Modelling extreme hot events using a non homogeneous Poisson process

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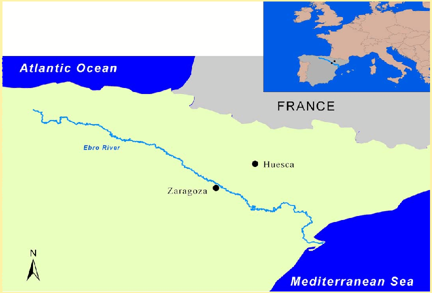
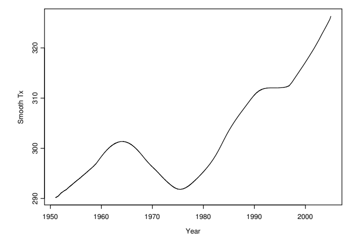
- To develop a **statistical model for extreme hot events** (based on Extreme value properties) to answer questions such as: 'Are those events changing in frequency or severity over time?' or 'How that changes depend on temperature evolution?'.
- To obtain **medium and long term projections for the expected evolution of the extreme hot events**, using the fitted statistical model and the temperature projection provided by a General Circulation model.

Data description

Daily summer maximum temperature series, **Tx**, from Zaragoza (Spain);
summer: June-July-August
Series record: 1951 to 2004
(Data from the Spanish National Meteorological Institute, INM.)

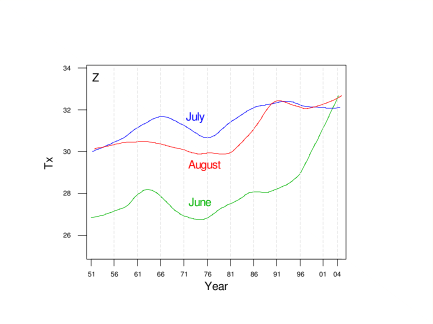


Tx evolution: smooth of the summer daily series (lowess 30%).



Increasing trend from the middle of the 70s

Tx evolution by month: smooth of the daily series, by month (lowess 30%)



In June, increasing slope becomes steeper from 1994

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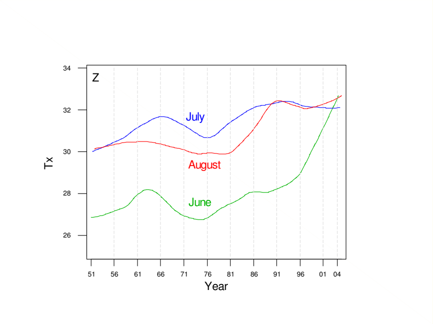

In July and August, temperature becomes more stable at the end

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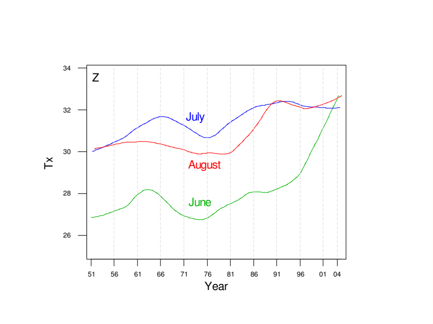

Temperature evolution is not homogeneous during the summer

Part II

Analysis of extreme hot events

1. Definition of extreme hot events

'Excess over threshold' approach: an EHE is defined as a run of consecutive days with temperature values over an extreme threshold.
Selected threshold: 95^{th} percentile of the summer temperature series for the interval 1971-2000; Zaragoza: $37^{o}C$

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We assign to each event:
- A point of occurrence (maximum intensity point)
- Three variables describing the event severity
  - **L**: the **length** of the spell
  - **Ix**: the **maximum intensity** over the threshold in the spell
  - **Im**: the **mean intensity** of the spell (accumulated exceedances of Tx over the threshold $/L$)

Descriptive analysis of the observed EHE (157 events)

