$. The following assumptions apply to the analysis;

(1) F represents the turbulent fluctuations of local instantaneous shear stress τ_f around the mean boundary shear stress, τ_o. F is stationary and applies uniformly to the area of the surface under consideration;

(2) F applies to all grains within a size fraction regardless of their position in the surface. Thus any effect due to localized non-uniformity in the near-bed flow is neglected;

(3) Similarly F applies to all size fractions so that all differential fractional sheltering effects are accounted for by variation in G^i;

(4) G_i represents the variation in critical entrainment shear stresses within the i^{th} size fraction due to local surface heterogeneity. It is stationary in the sense that the grain exchange process between the surface and bed-load is in complete equilibrium. The definition of equilibrium extends beyond considerations of the surface composition to include stability effects. Thus it is assumed that the entrainment of a grain in the i^{th} size fraction with a critical entrainment shear stress of τ_g is, on average, compensated by the deposition of a grain with similar size and equivalent stability;

(5) The influence of bed-forms on entrainment is sufficiently small that it may be sensibly neglected.

Note that as a consequence of these assumptions, particularly (4), the analysis only applies to conditions where transport is steady and uniform, precluding further grain sorting in the system. This is consistent with the operation of the sediment re-circulating flume (Parker and Wilcock, 1993) in which Wilcock's data for the 1ϕ mixture were obtained. However, it excludes systems which are not in equilibrium, for instance during the formation of static armors (Proffitt and Sutherland, 1983; Tait *et al.*, 1992).

For a given fluid shear stress distribution F, the probability of the instantaneous fluid shear stress τ_f lying within the interval $\delta\tau$ centered at τ_0 is,

$$P\left(\tau_0 - \frac{1}{2}\delta\tau \le \tau_f \le \tau_0 + \frac{1}{2}\delta\tau\right) = f_F(\tau_f)\delta\tau \tag{5}$$

and the probability of τ_g taking a value lower than τ_f is given by the $G^i(\tau_f)$ such that,

$$P\left(\tau_g < \tau_f\right) = \int_0^{\tau_f} f_G^i(\tau')d\tau' = G^i\left(\tau_f\right) \tag{6}$$

The probability of τ_f lying within the range $\tau_0 - \frac{1}{2}\delta\tau \le \tau_f \le \tau_0 + \frac{1}{2}\delta\tau$ and G^i taking a value less than τ_f gives the elemental risk of entrainment δE_i for the i^{th} size fraction,

$$\delta E_i\left(\tau_f\right) = f_F\left(\tau_f\right)\delta\tau_f G^i\left(\tau_f\right) \tag{7}$$

and therefore the risk of entrainment for the i^{th} size fraction is,

$$E_i = \int_0^{\infty} \delta E_i(\tau')d\tau' = \int_0^{\infty} f_F(\tau')G^i(\tau')d\tau' = \lim_{\delta\tau \to 0} \sum_{n=1}^{\infty} f_F\left(\tau_{f_n}\right)G^i\left(\tau_{f_n}\right)\delta\tau \tag{8}$$

where $\tau_{f_n} = \left(n + \frac{1}{2}\right)\delta\tau$.

Now suppose that the fractional entrainment rate $\phi_i \propto f_i E_i$, then Equation (4) yields,

$$\frac{p_i}{f_i} = \frac{\phi_i}{f_i \sum_{j=1}^{n} \phi_j} = \frac{E_i}{\sum_{j=1}^{n} f_j E_j} = kE_i \tag{9}$$

where k is constant at a particular mean boundary shear stress τ_0 and is given by $k = \left(\sum_{j=1}^{n} f_j E_j\right)^{-1}$.

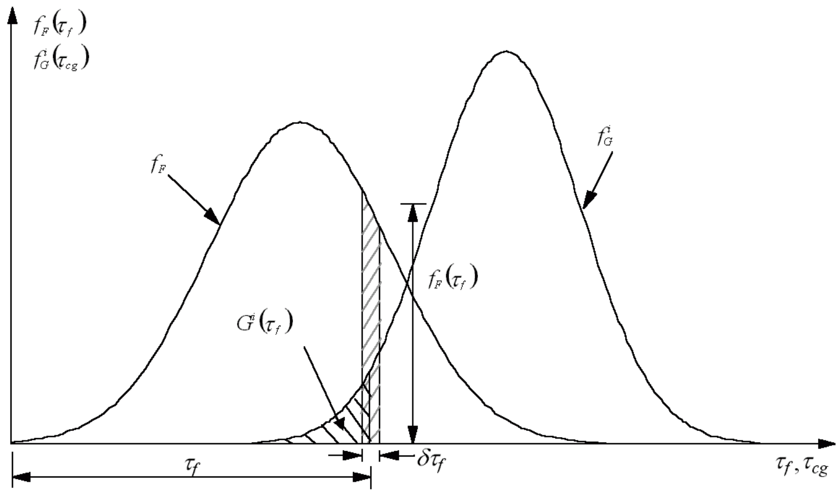

Figure 1: Schematic representation of the concept of Grass (1970).

RESULTS AND ANALYSIS

The Behavior of the 1ϕ Mixture in a Laboratory Flume (Wilcock, 1992)

The 1ϕ mixture is a unimodal sediment with an approximately log-normal size distribution (Fig. 2) with d_{50} = 1.83 mm and a standard deviation (σ_ϕ = 1ϕ, where σ_ϕ is the standard deviation of the sediment expressed in ϕ units). Wilcock (1992) contains a fairly complete description of the behavior of this mixture including fractional transport rate data which shows how bed-load composition changes with mean boundary shear stress. These data are shown in Fig. 3 which is derived from to Fig. 6.6b in Wilcock (1992). Such complete data sets are reasonably rare; therefore the 1ϕ mixture represents an unusually good test of the probabilistic method for mixed-grain-size sediments.

