The Economics of Systemic Risk

Casper G. de Vries

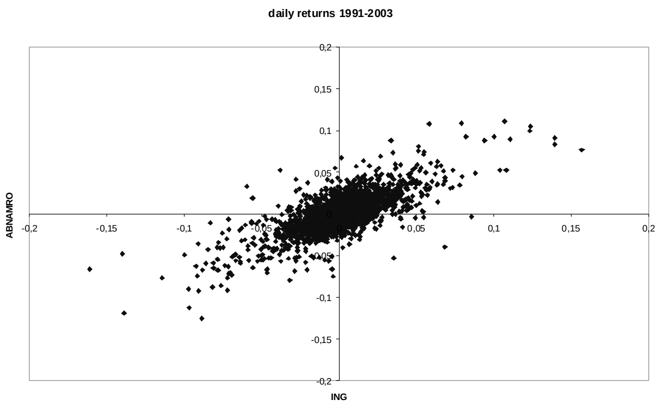
Jaap Geluk, Laurens de Haan

Erasmus University Rotterdam,
Tinbergen Institute

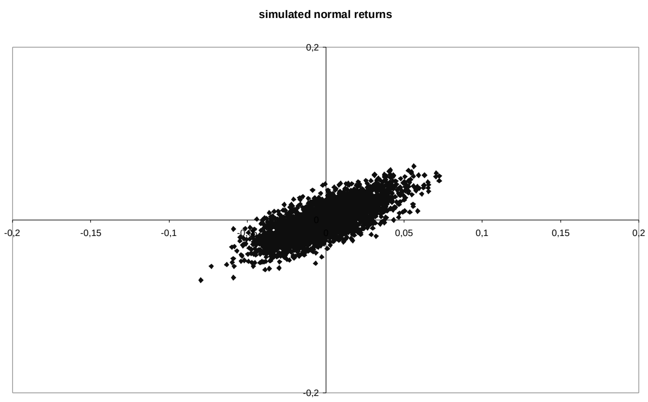
August 2005

Empirical Bank Dependence

Daily returns cross plot ABN-AMRO versus ING



Normal remake



Question

Should supervisors be worried?

Two issues

- univariate fat tail property
- multivariate dependency

Topics

1. banking linkages, affine portfolios
2. measures of dependency
3. discrete and continuous compounding
4. discrete loss returns
5. continuous loss returns
6. economic copula
7. options
8. sequence of banking networks
9. cross Atlantic evidence on banks

1. Banking Linkages

Relevance of Bank Stability

- externality to payment & clearing (joint product)
- transmission of monetary policy

∴ intensive supervision & regulation

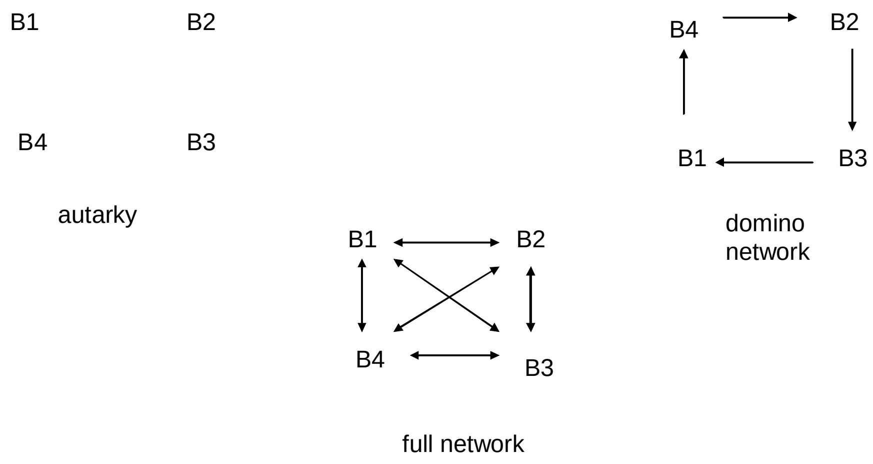
(more than for e.g. reinsurance)

causes of fragility

- deposit contract nature & high leverage
- sensitivity to macro policy shocks (interest rate decisions)
- high connectedness of banks through interbank market (contagion)
- similar portfolio exposures, syndicated loans
- Basle II micro orientation

  (the cynical supervisor only cares about isolated bank failures)

Network Structures



Measures of Connectedness

- correlation of bank stock returns
- conditional (on stress) correlation
- copula structure (stock returns)
- concentration measure interbank market
- correlation between distances to default
- evt based tail dependence

Disadvantages of correlation measure

- normal based (requires dgp otherwise)
- ultimate trade-off is between probability of loss and loss level
- is global measure, only need downside risk
- not applicable if moment failure
- zero correlation $\nRightarrow$ independence
- multivariate correlation matrix hard to interpret