Decade	Occurrence		L		Ix		Im	
	Ann.Mean	Ann.Max	Mean	P90	Mean	P90	Mean	P90
1951-60	1.2	3	1.4	2.7	1.3	3.0	1.1	2.2
1961-70	2.8	6	1.6	3.1	1.2	3.1	1.0	2.3
1971-80	2.1	3	1.4	2.8	0.8	1.6	0.6	1.6
1981-90	2.5	5	1.8	3.8	1.3	4.0	0.9	2.8
1991-00	3.9	7	1.9	3.0	1.3	2.5	1.0	1.8
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- **Seasonal component**
  Variables defining the part of an annual harmonic corresponding to the summer months
  $$\cos(2\pi t), \ \sin(2\pi t)$$
  with $t = 152/365, ..., 243/365$

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- **Seasonal component**
- **Temperature information**
  - *Long term temperature* signal: $TTx = Tx$ smooth (lowess 30%)
  - *Semi-local temperature* signal: $Tx_{m30} = Tx$ moving mean of the 15 previous and the 15 following days.
  - *Local temperature* signal: Tx
  - Quadratic temperature terms and seasonal-temperature interaction terms.

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\begin{aligned}
log(\lambda(t)) \quad = \quad & \beta_0 + \beta_1 \cos(2\pi t) + \beta_2 \sin(2\pi t) + \\
& f1(\text{temperature variables}; \beta_i) + \\
& f2(\cos(2\pi t) \text{ temperature variables}; \beta_j) + \\
& f3(\sin(2\pi t) \text{ temperature variables}; \beta_k)
\end{aligned}
$$

The model parameter estimation is performed by maximum likelihood.

- The likelihood function is derived taking into account the independence and the Poisson distribution of the number of events in every interval,

$$L(t_i;\beta) \simeq \exp[-\Lambda(\mathcal{A};\beta)] \prod_{i=1}^{n} \lambda(t_i;\beta)$$

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Fitted model resulting from the selection process:

$$\begin{aligned} log(\lambda(t)) &= -15.0 - 3.9\cos(2\pi t) - 1.3\sin(2\pi t) + \\ & \quad 0.006\,TTx + 0.045\,Tx_{m30} \end{aligned}$$

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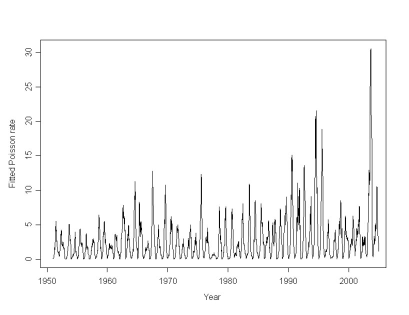
The EHE occurrence process shows a significant seasonal behaviour.

The temperature has a long term and semi-local linear effect.

Poisson rate fitted for the observed time interval 1951-2004



Increasing trend towards higher occurrence rates

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NHPP residuals

Procedure to check the validity of the model (Poisson character with the specified time-dependent rate) based on two properties,

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- The homogenous Poisson character can be checked by controlling the exponential distribution of their inter-event distances d_i; or, equivalently, the standard uniform character of $\exp(-d_i)$.

Checking procedure

1. The observed NHPP is transformed to a HPP of rate 1 using the previous property; thus, the original occurrence points t_i are transformed to

$$t_i^* = E[(0, t_i)] = \int_{(0,t_i)} \hat{\lambda}(t)dt$$

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2. The uniform behaviour of the sample $\exp(-d_i)$ is checked using a Kolmogorov-Smirnov goodness of fit test and a qqplot with a confidence band based on the beta distribution of the ordered uniform quantiles.

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- Usual residuals (observed-fitted values) can not be defined since the occurrence rates are not observed.
- Alternative residuals: difference between an empirical daily occurrence rate (calculated using observations of an interval t_l of length l around each day) and a fitted daily occurrence rate (calculated as $\int_{t_l} \hat{\lambda}(t)dt/l$); we consider $l = 3$ months.