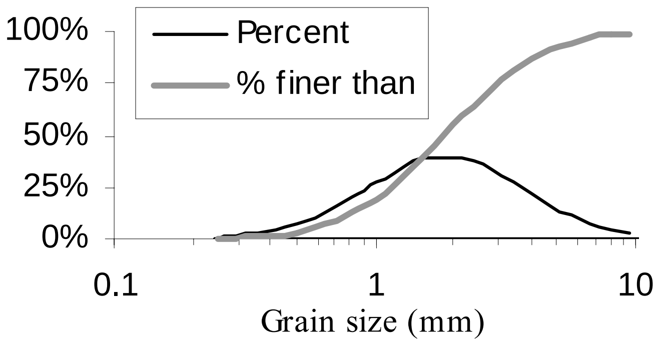

Figure 2: Size distribution for the 1ϕ sediment mixture.

Wilcock's data shows the 1ϕ mixture to have the following transport characteristics. The reference shear stress for this mixture was found to be $\tau_r \approx 1.00$ N/m^2 and to be independent of grain size (Wilcock, 1993). A consequence of this result is that approximate equal mobility ($p_i = f_i$) exists at $\tau_o = \tau_r$. This behavior is evident in Fig. 3 which shows that at the lowest shear stress $\tau_o \approx \tau_r = 1.14$ N/m^2 there is a tendency toward equal mobility (i.e. $p_i = f_i$). As mean boundary shear stress increases further above threshold, the finest and coarsest fractions are under represented in the bed-load relative to their proportions in the bulk mixture. This condition is maintained until $\tau_o \approx 2.41$ N/m^2 above which equal mobility is again approximately obtained, according to Wilcock (1992).

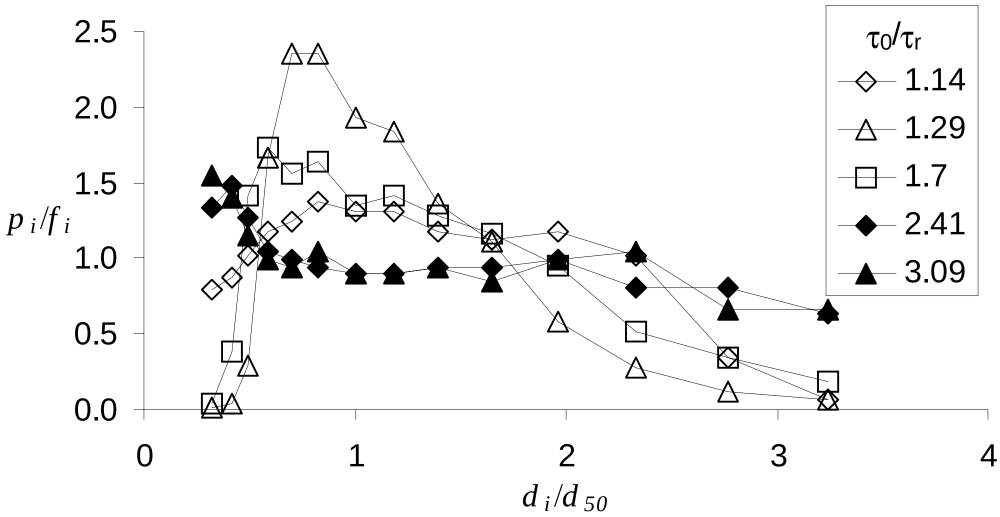

Figure 3: Fractional transport rate data for the 1ϕ mixture derived from Wilcock (1992)

Simulations of the 1ϕ Sediment Mixture

The simulated beds are formed using a discrete particle model in which individual grains in the sediment mixture are represented as spheres. A macroscopically flat bed is deposited under conditions of ballistic deposition. Particles are dropped from random position in a plane well above the surface and fall under gravity until they undergo a series of collisions with the static surface before coming to rest in a stable position. Once a bed has been formed the stability of surface grains is evaluated by a moment analysis which calculates the stream-wise force which would be required to destabilize the grain from its position. In this paper this critical entrainment force is expressed as a stress in order to compare the results with conventional measures of sediment activity. The critical entrainment shear stress is calculated for a single grain from the stream-wise destabilizing force and the plan area of the grain. It is therefore equivalent to the drag stress described by Buffington and Montgomery (1999) which is derived from instantaneous stream-wise velocity rather than an instantaneous Reynolds stress. A fuller account of the discrete particle model and the calculation of critical entrainment shear stresses is found in McEwan and Heald (2001).

Fig. 4 shows a small section of the numerically generated bed both in plan view and in perspective. Particle size is represented by three discrete gray levels so that coarser particles appear darker. Visual inspection of the surface indicates that groups or clusters of coarse particles seem to have formed. A similar tendency was found in natural stream beds by Church *et al.* (1998), also using visual inspection. There is scope for some sorting to develop during the deposition of the bed as particles are free to find preferred locations under the general influence of a gravity-driven process. However a more rigorous analysis of the surface arrangement would be required to quantify the degree to which the structure of the numerical surface is comparable to a water-worked bed (Goring *et al.*, 1999).

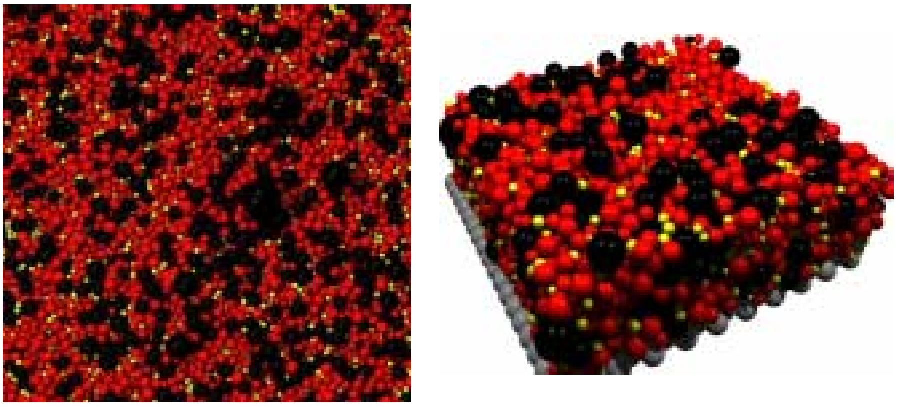

Figure 4: 100 x 100 mm plan view of the surface and an expanded view of the grain-scale topography.

