WAFO - A MATLAB toolbox for random waves and loads

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Outline

WAFO briefly described

Wave Analysis for Fatigue and Oceanography

- Statistical analysis and simulation of random waves
- Calculation of theoretical distributions related to characteristic wave parameters
- Applications to sea waves and fatigue analysis

Developed by the WAFO-group:
P.A. Brodtkorb, M. Frendahl, P. Johannesson, G. Lindgren, I. Rychlik, J. Rydén, E. Sjö + others

WAFO - Philosophy

MAKE SCIENTIFIC COMPUTATIONS REPRODUCIBLE!!!

- Available free of charge at the Internet
- More than 250 routines organised in modules related to applications
- Easy to find routines, easy to add new ones
- Help pages in nice html-interface
- Tutorial with many examples

Important module: algorithms and code for generation of results in selected articles

`www.maths.lth.se/matstat/wafo/`

Random sea waves

Modelling of sea waves as stationary transformed Gaussian processes.

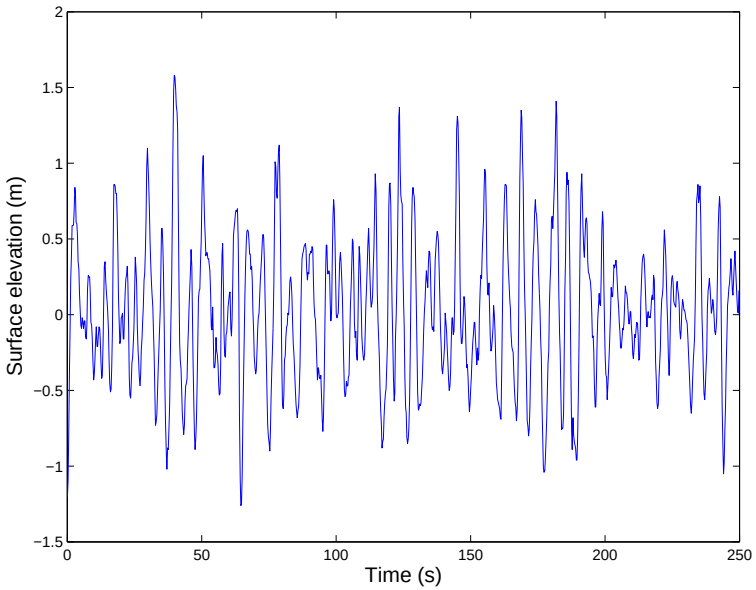
- Extraction of wave characteristics from data
- Estimation of spectrum
- Spectral simulation
- Calculations of exact distributions for characteristic wave parameters

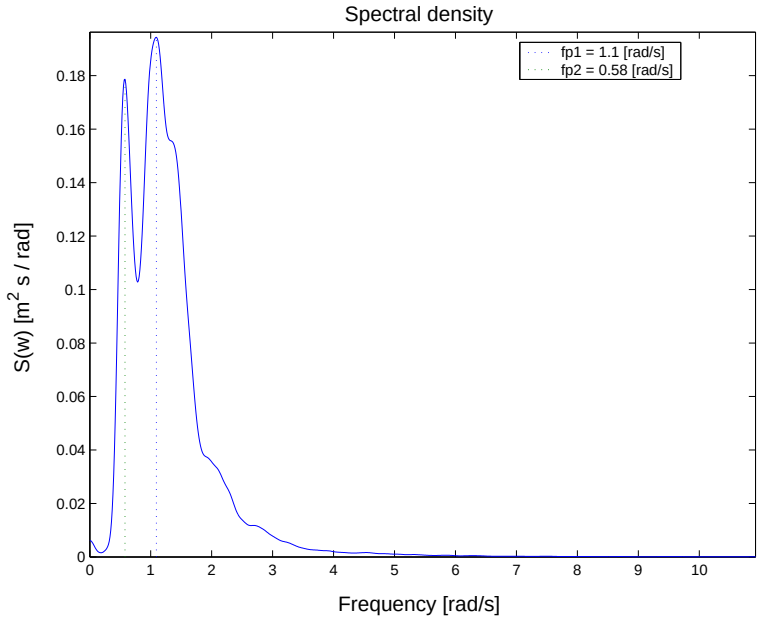
Example: Wave spectra

Estimation of spectrum from data.