Desired Measure

- measures dependence in loss area
- reflects trade-off between loss and probability directly
- handles multivariate case easily

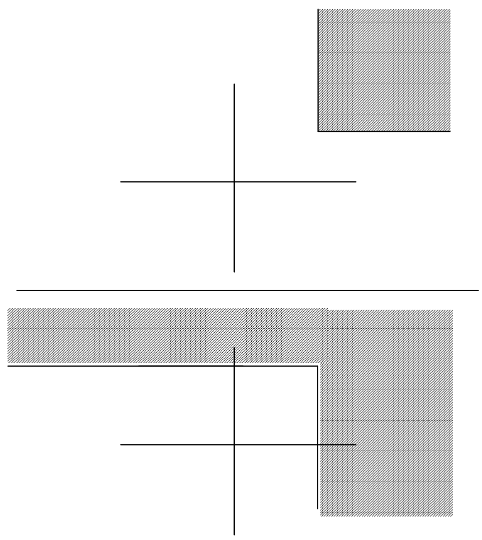
Systemic Risk Measure

Conditional Crash Probability:

probability that two markets crash given that at least one crashes

$$\frac{P\{X > t, Y > t\}}{1 - P\{X \le t, Y \le t\}}$$

=



Note

$$\frac{P\{X > t, Y > t\}}{1 - P\{X \le t, Y \le t\}} =$$

$$\frac{P\{X > t\} + P\{Y > t\} - [1 - P\{X \le t, Y \le t\}]}{1 - P\{X \le t, Y \le t\}}$$

$$= \frac{P\{X > t\} + P\{Y > t\}}{1 - P\{X \le t, Y \le t\}} - 1$$

We found **the conditional probability**

$$\frac{P\{X > s, Y > s\}}{1 - P\{X \le s, Y \le s\}} = \frac{P\{X > s\} + P\{Y > s\}}{1 - P\{X \le s, Y \le s\}} - 1$$

Note that the **expected** number of market crashes given that one market crashes is

$$1\frac{P\{X > s, Y \le s\} + P\{X \le s, Y > s\}}{1 - P\{X \le s, Y \le s\}} +$$

$$2\frac{P\{X > s, Y > s\}}{1 - P\{X \le s, Y \le s\}}$$

$$= \frac{P\{X > s\} + P\{Y > s\}}{1 - P\{X \le s, Y \le s\}}$$

Thus the **expected** number κ of market crashes given that one market crashes $E[\kappa \mid \kappa \ge 1]$, is

$$E[\kappa \mid \kappa \ge 1] = \frac{P\{X > s\} + P\{Y > s\}}{1 - P\{X \le s, Y \le s\}}$$

Use this **conditional expected number of market crashes** as measure of systemic risk

Carries over to higher dimensions.

Advantages of proposed systemic risk measure

- has simple connection to univariate VaR (Value at Risk) measure
- directly focuses on systemic risk area (no center bias)
- provides trade-off between risk and loss level
- can be easily extended to n-dimensions

Asymptotic Dependence and Independence

Independence

$$P\{X > t, Y > t\} = P\{X > t\}P\{Y > t\}$$

$$\lim_{s \to \infty} E[\kappa \mid \kappa \geq 1] = 1$$

Asymptotic Dependence (Systemic Risk)

$$P\{X > t, Y > t\} = O(P\{X > t\} + P\{Y > t\})$$

$$\lim_{s \to \infty} E[\kappa \mid \kappa \geq 1] > 1$$

Asymptotic Independence

$$P\{X > t, Y > t\} = o(P\{X > t\} + P\{Y > t\})$$

&

$$P\{X > t\}P\{Y > t\} = o(P\{X > t, Y > t\})$$

$$\lim_{s \to \infty} E[\kappa \mid \kappa \geq 1] = 1$$

Full dependence (Systemic Risk)

$$\lim_{s \to \infty} E[\kappa \mid \kappa \geq 1] = 2$$

distribution is singular

Fragility Index

For correlated normals e.g.

$$\lim_{s\to\infty} E[\kappa \mid \kappa \geq 1] = 1$$

In case of asymptotic independence apply the finer scale of Ledford and Tawn. Assume the domain of attraction condition

$$\lim_{t\to\infty} t\{1 - F(tx, ty)\} = -\log G(x,y)$$

with the ‘second order’ condition:

$$\lim_{t\to\infty} \frac{t(1 - F(tx,ty)) + \log G(x,y)}{A(t)} \to H(x,y)$$

for $0 < x,y \leq \infty$. One shows A is regularly varying with index $\rho \leq 0$, $P\left\{X > tx \; or \; Y > ty\right\}$ is asymptotically of order t^{-1} (has index -1), whereas $P\left\{X > tx \; and \; Y > ty\right\}$ is asymptotically of order $t^{-1}A(t)$ (index $\rho - 1$), so that $1/(1-\rho) \in [0,1]$.