Critical Entrainment Shear Stress Distributions for the 1ϕ Mixture

The cumulative critical entrainment shear stress distributions $[G^i]$ for the 1ϕ mixture are shown in Fig. 5. The general form of the cumulative distributions is that at lower shear stresses they appear to be approximately coincident, following a similar path until they begin to diverge at moderate shear stresses with the coarser fractions being less susceptible to entrainment. As shear stresses increases further the distributions become roughly parallel, each having a common slope close to $^3/_2$ in log-log space. At very high shear stresses, $G^i \rightarrow 1$ for all size fractions, indicating that the surface would be fully mobilized.

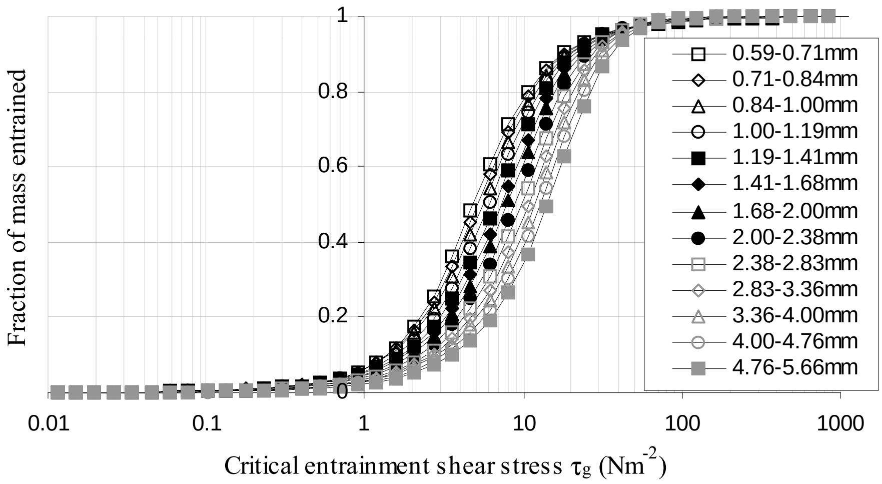

Figure 5: Critical entrainment shears stress distributions for the 1ϕ mixture.

There is no indication in Fig. 5 of a distinct threshold of motion for any of the size fractions. The cumulative distributions demonstrate that grains are unstable at shear stresses which would, in terms of the traditional analyses of entrainment, be well below the threshold of motion. Wilcock (1992, 1993) found little variation in the threshold shear stress obtained from the reference shear stress method. This is consistent with the similar path followed by the distributions for all fractions at low shear stresses in Fig. 5. The physical meaning of a distinct single-valued threshold has been challenged before

(Paintal, 1971a) but the concept seems to have been so strongly embodied in our thinking about sediment problems that we are reluctant to manage without it. Paintal (1971a) conducted a systematic series of experiments with a variety of sediment types at very low shear stresses and found that, if sufficiently long observation periods were used, transport could be measured at shear stresses well below what is conventionally considered to be threshold.

McEwan and Heald (2001) demonstrated, that the critical entrainment shear stress distributions for uniformly-sized sediments of different grain sizes can be collapsed using a Shields scaling. Fig. 6 shows the scaled distributions G^i for the 1ϕ mixture together with the non-dimensional curve obtained for uniformly-sized sediments (McEwan and Heald, 2001). It is evident that, relative to their behavior as uniformly sized sediments, the coarser particles in the 1ϕ mixture are more mobile and the finer particles are less mobile. The mobility of the size fraction containing the d_{50} grain size is roughly unchanged with respect to its behavior as a uniformly-sized sediment. Therefore the simulations, reproduce the effects of hiding which, as is well known, tend to reduce mobility differences between the different size fractions in a sediment mixture.

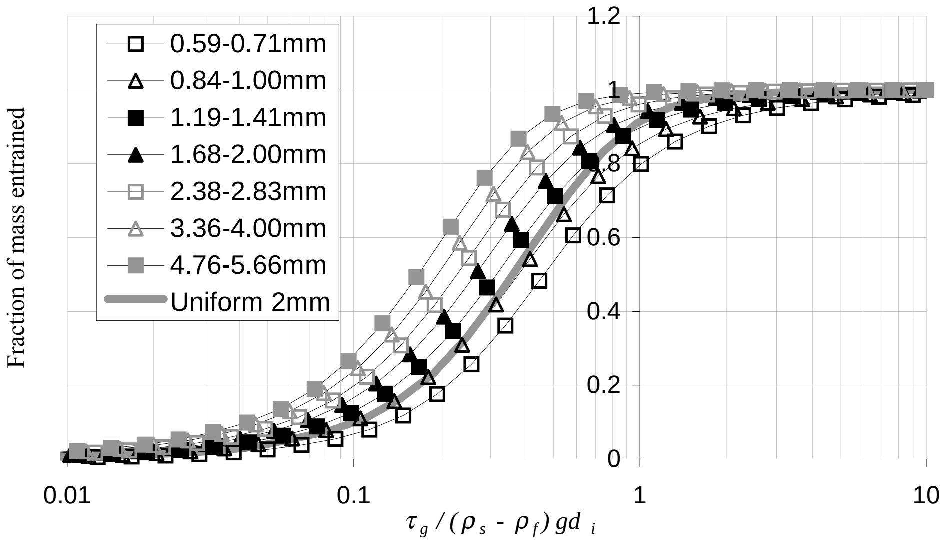

Figure 6: Non-dimensional critical entrainment shear stress distributions for the 1ϕ mixture shown with the equivalent curve for a uniformly-sized sediment.