>> Sest = dat2spec(data,200);

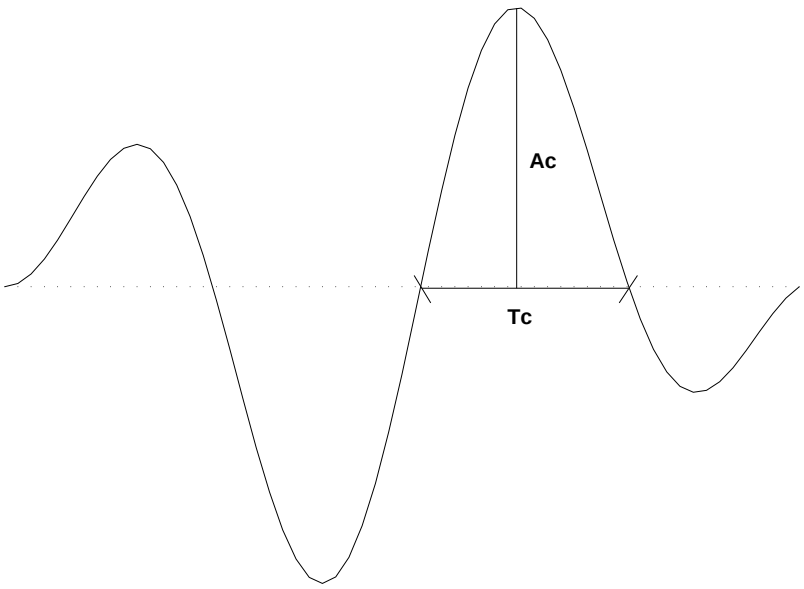
>> wspecplot(Sest);



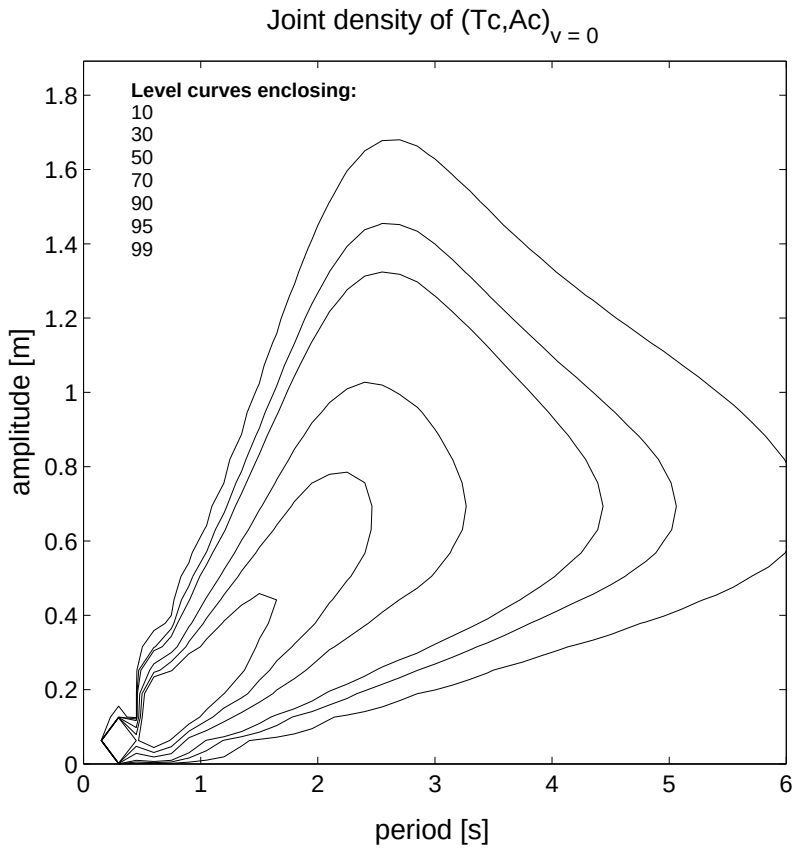


Joint distribution of wave characteristics

Definition of crest length and crest amplitude.