Combine the two scales into the **Fragility Index** FI:

$$FI = \begin{cases} \lim_{s\to\infty} E\{\kappa_s | \kappa_s \geq 1\} \\ \frac{1}{2} \lim_{s\to\infty} \frac{\log P\{X>s\}+\log P\{Y>s\}}{\log P\{X>s, Y>s\}} \end{cases}$$

Discrete and Continuous Compounding

$Q(t)$ - price of bank stock at time t

Discrete Return

$$Y(t) = \frac{S(t)}{S(t-1)} - 1$$

Continuous Return

$$X(t) = \ln \frac{S(t)}{S(t-1)}$$

Portfolios are Affine

Syndicate Loan Case

Consider two investment banks who participate in two independent syndicated loans with returns X and Y. Let bank one hold the portfolio

$$Q = (1-\gamma)X + \gamma Y,$$

while the loan portfolio of bank two is

$$W = \gamma X + (1-\gamma)Y.$$

The correlation between the portfolios is

$$\rho = \frac{2\gamma(1-\gamma)}{\gamma^2 + (1-\gamma)^2} > 0$$

for $\gamma \epsilon [1/2, 1)$.

Main Results

For portfolios $W = \gamma X + (1-\gamma)Y$ and $Q = (1-\gamma)X + \gamma Y$:

1. If X, Y are Normal, then W, Q are correlated but asymptotically **in**dependent ⇒ **No** systemic risk
2. If X, Y are Uniformly distributed, then W, Q are correlated and asymptotically dependent ⇒ **Systemic** risk
3. If X, Y are Pareto distributed, then W, Q are correlated and asymptotically dependent ⇒ **Systemic** risk

We investigate the systemic risk features under different tail assumptions.

Continuous Compounding

Returns with Light Tails

Suppose that the two project returns X and Y are i.i.d. standard normally distributed. It is immediate that the portfolios Q and W are multivariate normally distributed. Let κ be the # of bank stocks.

Proposition. If X and Y follow independent standard normal distributions and $\gamma \epsilon (1/2, 1)$, then $\lim_{s \to \infty} E\{\kappa | \kappa \geq 1\} = 1$, so that the fragility is weak.

Proof Sibuya (1960).

Returns with Heavy Tails

In paper subexponential, here regular variation

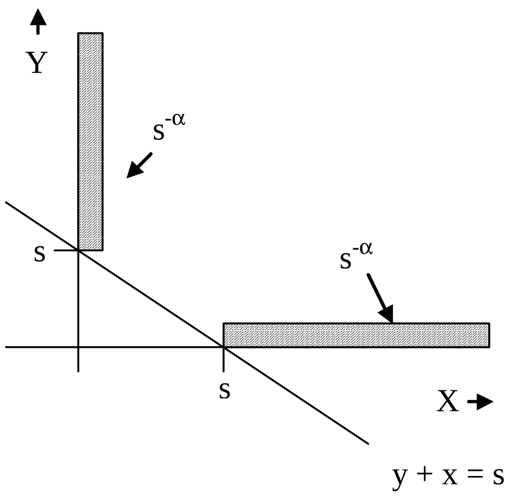
We first need the Feller convolution result: If

$$P\{X > s\} = P\{Y > s\} = s^{-\alpha}L(s),$$

as $s \to \infty$, then

$$P\{X + Y > s\} \approx 2P\{X > s\}.$$

Intuition: For a large threshold s the probability mass on any area far away from the origin is determined by where such an area cuts the axes, and the marginal probability mass that is loaded along these axes above such points.