Bed-load Composition Predictions

These distributions are then used, together with the probabilistic model, to predict the dependency of bed-load composition on mean boundary shear stress, τ_o. Calculations are made for two forms of the fluid shear stress distribution termed Case (A) and Case (B). The fluid shear stress density function for Case (A) is a delta function with one possible value, τ_o, of the instantaneous shear stress thereby eliminating any effect due to turbulent fluctuations. For Case (B) the influence of turbulent fluctuations is introduced by representing the fluid shear stress as a normal distribution with a mean of τ_o and standard deviation of $\sigma_\tau = \tau_o/4$, a value estimated from turbulence measurements in Cunningham (2000). The results for Case (A) are shown in Fig. 7 and for Case (B) in Figs. 8a and 8b. Although there are some small differences between Cases (A) and (B) the trends are very similar indicating that turbulent fluctuations play a minor role in determining bed-load composition. This point will be examined later but for now the common features of both cases will be discussed in terms of the Case (B) results in Figs. 8a and 8b.

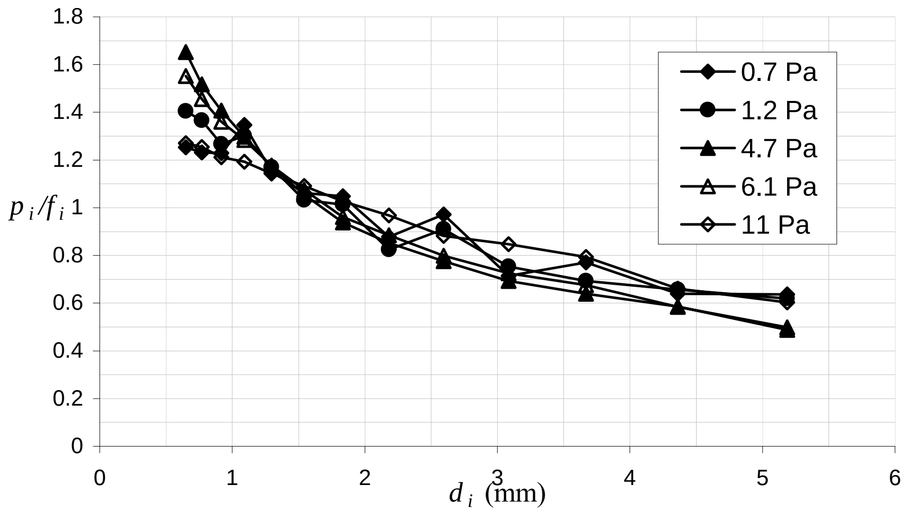

Figure 7: Shows p_i/f_i against grain size evaluated for Case (A) for shear stresses in the range 0.7 – 11 Pa.

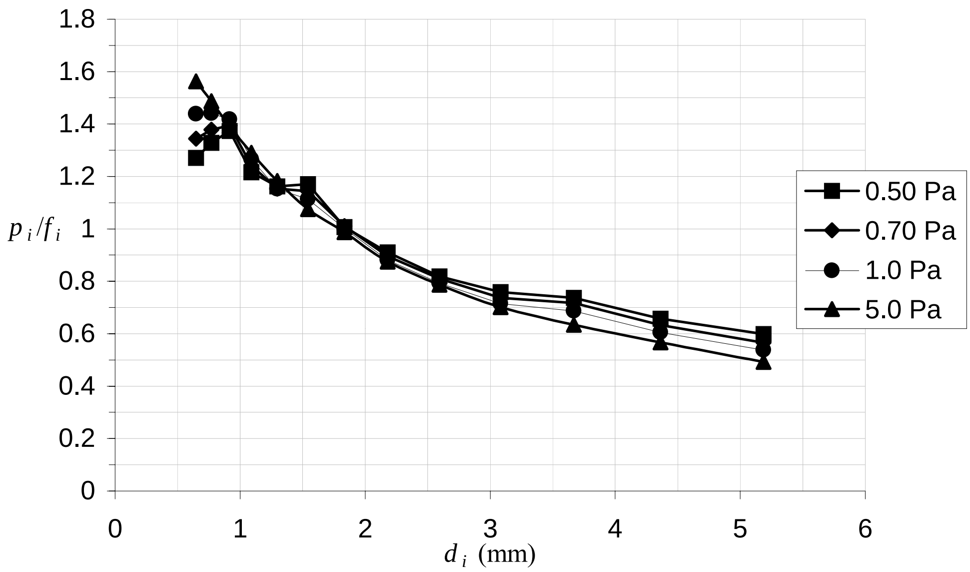

Figure 8a: Shows p_i/f_i against grain size evaluated for Case (B) for mean boundary shear stresses in the range 0.5 – 5.0 Pa, $\sigma_\tau = \tau_o/4$.