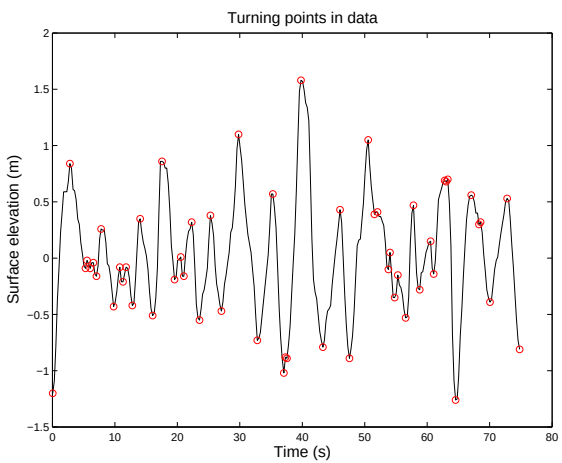


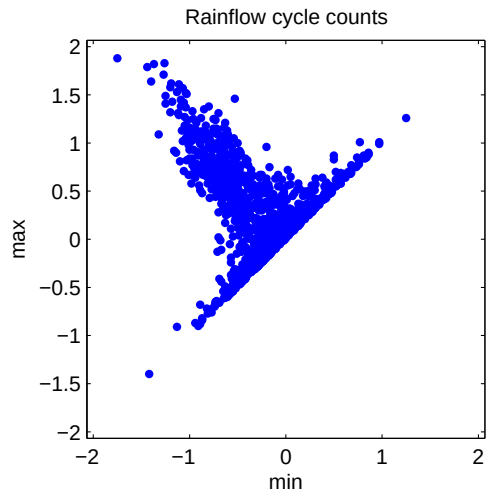
>> f = spec2thpdf(Sest,...);

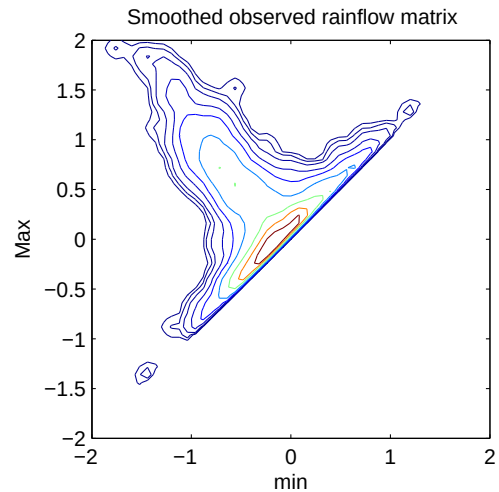


Routines related to random loads and fatigue

- Extraction of rainflow cycles from data
- Calculation of expected rainflow matrix
- Switching Markov loads
- Visualization of cycle counts etc.