Convolution

Therefore, for large s

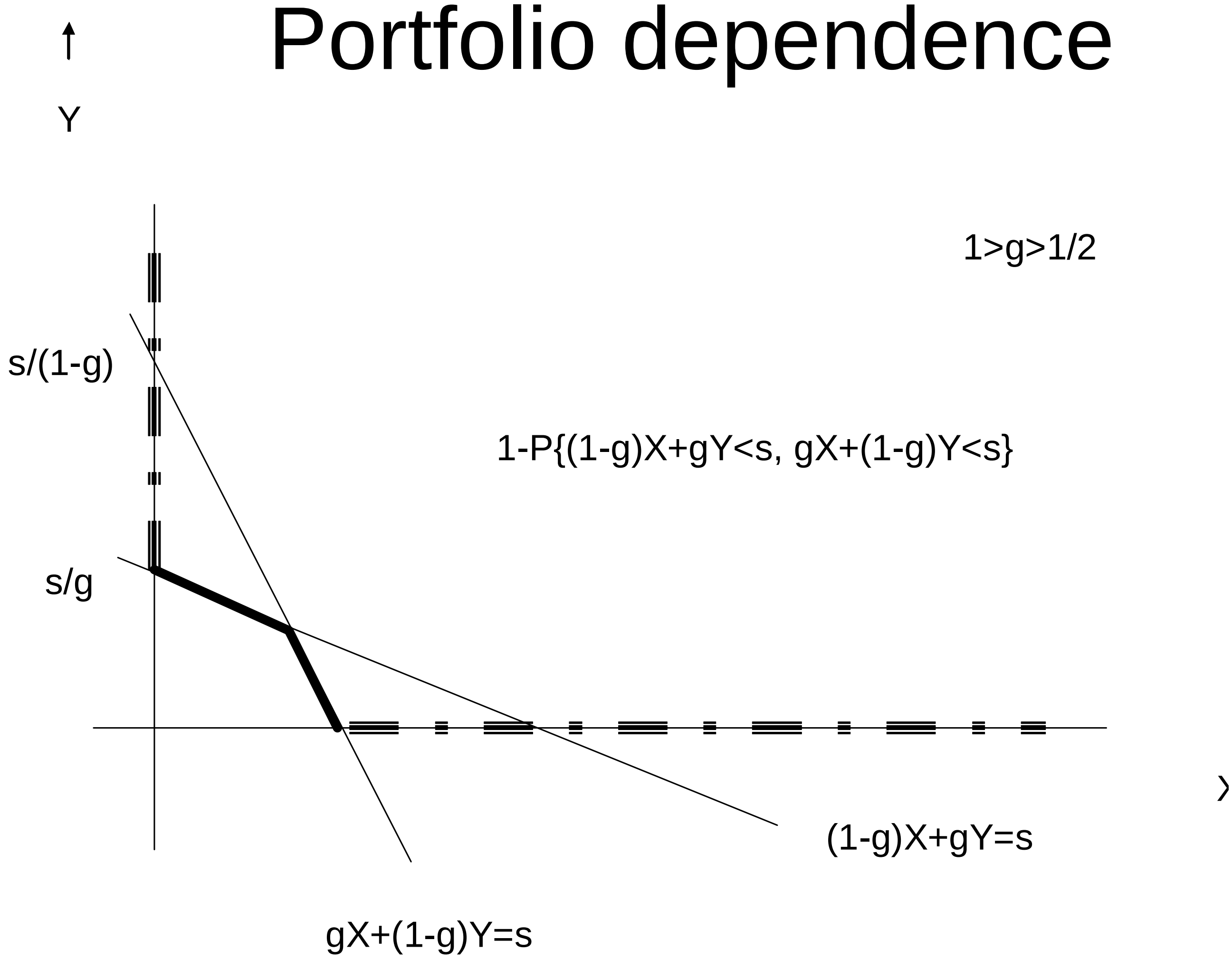
$$P\{X \le s, Y \le s\} = P\{X + Y \le s\} = 1 - 2s^{-\alpha}$$

The eventual concentration of probability mass along the axes implies:

Corollary. Let X and Y be i.i.d. random variables with fat tails $s^{-\alpha}L(s)$ as $s \to \infty$. Let $\gamma \in [1/2, 1]$. Then for the joint probability as $s \to \infty$

$$P\{(1-\gamma)X + \gamma Y \leq s, \gamma X + (1-\gamma)Y \leq s\}$$
$$= 1 - 2\gamma^{\alpha} s^{-\alpha} L(s).$$

Portfolio dependence



Proposition: **Systemic Risk**. Let X and Y be i.i.d. random variables with regularly varying tails $s^{-\alpha}L(s)$ as $s \to \infty$. Then for $\gamma \in [1/2, 1]$

$$\lim_{s\to\infty} E\{\kappa|\kappa \geq 1\} = 1 + (\frac{1}{\gamma} - 1)^{\alpha}.$$

Proof. By definition

$$E\{\kappa|\kappa \geq 1\} =$$

$$\frac{P\{(1-\gamma)X + \gamma Y > s\} + P\{\gamma X + (1-\gamma)Y > s\}}{1 - P\{(1-\gamma)X + \gamma Y \leq s, \gamma X + (1-\gamma)Y \leq s\}}$$

Use the Corollary for the denominator. For the numerator adapt Feller's convolution theorem

$$P\{(1-\gamma)X + \gamma Y > s\} = P\{\gamma X + (1-\gamma)Y > s\}$$

$$\simeq [\gamma^{\alpha} + (1-\gamma)^{\alpha}]s^{-\alpha}L(s).$$

Thus

$$\lim_{s\to\infty} E\{\kappa|\kappa \geq 1\} =$$

$$\lim_{s\to\infty} \frac{2[\gamma^{\alpha} + (1-\gamma)^{\alpha}]s^{-\alpha}L(s)}{2\gamma^{\alpha}s^{-\alpha}L(s)} = 1 + (\frac{1}{\gamma} - 1)^{\alpha}.$$

QED.