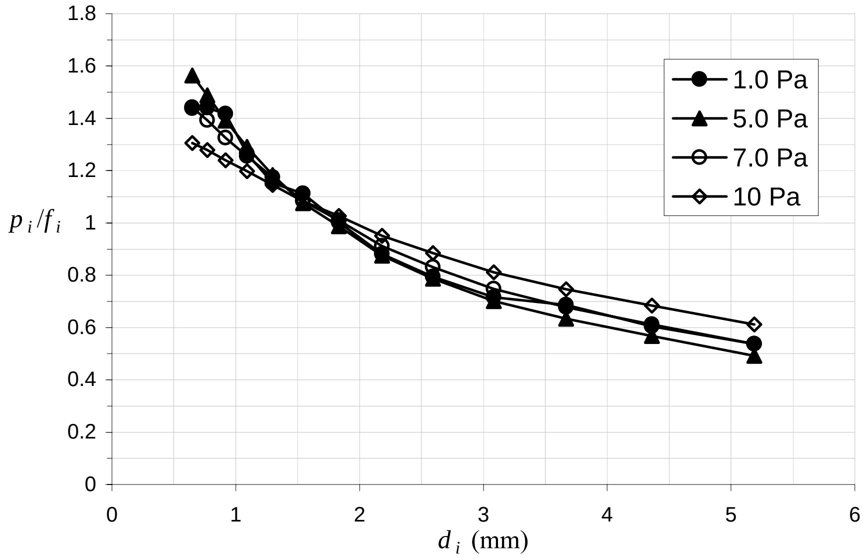

Figure 8b: Shows p_i/f_i against grain size evaluated for Case (B) for mean boundary shear stresses in the range 1.0 – 10.0 N/m^2, $\sigma_\tau = \tau_o/4$.

The probabilistic modeling displays a progression towards equal mobility as mean fluid shear stress is decreased (Fig. 8a), signified by the p_i/f_i curves becomes progressive flatter as shear stress decreases from 5.0 Pa to 0.5 Pa,. Grain size independence at low shear stress may be interpreted in terms of the critical entrainment shear stress distributions, which, for very low shear stresses, follow a similar path. Encouragingly, this result has also been obtained from critical entrainment shear stress distributions obtained from water-worked beds in the laboratory (Buffington *et al.,* 1992) and from in-situ measurements in the field by Johnston *et al.* (1998). Thus, it seems evident that the empirical result of the size-independence of the reference shear stress for unimodal and weakly bi-modal sediments (Wilcock, 1993) may be attributed to the critical entrainment shear stress distributions of the constituent size fractions approaching a common origin at low shear stresses.

As the fluid shear stress is increased beyond the apparent threshold (or reference shear stress) distinct changes in the bed-load composition are predicted by the probabilistic method. Considering the effects of increased shear stress upon the coarser fractions, it can be seen that they are predicted to become less mobile so that the medium size fractions are transported preferentially. This reproduces the trend that Wilcock (1992) obtained for the 1ϕ mixture and may be associated with the divergence and gradient change of the critical entrainment shear stress distributions which occurs somewhat above threshold. However, the model's bed-load composition predictions contradict Wilcock's (1992) findings for the finer sediment fractions which become more abundant with increasing shear stress. This discrepancy can be accounted for in the assumption that λ is constant for all grain sizes at a given shear stress. The results presented in McEwan and Heald (2001) support this notion as a first approximation only for $d_i > d_{50}$. It is also noteworthy that the Wilcock (1992) results are plotted as $(p_i/f_i)q_{bi}$, and not p_i/f_i, against d_i. It is proposed that the reduced mobility of the finer fractions due to surface roughness is the reason for their reduced presence evident in Fig. 3 at low transport stage.

A range of mean shear stresses can be identified over which divergence and gradient change in the fractional critical shear stress (fig. 5) takes place producing a strong variation in p_i/f_i for the finest tail of the mixture. At mean shear stresses greater than $\tau_0 \approx$ 5.0 Pa, there is monotonic decrease in p_i/f_i between the finest and coarsest size fractions, which at $\tau_0 = 5.0$ Pa is roughly three-fold. As mean fluid shear stress is increased further, the difference in p_i/f_i between the finest and coarsest fraction is reduced so that at $\tau_0 = 10$ N/m^2 it is close to a two-fold difference and by $\tau_0 = 50$ Nm^{-2} a condition of near equal mobility is approached. Thus, the probabilistic data suggest that the transport behavior of the 1ϕ mixture approaches equal mobility ($p_i \rightarrow f_i$) as shear stress increases.

Wilcock's (1992) data include two shear stresses ($\tau_0 = 2.41$ N/m^2 and $\tau_0 = 3$ N/m^2) which may be considered to be representative of the lower part of the range of shear stresses. In both cases there is an approximately two-fold decrease in the ratio p_i/f_i from the finest to the coarsest fractions. Interpreting this behavior in terms of the critical entrainment shear stress distributions, it is associated with a range of intermediate shear stresses in which the distributions are characterized by roughly uniform and parallel gradients in log-linear space (fig. 9). This behavior is not, in a strict sense, equal-mobility; rather, as mean shear stress increases, the difference between the cumulative stress distributions becomes less influential and equal-mobility is approached asymptotically ($p_i \rightarrow f_i$).

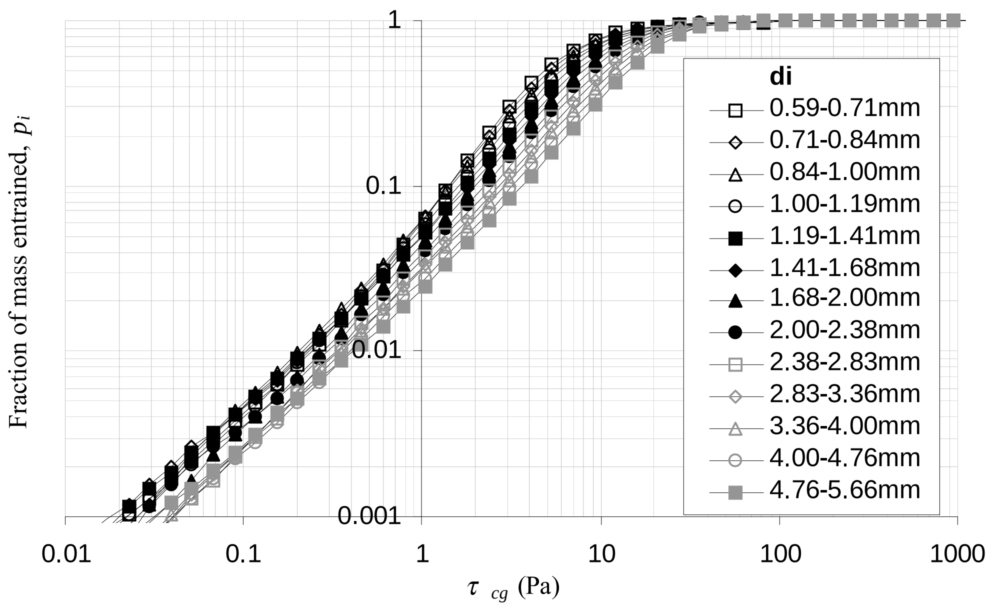

Figure 9: Critical entrainment shear stress distributions for the 1ϕ mixture plotted on log-linear axes.