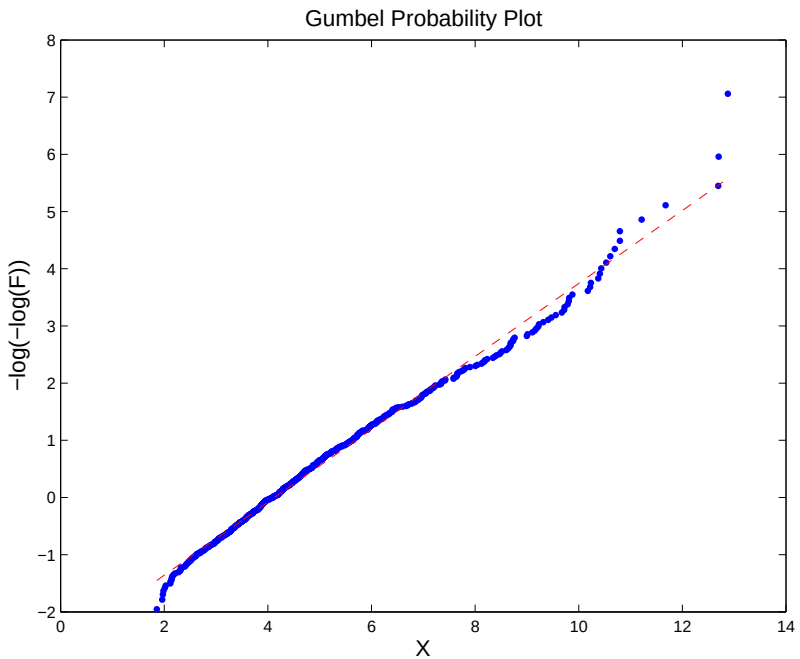


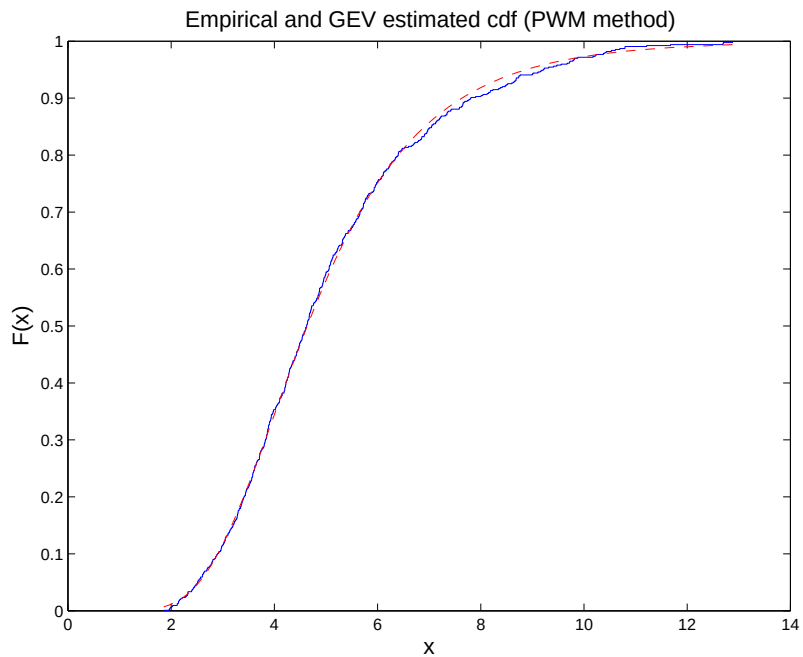


Extreme value analysis

WAFO contains a module for extreme value analysis. For example one can

- estimate parameters in the Generalized Extreme Value distribution and the Generalized Pareto distribution
- simulate from the GEV and GPD
- make probability and quantile plots

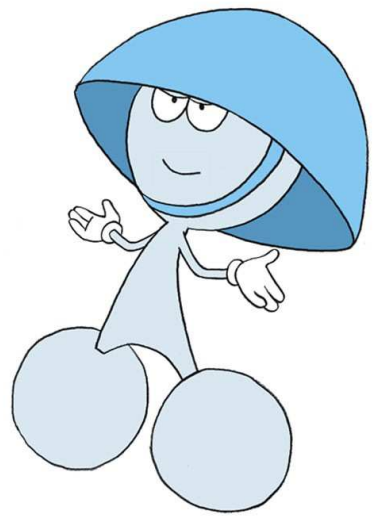




Environmental standard for ozone

Ozone is an airpollutant which may cause severe lung damage. In the U.S. the 1-hour air-quality standard for ozone is 0.12 ppm. If we get concentrations above 0.18 ppm it is considered to be a serious violation of the clean air act.

Can it be assured that people are not exposed to dangerous concentrations even though the standard is attained?



Experiment

Suppose that a measurement is obtained exactly at the standard level. Define a region in the plane such that it consists of all points correlated at least 0.7 with the monitoring site.