Economics

Note that since $dE\{\kappa|\kappa \geq 1\}/d\gamma < 0$, the asymptotic dependency is increasing in the amount of diversification. Thus **systemic risk increases due to diversification**. Financial conglomorates may be a bad idea.

The asymptotic dependence is declining as the tail thickness declines, since $dE\{\kappa|\kappa \geq 1\}/d\alpha < 0$. This corroborates the results for the normal and exponential distributions.

Discrete Returns

Lemma *Suppose $H(x)$ is in the domain of attraction of the Weibull extreme value distribution, i.e.*

$$\lim_{t \downarrow 0} H(tx)/H(t) = x^{\alpha}, \alpha > 0$$

Then the two convolution H^{2*} of $H(x)$ is again in the domain of attraction of the Weibull extreme value distribution and satisfies

$$\lim_{t \downarrow 0} \frac{H^{2*}(tx)}{H^{2*}(t)} = x^{2\alpha}.$$

Thus the class is closed under addition, but the index of regular variation changes.

Let X and Y be uniformly distributed on $[0,1]$, then

$$P\{Q > t\} = P\{W > t\} = \frac{1}{2}\frac{1}{\gamma(1-\gamma)}(1-t)^2$$

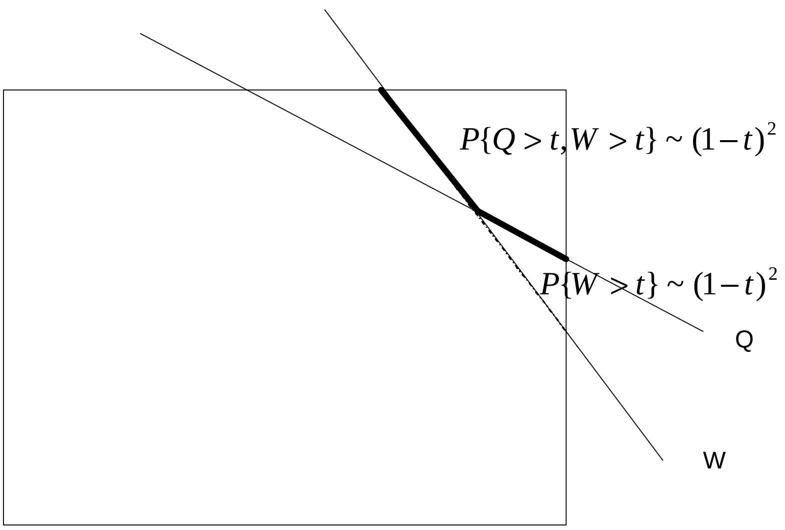
and

$$P\{Q > t, W > t\} = \frac{1}{\gamma}(1-t)^2.$$

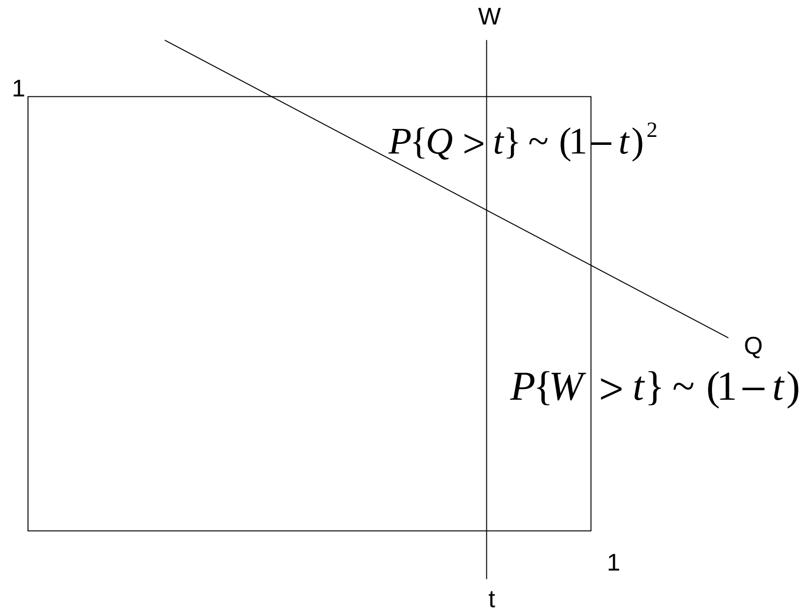
Thus

$$\lim_{t \uparrow 1} E\{\kappa | \kappa \geq 1, s\} = FI = \frac{1}{\gamma} > 1$$

as $\gamma \epsilon (1/2, 1)$. The two portfolios are asymptotically dependent and there is systemic risk.



Alternatively, s'pose the portfolio W is $W = X$. Thus, $P\{W > t\} = 1 - t$, while $P\{Q > t\}$ remains ~$(1 - t)^2$. Then asymptotic independence (the *FI*=3/4) and no systemic risk.