Strictly, equal mobility is not attained until the significant portion of the fluid shear stress distribution is located well beyond the critical entrainment shear stress necessary for all grain sizes to exhibit $G^i \approx 1$, although departures from $p_i = f_i$ become small well before this limit is attained. The impact of this behavior on bed-load composition is seen in fig. 9b which shows the difference in relative mobility (p_i/f_i) for fractions decreasing as shear stress increases. Significantly, Diplas (1987, 1992) obtained the same result from his analysis of the Oak Creek data and this is reflected in the behavior of the hiding function he derived from that data (Diplas, 1992).

The Grain Mechanics Perspective Revisited

Four distinct compositional regimes of bed-load transport have been identified for the uni-modal sediment analysed in this paper. These are described below and shown schematically in Fig. 10.

(1) Behavior approaching equal mobility as shear stress is reduced caused by the convergence of the critical entrainment shear stress distributions for each fraction (G^i).

(2) An intermediate partial transport regime in which medium-sized particles are over represented in the bed-load and the fine and coarse particles are under-represented. This may be associated with the divergence of the critical entrainment shear stress distributions at low and intermediate shear stresses.

(3) A weak partial transport regime, marked by a rapid decrease in the transport of fine particles and an increased mobility of the coarse particles. This regime is associated with the intermediate log-linear section of the critical entrainment shear stress distributions. In this regime $p_i \rightarrow f_i$ as shear stress increases because an increasing proportion of the surface is mobilized.

(4) Strictly equal mobility is attained only when the surface is fully mobilized. This occurs at higher shear stresses than has previously been recognized. Note that few flume data are available for this regime and that the validity of the simulated results (for a fully mobile bed) should be treated with some caution. This restriction will be discussed in the next section.

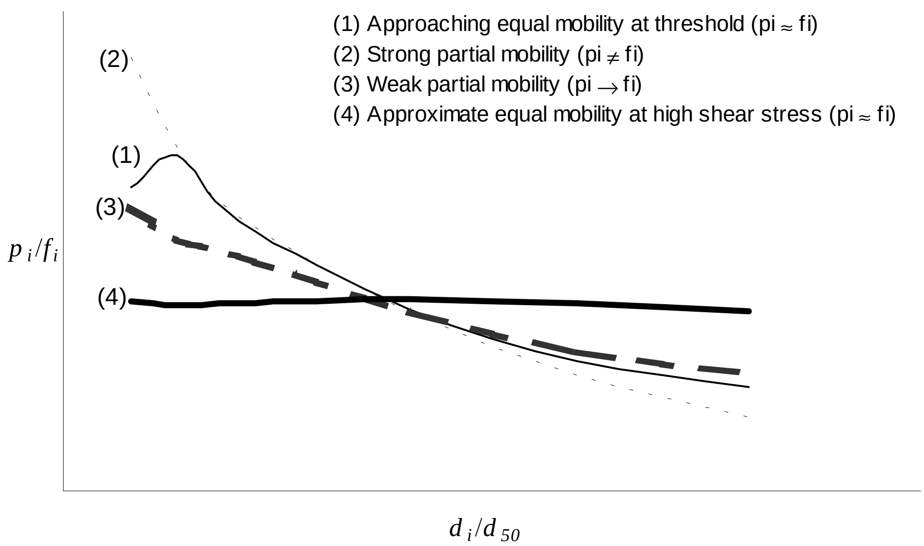

Figure 10: A schematic representation of the four compositional regimes of bed-load transport

When describing the behavior of various sediment mixtures in the laboratory and the field data from Oak Creek Wilcock (1992b) stated that these sediments, “show a similar pattern of distinctly non-equal mobility in the range $\tau_0/\tau_c < 2$, and approach equal mobility for $\tau_0/\tau_c > 2$. It is likely that at still higher transport rates (achieved in extreme floods, but difficult to produce in the laboratory) the fractional transport rates would achieve equal mobility (Einstein, 1950)." The predictions of bed-load compositions as a function of mean boundary shear stress provide strong support for this conceptual model.

Somewhat surprisingly the results indicate turbulence plays a rather limited role in conditioning bed-load composition as evidenced by the slight differences between Cases (A) – constant fluid shear stress and Case (B) - normally distributed fluid shear stress. Thus it is concluded that compositional changes are, in most cases, dominated by the critical entrainment shear stress distributions rather than by turbulence. Note that this does not mean that the magnitude fractional transport rates are not influenced by turbulent fluctuations. In fact there is considerable scope for the magnitude of the fractional transport rates to depend on the form of the fluid shear stress distribution. Nelson *et al*. (1995) demonstrated that bed-load motion correlates well with near-bed stream-wise velocity. Many sediment transport prediction methods have been developed and calibrated in conditions of two-dimensional flow in a well developed boundary-layer in which the scale for near-bed velocity fluctuations is the shear velocity, u_*. Nelson *et al*. (1995) noted that, in conditions for which the near-bed flow departs from these conditions, standard prediction methods are unlikely to provide a good model of events. This provides a compelling argument in support of developing prediction methods based more firmly on the physical processes with less dependence on empiricism. For instance, the method developed here has not been developed for, or calibrated with, data obtained in uniform, two-dimensional flow. Therefore its application is not, a priori, restricted to such flows and can, in principle, be adapted for other conditions provided $f_F(\tau_o)$ is known.