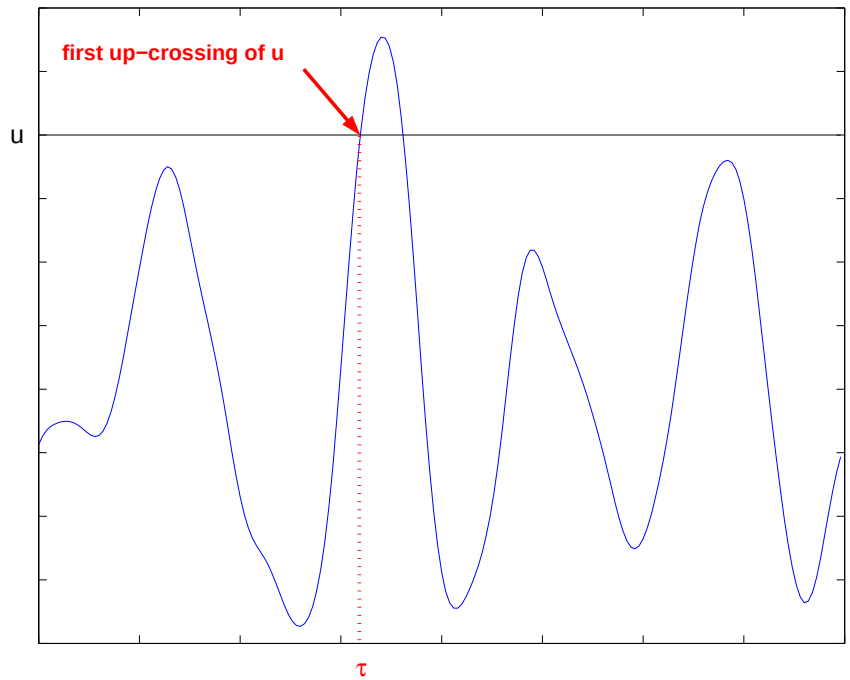
Given the observation, what is then the distribution of the maximum over this region?

One dimensional counterpart

Let $\{X(t), t \in [0, T]\}$ be a random process. Then

$$P(\max_{t\in[0,T]} X(t) > u) = P(X(0) > u) + P(X(0) \leq u, \max_{t\in[0,T]} X(t) > u)$$
$$= P(X(0) > u) + P(X(0) \leq u, N_T^+(u) \geq 1),$$

where $N_T^+(u)$ is the number of up-crossings of level u of the process $X(t)$ in the interval $[0, T]$.



Rice bounds

Classical Rice upper bound:

$$P(X(0) \leq u, N_T^+(u) > 0) \leq E(N_T^+(u))$$
$$= \int_0^T E(X'(t)^+ | X(t) = u) f_{X(t)}(u) dt$$

Let τ be the first time the process crosses level u. Then by using the first passage density

$$P(X(0) \leq u, \max_{t \in [0,T]} X(t) > u) = P(\tau \in [0, T])$$
$$= \int_0^T E(X'(t)^+ \{X(s) < u, \forall s < t\} | X(t) = u) f_{X(t)}(u) dt$$

Two dimensions

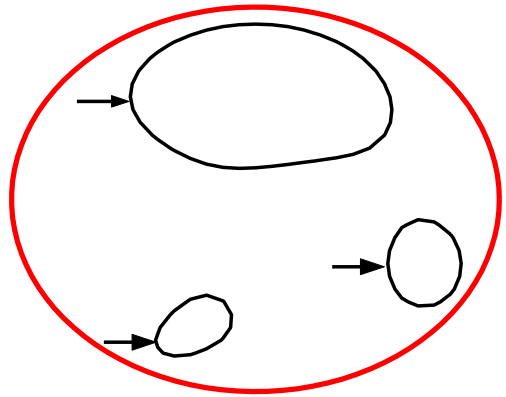
Let $W(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^2$ be a random field and let $\mathbf{S}$ be a bounded region in $\mathbb{R}^2$ with boundary $\partial\mathbf{S}$. Then

$$P(\max_{\mathbf{x}\in\mathbf{S}} W(\mathbf{x}) > u)$$
$$= P(\max_{\mathbf{x}\in\partial\mathbf{S}} W(\mathbf{x}) > u) + P(\max_{\mathbf{x}\in\partial\mathbf{S}} W(\mathbf{x}) \leq u, \max_{\mathbf{x}\in\mathbf{S}} W(\mathbf{x}) > u).$$

Needs an analogue to the up-crossings in one dimension!!

Analogue to up-crossings

Use up-crossings in the x-direction.



These points satisfy

$$W(\mathbf{x}) = u,\ W_{01}(\mathbf{x}) = 0,\ W_{02} < 0,\ W_{10} > 0$$

Denote the number of such points by $N_{\mathbf{S}}(\mathbf{u})$.