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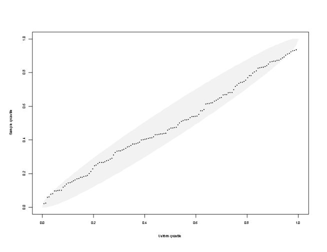
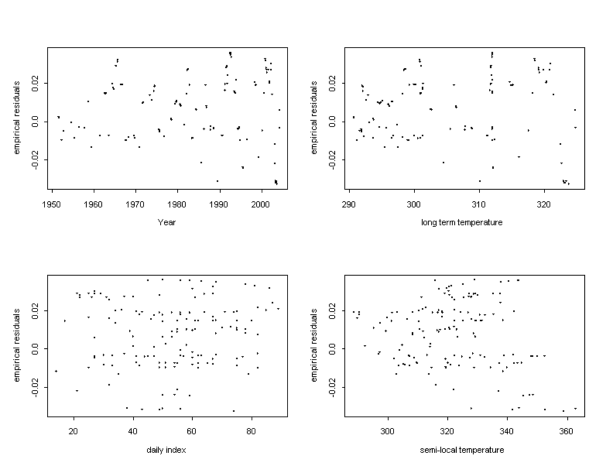
Results. NHPP residuals
Uniform qqplot with beta confidence bands



P-value of the Kolmogorov-Smirnov goodness of fit test: 0.48

Results. Empirical residuals



3. Modelling EHE severity

To complete the description of the EHE, we model the three variables describing their severity (L, Ix and Im) using adequate probability distributions.

To allow these distributions to be dependent on influential variables, we use **Generalized Linear Models**, GLM, selecting an adequate error family for each case; the same variables used for the occurrence model are considered again.

Fitted models

Length (shifted Poisson): $\log(L) = -8.6 + 0.027\,TTx$
Maximum Intensity (Gamma): $\log(Ix) = 5.9$
Mean Intensity (Gamma): $\log(Im) = 2.2$

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L is affected by long term temperature; no seasonal behaviour

Distribution of the intensity variables remains stable in time; no seasonal behaviour.

Part III

Projecting EHE evolution in a climate change scenario

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- In order to validate the methodology to be used, we analyse the behaviour of the fitted model using the GCM projected temperature **for the observed interval 1951-2004.**
- If this validation is satisfactory, we project the EHE for the future.

2. General Circulation Models

General circulation model: numerical model to simulate changes in different climate signals, such as temperature, under possible scenarios resulting from slow changes in atmospheric concentrations of greenhouse-gases, etc.

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Selected scenario: A2, which represents a world with continuously increasing population and regionally oriented economic growth.

2.1 GCM data

Output series from the GCM: monthly mean series of Tx from 1951 to 2050.
Input temperature variables of the statistical model: TTx and Tx_{m30}; they are estimated from the monthly mean series of Tx provided by the GCM.

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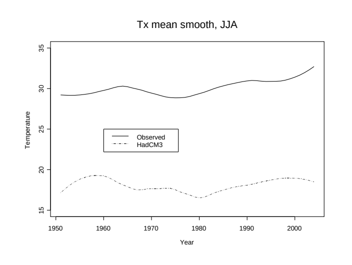
- Output series from a GCM integrates the values over all the area associated to a point of the spatial grid (heights varying from 100 to 2500m.).
  Even if the evolution of the temperature can be considered homogeneous over the area, the series for a point location has to be scaled to fit the mean level of that location.

Smooth of monthly mean Tx: observed and projected series. Zaragoza



Mean level of the projected series is lower than the observed one

Scaling GCM series to fit the mean level

1. Selection of a time interval where the evolution of the observed and simulated signals is parallel: 1971-2000.

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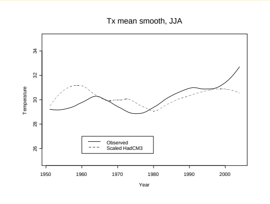
2. GCM series is scaled in order to get that, in that time interval, both series have the same mean value and standard deviation, in each summer month.

3. Validating the projection procedure

1. Validation of the temperature signal provided by the GCM **for the observed interval** 1951-2004
Smooth of monthly mean Tx, observed and projected series



Differences: the initial interval 1951-60 and from 1996, where only the observed signal is increasing

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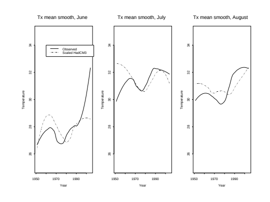

Differences by month: in June, the observed signal becomes steeper than the fitted one from about 1996

2. Validation of the EHE projection: we compare the observed number of events and the number calculated from the fitted occurrence rate using as input the GCM temperature **for the observed interval** 1951-2004, by month and decade.