The result that turbulence has a limited role in determining bed-load composition should be considered in the context of the assumption that the distribution of F applies to all size fractions so that all differential fractional sheltering effects are accounted for by the variation in G^i. It is possible that this assumption could be untenable in the case of well-sorted surfaces which exhibit strong local patchiness and hence local variability in roughness and near-bed turbulence properties. Such conditions have been reported for strongly bi-modal sediments (Wilcock, 1993).

IMPLICATIONS FOR TRANSPORT RATE PREDICTION

The method introduced in this paper for predicting bed-load composition faithfully reproduces some of the observed changes in bed-load composition with shear stress for the 1ϕ mixture. The results are encouraging as it suggests that probabilistic methods based, more firmly on the grain mechanics, can lead to improvements in the accuracy of predictions of bed-load composition in mixed-grain-size sediments. The key to doing this is a knowledge of the critical entrainment shear stress distributions for the various size fractions in a mixture. The present success in modeling bed-load composition may be almost entirely attributed to the relaxation of the assumption that all grains in a size fraction behave in a like-manner. Historically, the widespread use of this assumption may be attributed to a flawed concept of threshold in which it has been generally regarded as a discrete transition between no-motion and motion. This may be tenable in uniformly-sized sediments but it certainly cannot be justified for sediment mixtures.

Furthermore, our continued adherence to this concept as a basis for prediction is apparently responsible for the difficulties that have been experienced in making reliable predictions of bed-load composition.

Much work remains to be done before a generalized probabilistic model suitable for predicting fractional transport rates in mixtures can be developed. In order to structure the discussion of the steps required to attain this goal it is necessary to distinguish between predictions for equilibrium conditions (an analogue for an equilibrium system is a re-circulating sediment flume) and prediction for non-equilibrium predictions which typically encompass long time periods during which sorting effects are influential. The treatment of bed-load composition in this paper has been restricted to equilibrium conditions and so, firstly, the steps that will lead to a generalized probabilistic transport rate model for mixtures in equilibrium conditions are described. Following this the additional difficulties that the non-equilibrium case presents will be idenified.

The approach that has been introduced is founded on knowledge of the critical entrainment shear stress distributions for the various size fractions. Good progress has been made in describing these over the last decade (Kirchner *et al.* 1990; Buffington *et al*., 1992 Johnston *et al*., 1998 and McEwan and Heald, 2001). Nevertheless, to date, these descriptions are entirely empirical (or numerical) and their acquisition requires considerable effort in either the laboratory, field or computational resources. Clearly a less demanding means to derive these distributions for a particular sediment is desirable and research effort should be expended to develop such methods. The demonstration that these distributions collapse with a Shields scaling (McEwan and Heald, 2001) is an important first step in this regard but very little is known about how different mixtures interact in changing the form of these distributions. A combination of discrete particle modeling and particle stability measurements has the potential to contribute to this development. Furthermore methods, termed texturing, have been developed to characterize the details of the surface topography, at a grain scale (Willetts *et al*., 1998; Marion *et al*., 1997, Tait *et al*., 1998). Cunningham (2000) describes some preliminary attempts at using such scans (or digital elevation models) as an alternative means to derive critical entrainment shear stress distributions during re-circulating flume experiments with mixed-grain-size sediments.

Some models of mixed-grain-size sediment transport (e.g. Willetts *et al*., 1987) adopted a strategy whereby a total transport rate was calculated based on the capacity of the flow to move sediment and then the composition of the bed-load was determined with the assistance of some hiding function. While the probabilistic method for predicting bed-load composition described here could be deployed in this manner, it seems more appropriate to attempt to extend it to predict fractional transport rates directly rather than by partitioning of the total mass transport rate. To do this, the fractional entrainment rate ϕ_i and the mean step length λ_i (as opposed to Φ_i and Λ_i) must be determined as functions of mean boundary shear stress. It is considered that much of the information required to specify the function $\phi_i(\tau_o)$ is already implicit in the critical entrainment shear stress distributions (Fig 5b). By restricting the treatment of sediment mixtures to bed-load composition only the relative magnitudes of G^i as a means of obtaining p_i at a particular shear stress are have considered. Additional information concerning the variation of fractional entrainment rates is, however, available if the absolute values of G^i are accounted for as shear stress increases. Intriguingly the slope of the distribution in log-log space in the weak partial transport regime is roughly $^3/_2$ (McEwan and Heald, 2001). However, this observation has not been exploited in the present paper because there is no corresponding description of how the mean step length (λ) varies with shear stress and the extension of the probabilistic treatment to predict fractional transport rates would also require knowledge of the function $\lambda(\tau_o)$. McEwan *et al*. (2000) describe a combination of discrete particle modeling and field tracer studies in studying the behavior of mean step length. This, and other studies (e.g. Church and Hassan, 1992), will be an essential ingredient in the development of a more complete probabilistic account of mixed-grain-size sediment transport and will, it is hoped, lead to a complete probabilistic method for predicting mixed-grain-size sediment transport in equilibrium conditions.