Upper bound in two dimensions

As in one dimension bound the probability that the number of such points is greater than one with an expectation. This gives, after some calculations :-)

$$P(\max_{\mathbf{x}\in\partial\mathbf{S}} W(\mathbf{x}) \leq u, \max_{\mathbf{x}\in\mathbf{S}} W(\mathbf{x}) > u) = P(\max_{\mathbf{x}\in\partial\mathbf{S}} W(\mathbf{x}) \leq u, N_{\mathbf{S}}(\mathbf{u}) \geq 1)$$

$$\leq \int_{\mathbf{S}} E(W_{02}(\mathbf{x})^{-} W_{10}^{+}\{W(\mathbf{s}) \leq u, \forall \mathbf{s} \in \partial\mathbf{S}\} | W(\mathbf{x}) = u, W_{01}(\mathbf{x}) = 0) f_{W(\mathbf{x}),W_{01}(\mathbf{x})}(u, 0) d\mathbf{x}$$

rind

To compute an upper bound for $P(\max_{\mathbf{x}\in\mathbf{S}} W(\mathbf{x}) > u)$ we need to compute multivariate normal expectations of the form

$$E\left(|Xd(1)\cdot\ldots\cdot Xd(Nd)|\{a_i < Xd(i) < b_i, c_j < Xt(j) < d_j\}|Xc = xc\right) f_{Xc}(xc)$$

The WAFO-function `rind` is custom made for these type of calculations under the Gaussian assumption!! The input is just the mean and covariance matrix of the variables.

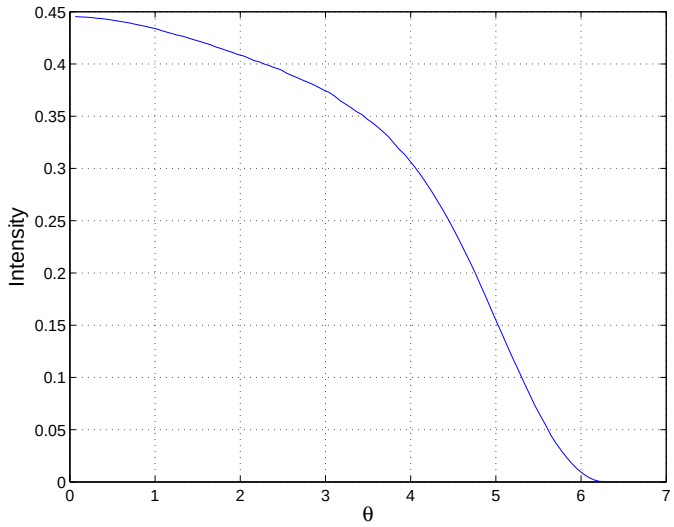
Ozone example revisited

Model and assumptions

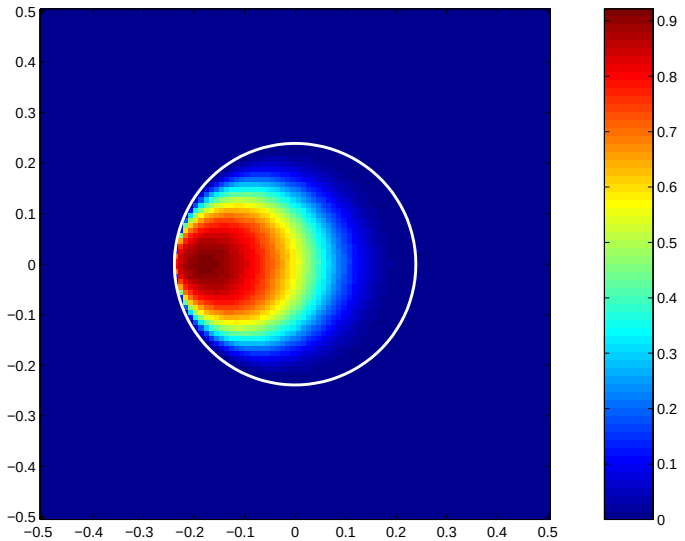
- $\xi(\mathbf{x})$ square root of true underlying ozone field. Assumed to be Gaussian.
- observation $z(\mathbf{x}_0) = \xi(\mathbf{x}_0) + \epsilon(\mathbf{x}_0)$
- conditional process $W(\mathbf{x}) = \{\xi(\mathbf{x}) | \xi(\mathbf{x}_0) + \epsilon(\mathbf{x}_0) = z(x_0)\}$
- $E(\xi(\mathbf{x})) = 0.235$, $Var(\xi(\mathbf{x})) = 0.064^2$, $Var(\epsilon(\mathbf{x}_0)) = 0.032^2$
- Covariance of $\xi(\mathbf{x})$ is of the squared exponential type