If portfolios contain an equal number of assets, the portfolios have the same index of regular variation, while if two portfolios differ with respect to the number of assets, their indices differ.This determines whether or not the portfolios are asymptotically dependent or independent.

Economics

Financial Copula

If asset returns X, Y vary regularly at infinity

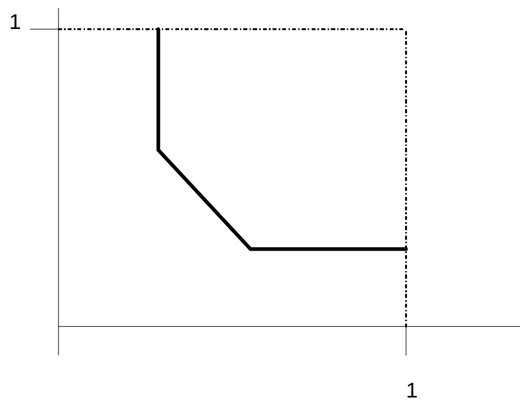
Suppose $\gamma \in (1/2, 1)$. For the joint portfolios distribution $P\{Q \leq s, W \leq t\}$, one has as $s, t \to \infty$

$$P\{Q \leq s, W \leq t\} =$$

$$\begin{cases} 1 - [\gamma^{\alpha} + (1-\gamma)^{\alpha}]t^{-\alpha}L(t) & \text{as } \frac{s}{t} > \frac{\gamma}{1-\gamma} \\ 1 - \gamma^{\alpha}[s^{-\alpha}L(s) + t^{-\alpha}L(t)] & \text{as } \frac{1-\gamma}{\gamma} \leq \frac{s}{t} \leq \frac{\gamma}{1-\gamma} \\ 1 - [\gamma^{\alpha} + (1-\gamma)^{\alpha}]s^{-\alpha}L(s) & \text{as } \frac{s}{t} < \frac{1-\gamma}{\gamma} \end{cases}$$

Implied **Limit Tail Dependence Copula** contour plot (level set)

Limit Tail Dependence Copula



This copula is derived from **economic** principles, rather than chosen for analytical or estimation convenience

Options

Case I. At the money call held until expiration

expiration time T

$S(t)$ time t share price,

Gross Return $S(T)/S(t)$

$C(t)$ time t call premium, Gross Return

$$\max[0, \left(\frac{S(T)}{S(t)} - 1\right)\frac{S(t)}{C(t)}]$$

option has positive probability to end out of the money, so point mass at zero

$\therefore$ asymptotic independence

Case II. Two at the money calls on two different *Independent* stocks

calls have positive probability to end out of the money, so have point mass at zero

$\therefore$ calls are asymptotically dependent, even though the underlying stocks are not

Sequence of Netwotionsrks

Four projects: $4U$, $4X$, $4Y$, and $4T$.

Four distinct banks: B_1, B_2, B_3, and B_4.

Case 1. Each bank finances one project

$$B_1 = 4U,\ B_2 = 4X,\ B_3 = 4Y,\ B_4 = 4T,$$

Case 2. Each bank participates in two projects

$$B_1 = 2U + 2X, B_2 = 2X + 2Y,$$

$$B_3 = 2Y + 2T,\ B_4 = 2T + 2U.$$

Case 3. Further asymmetric diversification

$$B_1 = 2U + X + Y, B_2 = 2X + Y + T,$$

$$B_3 = 2Y + T + U,\ B_4 = 2T + U + X.$$

Case 4. Bank portfolios are fully diversified:

$$B_i = U + X + Y + T, \text{ for } i = 1, \ldots, 4.$$

normal

Suppose the project returns U, X, Y, and T are standard normally distributed. In that case the correlation matrix C is a natural representation of the network dependencies. As a summary measure D for the dependencies one could use

$$\mathrm{D} = \mathrm{trace}\, CC^T.$$

We show this measure reflects the increases in network connectedness as we move from one case to the other.

Compare the sequence of networks by their ranking of systemic dependencies. Rankings for Normal based on $\mathrm{D} = \mathrm{trace}\, CC^T$ and fat tail using $E\{\kappa|\kappa \geq 1, s\}$ differ! (if $\alpha > 1$).