Non-equilibrium conditions pose an even greater challenge. There are two processes which contribute to this. First, non-equilibrium conditions are often associated with changes in the surface composition. It is well know that these are difficult to track especially over longer prediction periods. Second, recent work has suggested a more subtle effect which, the evidence suggests, is very influential in some non-equilibrium transport systems. Marion *et al*. (1997) and Pender *et al*. (2000) found that during the formation of a static armor, the transport rate decreased to near-zero over time as expected but that the bed-load composition and the surface composition did not change greatly during the experiment. Clearly this contrasts with the classic concept of winnowing whereby fine particles are preferentially transported until the surface coarsens sufficiently to inhibit transport. Texture scans of the sediment surface were obtained at intervals throughout these experiments and these scans indicate that significant changes in the bed topography take place in the early phase of the degradation which is also associated with the rapid decline in transport rate. Marion *et al*. (1997) and Pender *et al*. (2000) suggest that this effect produces an increase in bed stability due to the re-arrangement of the surface grains and, for these particular conditions, the re-structuring of the surface dominated over the effect of changing surface composition. These observations have important implications for extending the probabilistic method to non-equilibrium conditions as they suggest that the critical entrainment shear stress distributions are not only a function of surface composition but of surface history as well. Thus, while they provide an appropriate theoretical framework in which to describe the changes in surface stability found by Marion *et al*. (1997) and Pender *et al*. (2000), quantifying their behavior in such conditions will inevitably be more complex than the equilibrium case. Research effort towards this goal is justified.

EVALUATION OF THE DISCRETE PARTICLE MODELING

The probabilistic analysis of the critical entrainment shear stress distributions derived from the discrete particle modeling has reproduced the “flume-scale” experimental results with remarkable fidelity. This is very encouraging, not least because the discrete particle modeling is itself subject to simplifications of some very complex physical processes. The success therefore suggests that significant improvements in our predictive capability can result from even rough approximations to actual grain mechanics of entrainment. It is considered that this is an incentive to vigorously pursue further research into the grain mechanics (Wilcock, 2000) and should not be taken as a reason to curtail our efforts in this regard.

The discrete particle modeling is based on a number of assumptions. Firstly, a numerical bed consisting of frictionless spherical grains, deposited under artificial conditions, has been compared to a water worked bed of irregular particles. Secondly, the analysis of the numerical bed presumes that lift forces do not play a significant role in entrainment. Thirdly, the structure of the surface would be changed by high levels of entrainment as shear stress increases, so the upper ends of the distributions presented in Fig. 5 are unlikely to represent actual conditions during bed load transport. Fourthly, in comparing the composition of the entrained particles to the bed-load composition measured by Wilcock (1992), it has been assumed that the excursion length is independent of grain size. Fifthly, bedforms were noted as being present in some of the physical experiments which are not present in the simulations. In general terms, the success of the predictions suggests that the above assumptions are not crucial to the success of the method. However, it is intended that future modeling will examine the influence of some of these assumptions.

In addition to these, it is noted that a further restriction to the range of sediment mixtures that can sensibly be treated with the discrete particle model. The simulated 1ϕ mixture has a limited range of particle sizes so that the ratio of the largest to the smallest grains is almost one order of magnitude. This is considered to be a sensible limit to the type of mixture that can modeled for the following reason. A fundamental part of the model is positioning and stability of particles. Each particle, when at rest, is supported by three, and only three, other grains forming a tetrahedral unit. For sediment mixtures with a large range of sizes, say two orders of magnitude, some larger grains will have supporting contacts with many more than three grains. Thus, the simple tetrahedral structure currently employed in the simulations cannot properly represent the supporting arrangements for the coarse size fractions in mixtures containing size ranges over several orders of magnitude. Considerable effort would be required in developing algorithms, to extend the method to treat such mixtures. Such effort will be worthwhile as many important sediment problems are concerned with mixtures with such size ranges. However, the required development is not trivial and it is suggested that at present, one order of magnitude, is a sensible restriction to place on the range of sediment mixtures that can be treated.

CONCLUSIONS

The threshold shear stress is an arbitrarily chosen point located somewhere on the lower tail of the distribution of critical entrainment stresses of individual grains. Transport in fact continues at lower stresses, at small but finite levels which require extended periods to measure (Paintal, 1971a). Theoretically, threshold has no special significance. Practically, it is a useful, but essentially arbitrary, guide to bed stability. The entrainment process, when considered as the sediment source for bed-load particles during transport, seems to have no obvious relation to the empirical threshold of motion determined for a particular size fraction. This has important ramifications for the parameterization of bed-load transport equations which include a single-valued threshold to regulate the transport rate at low shear stresses. In accordance with Wilcock (1997) it is concluded that such an approach is a poor basis for modeling fractional transport rates in mixed-grain-size sediments. It is suggested that improvements in the prediction of bed-load composition will result from the relaxation of this assumption.

The manner in which the critical entrainment shear stress distributions capture many of the features of the experimental data suggests that they have the potential to become a powerful tool in representing transport processes in graded sediments. It is suggested that a probabilistic formulation of transport rate based on the interaction of particles with turbulence is a promising approach meriting a concerted development effort from researchers.

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FIGURE CAPTIONS

Figure 1 Schematic representation of the concept of Grass (1970).

Figure 2 Size distribution for the 1ϕ sediment mixture.

Figure 3 Fractional transport rate data for the 1ϕ mixture over five mean shear stresses derived from Wilcock (1992).

Figure 4 100 x 100 mm plan view of the surface and an expanded view of grain-scale topography.

Figure 5 Critical entrainment shear stress distributions for the 1ϕ mixture.

Figure 6 Non-dimensional critical entrainment shear stress distributions for the 1ϕ mixture shown with the equivalent curve for a uniformly-sized sediment.

Figure 7 Shows p_i/f_i against grain size evaluated for Case (A) for shear stresses in the range 0.53 – 55 Pa.

Figure 8a Shows p_i/f_i against grain size evaluated for Case (B) for mean boundary shear stresses in the range 0.5 – 5.0 Pa, $\sigma_\tau = \tau_o/4$.

Figure 8b Shows p_i/f_i against grain size evaluated for Case (B) for mean boundary shear stresses in the range 1.0 – 50.0 N/m^2, $\sigma_\tau = \tau_o/4$.

Figure 9 Critical entrainment shear stress distributions for the 1ϕ mixture plotted on log-linear axes.

Figure 10 A schematic representation of the four compositional regimes of bed-load transport