Intensities computed by `rind`

Intensity on the boundary:

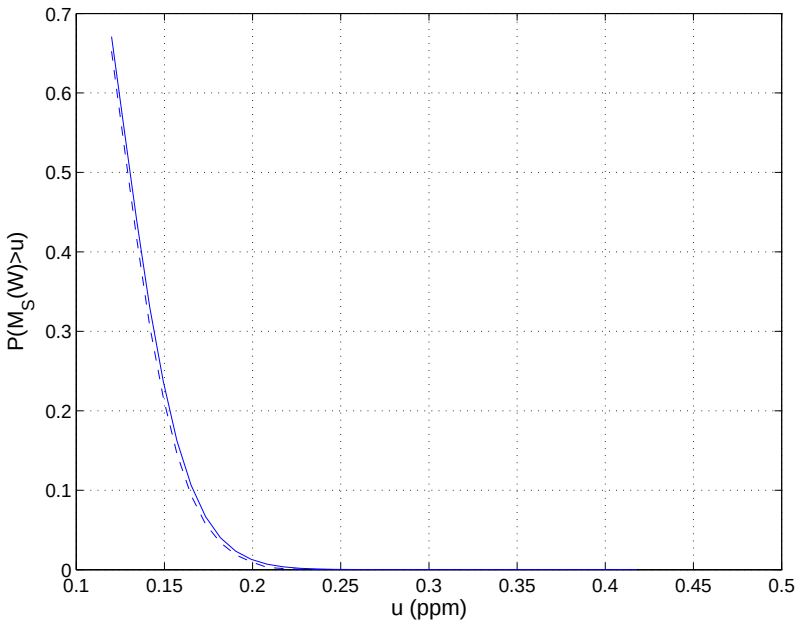


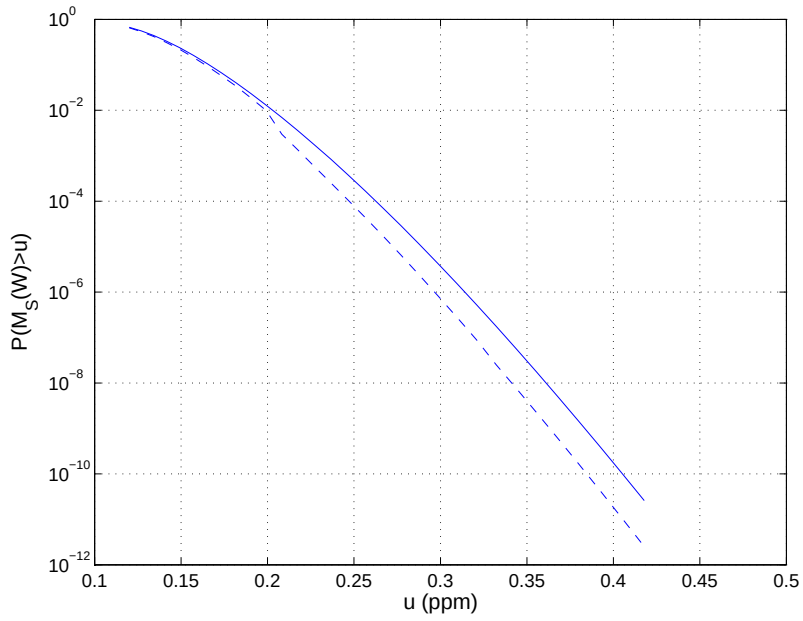
Intensity within the region:



Distribution of the maximum

Ditribution of the maximum in the 0.7-correlation region on a linear and log-scale respectively:





Note that there is a 5% risk of serious violation!!

Reminder!

The homepage of WAFO on the internet

`http://www.maths.lth.se/matstat/wafo/`

Versions to download are available for Windows and Unix.