	Fitted number (decade)			Observed number (decade)		
Decade	June	July	August	June	July	August
1951-60	3.1	27.1	14.2	1	9	2
1961-70	3.0	19.3	8.8	5	12	11
1971-80	2.0	15.3	10.2	0	13	8
1981-90	2.4	17.2	8.2	2	15	8
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- *Model able to reproduce the observed seasonal behaviour.*
- *Fitted numbers for the 50s are higher than the observed ones*.
- *The model reproduces satisfactorily the other rates; the only discrepancy appears in June 2001-04.*

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	Fitted number (decade)			Observed number (decade)		
Decade	June	July	August	June	July	August
1951-60	3.1	27.1	14.2	1	9	2
1961-70	3.0	19.3	8.8	5	12	11
1971-80	2.0	15.3	10.2	0	13	8
1981-90	2.4	17.2	8.2	2	15	8
1991-2000	2.9	23.3	13.4	2	23	14
2001-04	1.4	7.0	5.6	8	8	5

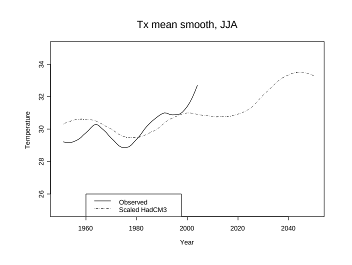
- *Model able to reproduce the observed seasonal behaviour.*
- *Fitted numbers for the 50s are higher than the observed ones*.
- *The model reproduces satisfactorily the other rates; the only discrepancy appears in June 2001-04.*

Origin of the discrepancies: bad GCM temperature projections

4. EHE projection until 2050

GCM temperature projection

Smooth of monthly mean Tx: observed and projected series, 2050



Increasing evolution from 2030

4.1 Projection of the EHE occurrence

Mean values of the projected occurrence rate, by month and decade

Decade	June	July	August
2001-10	0.387	2.209	1.337
2011-20	0.316	2.210	1.397
2021-30	0.342	2.255	1.878
2031-40	0.993	10.836	5.578
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From 2030, projection for July is about 11 EHE; for scenario A2, almost all days would be under extreme conditions

4.2 Projection of EHE length

To get projections of the EHE length, we use the fitted Poisson distribution (intensity depending on the long term temperature signal).

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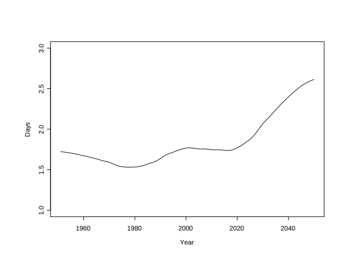
Validation (comparison between observed and fitted values for 1951-2004)

Decade	1951-60	1961-70	1971-80	1981-90	1991-2000	2001-4
Fitted	1.7	1.6	1.5	1.6	1.7	1.7
Observed	1.4	1.6	1.4	1.8	1.9	2.5

Both fitted and observed values show an increasing evolution from 1971; only the mean length for 2001-04 is under-fitted.

Projection of the EHE length until 2050



Decade	2001-10	2011-20	2021-30	2031-40	2041-50
Fitted	1.8	1.8	1.9	2.6	2.5

The main increase of the EHE length appears from 2030, following the evolution of the simulated GCM temperature

For scenario A2: a length increase of almost 1 day in 2050

Conclusions

- The fitted NHPP allows us to study the EHE ocurrence: their seasonal behaviour and their evolution in time, through their relationship with temperature.

  Zaragoza: occurrence is related to long term and semi-local temperature and has a seasonal behaviour inside the summer.
  EHE severity: we do not find seasonal behaviour and only length depends on temperature while intensity measures are stable in time.

- Combining the EHE statistical model with the GCM temperature output provides an adequate projection procedure, given that the GCM projections reproduce properly the temperature evolution.

  In order to get more reliable results covering different possible future situations, a wide range of projections under different scenarios and using information from different GCMs should be provided.

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