B_1 portfolio	Normal D	Fat Tail $E\{.|.\}$
$4U$	4	1
$2U + 2X$	6	2
$2U + X + Y$	$7\frac{7}{9} > 6$	$1 + \frac{1}{2^{\alpha-1}} < 2$
$U + X + Y + T$	16	4

Systemic Risk Recap

explanation uses two ingredients of bank stock return behavior

- linearity regarding exposures
- marginal fat tails (continuous returns) or discrete returns

Linearity

- syndicated loans
- interbank exposures
- portfolio theory (exposure to same macro factors)

estimation

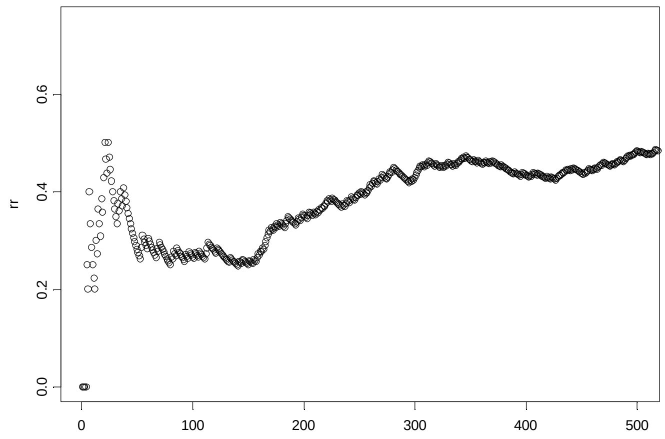
Suppose one wants to estimate

$$E\{\kappa|\kappa \geq 1\}.$$

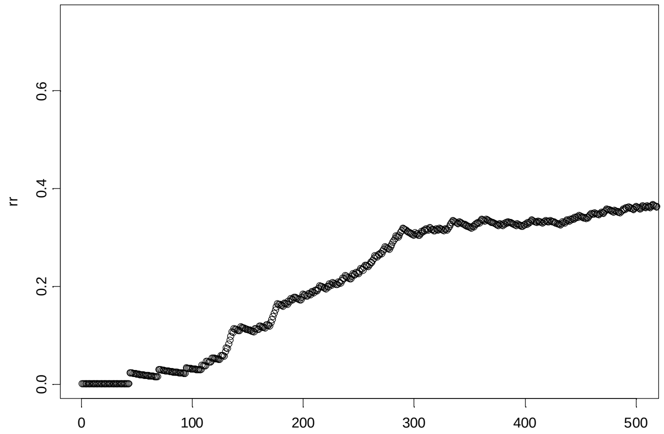
A simple approach is

$$\begin{aligned}
&E\{\kappa|\kappa \geq 1\} \\
&\quad = \frac{P\{Q > s\} + P\{W > s\}}{1 - P\{Q \leq s, W \leq s\}} \\
&= \frac{P\{\max[Q,W] > s\} + P\{Q > s, W > s\}}{P\{\max[Q,W] > s\}} \\
&\quad = 1 + \frac{P\{\min[Q,W] > s\}}{P\{\max[Q,W] > s\}} \\
&\quad \approx 1 + \frac{\#\min[Q,W] > s}{\#\max[Q,W] > s}.
\end{aligned}$$

Estimates of $\frac{\#\min[Q,W]>s}{\#\max[Q,W]>s}$ using 5000 observations on X and Y

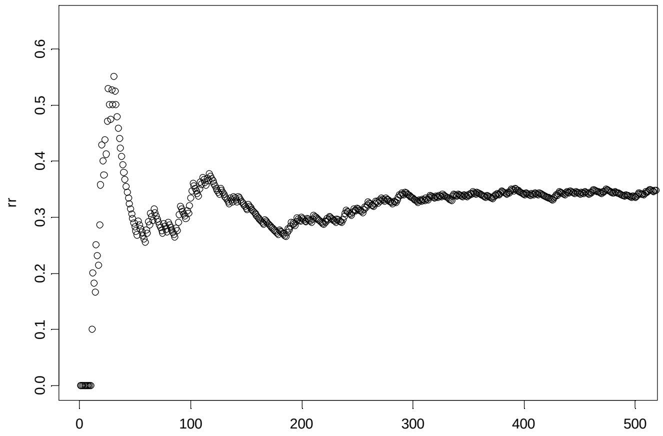


Correlated Student-t



Correlated Normal

Using the daily returns from the ABNAMRO and ING bank, we obtain a plot which resembles the simulated Student-t portfolios



First 500 bank data

Empirical Bank Risk

Bank	$\lvert \text{loss} \rvert$ (%)	$\hat{\alpha}$	$\lvert \widehat{\text{loss}(p)} \rvert$	
	$X_{1,n}$ (date)		p$=.05$	p$=.02$
DEUTSCHE	12.4 (01)	3.3	13.8	18.2
HYPO	17.3 (02)	3.1	17.9	24.0
DRESDNER	11.1 (97)	3.2	16.1	21.5
COMMERZ	13.3 (01)	2.9	15.9	21.9
BGBERLIN	37.9 (01)	2.4	23.4	34.2
DEPFA	16.5 (00)	3.2	13.4	17.6
BNPPAR	12.5 (98)	3.0	15.4	20.8
CA	19.6 (01)	2.4	13.3	19.4
SGENERAL	12.5 (98)	2.7	17.1	23.6
NATEXIS	13.6 (97)	3.6	9.6	12.3
BANK INDEX	6.9 (01)	2.5	11.2	16.1
STOCK INDEX	6.3 (01)	3.2	7.7	10.2

Bank	$\lvert \text{loss} \rvert$ in %	$\hat{\alpha}$	$\lvert \widehat{\text{loss}(p)} \rvert$	
	$X_{1,n}$ (date)		p$=.05$	p$=.02$
CITIG	17.1 (02)	3.3	13.7	18.0
JP MORGAN	20.0 (02)	3.7	12.9	16.6
BAMERICA	11.6 (98)	3.6	12.0	15.5
WACHOVIA	9.2 (00)	3.5	10.9	14.1
FARGO	9.2 (00)	3.7	9.6	12.3
BONE	25.8 (99)	3.0	13.5	18.4
WASHING	11.7 (01)	3.5	12.7	16.5
FLEET	11.2 (02)	3.7	11.7	15.0
BNYORK	16.9 (02)	3.4	12.6	16.5
SSTREET	19.7 (93)	3.0	14.8	20.0
BANK INDEX	7.0 (00)	3.4	9.1	12.0
STOCK INDEX	7.0 (98)	3.7	6.3	8.0

Largest bank	$\widehat{P}_1$	$\widehat{P}_2$	$\widehat{P}_3$	$\widehat{P}_4$
	Conditioning banks: German			
Germany	0.224	0.651	0.743	0.727
Netherlands	**0.265**	**0.541**	**0.701**	**0.430**
France	0.082	0.252	0.358	0.310
Spain	0.112	0.174	0.242	0.441
Italy	**0.075**	**0.136**	**0.129**	**0.075**
Belgium	0.161	0.442	0.426	0.285
Ireland	0.040	0.055	0.054	0.247
Portugal	0.077	0.136	0.217	0.251
Finland	0.009	0.017	0.023	0.040
Greece	0.009	0.014	0.013	0.013
	Conditioning banks: French			
France	0.029	0.359	0.766	
Germany	0.031	0.239	0.695	
Netherlands	0.082	0.487	0.718	
Italy	0.015	0.075	0.131	
Spain	0.033	0.274	0.701	
Belgium	0.067	0.380	0.563	

Note: The table reports conditional co-crash probabilities $\widehat{P}_i$ for the largest bank stock in each country conditional upon a set of banks from either the same country or other countries. The number of conditioning banks varies from 1 to 5 for Germany (upper panel) and 1 to 3 for France. For example, the $\widehat{P}_2$ column stands for the crash probability of the largest bank in each country, conditional on a crash in the 2nd and 3rd largest bank in Germany (top panel), France (second panel).

Country/Area		Estimates
		$\widehat{P}$
United States	(N=25)	2.77E-6
Euro area	(N=25)	6.73E-17
Germany	(N=6)	1.54E-5
France	(N=4)	1.41E-4
Italy	(N=4)	5.50E-3

Note: The table reports $\widehat{P}$ which gives the probabililty that all banks of a specific country/area crash given that one of them crashes. The univariate crash probability is chosen to be p=0.05.

Bank	bank index	stock index
DEUTSCHE	.511	.350
HYPO	.223	.208
DRESDNER	.379	.277
COMMERZ	.395	.308
BGBERLIN	.028	.016
DEPFA	.062	.073
BNPPAR	.421	.302
CA	.093	.067
SGENERAL	.458	.300
NATEXIS	.018	.019
average	.254	.194
st. dev.	.188	.145

Table gives estimates for the conditional probability of loss of bank stock given that the index has moved down (at p-level=0.05).

Bank	bank index	stock index
CITIG	.411	.265
JP MORGAN	.394	.180
BAMERICA	.377	.124
WACHOVIA	.272	.096
FARGO	.171	.071
BONE	.310	.140
WASHING	.095	.028
FLEET	.388	.131
BNYORK	.252	.129
SSTREET	.268	.190
average	.262	.113
st. dev.	.085	.044

Table gives estimates for the conditional probability of loss of bank stock given that the index has moved down (at p-level=0.05).

Conclusion

- syndicated loans, interbank deposit market and portfolio theory all imply that the dependeny between bank returns stems from different linear combinations of bank exposures to the same risk factors
- If bank returns are normally distributed, there does not exist systemic risk
- Under continuous compounding and if returns are heavy tailed distributed, or under discrete compounding, the dependency does not go away as the loss levels are increased
- Financially relevant tail copula were derived
- Bank structure in the sense of systemic risk in each EU country is similar to US
- Due to the segmentation in the EU banking industry along national border lines, the EMU systemic banking risk is still lower
- It remains an open question whether it pays or rather by how much to regulate a null event