Space time analysis of extreme values [a]

Gabriel Huerta

Department of Mathematics and Statistics
University of New Mexico
Albuquerque, NM, 87131, U.S.A.
http://www.stat.unm.edu/∼ghuerta

4th EVA conference, Gothenburg, August 15-19 2005

[a] Joint paper with Bruno Sanso, University of California, Santa Cruz, U.S.A

Summary Points

Summary Points

- Models for non-stationary extreme values.

Summary Points

- Models for non-stationary extreme values.
- Space-time formulation for the GEV distribution.

Summary Points

- Models for non-stationary extreme values.
- Space-time formulation for the GEV distribution.
- Dynamic Linear Model (DLM) framework for temporal components.

Summary Points

- Models for non-stationary extreme values.
- Space-time formulation for the GEV distribution.
- Dynamic Linear Model (DLM) framework for temporal components.
- Spatial elements through process convolutions.

Summary Points

- Models for non-stationary extreme values.
- Space-time formulation for the GEV distribution.
- Dynamic Linear Model (DLM) framework for temporal components.
- Spatial elements through process convolutions.
- Model fitting via customized Markov chain Monte Carlo (MCMC) methods.

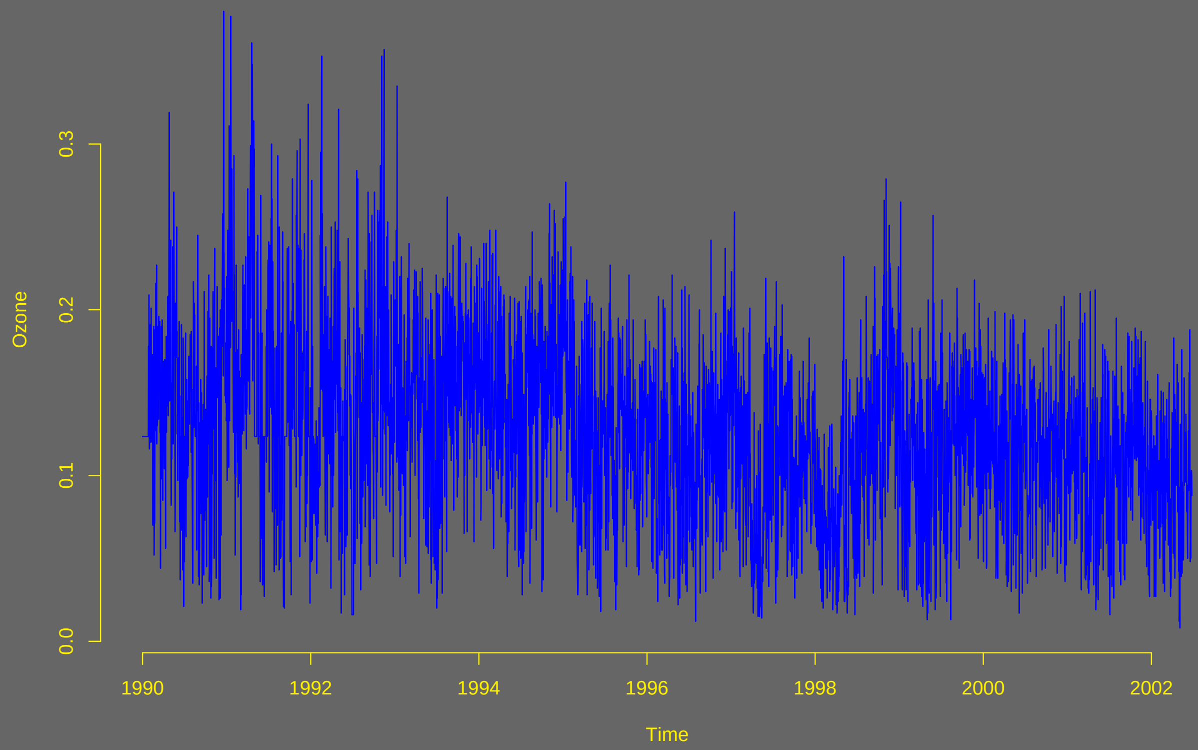
Summary Points

- Models for non-stationary extreme values.
- Space-time formulation for the GEV distribution.
- Dynamic Linear Model (DLM) framework for temporal components.
- Spatial elements through process convolutions.
- Model fitting via customized Markov chain Monte Carlo (MCMC) methods.
- Extreme values of ozone levels in Mexico City.

Summary Points

- Models for non-stationary extreme values.
- Space-time formulation for the GEV distribution.
- Dynamic Linear Model (DLM) framework for temporal components.
- Spatial elements through process convolutions.
- Model fitting via customized Markov chain Monte Carlo (MCMC) methods.
- Extreme values of ozone levels in Mexico City.
- Extreme values of rainfall in Venezuela.

Figure 1: Daily maximum values of ozone levels.



Extreme Value Modeling

Extreme Value Modeling

- The traditional approach is based on the *Generalized Extreme Value* (GEV) distribution function:

$$H(z) = \exp\left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]_{+}^{-1/\xi} \right\}$$

Extreme Value Modeling

- The traditional approach is based on the *Generalized Extreme Value* (GEV) distribution function:

$$H(z) = \exp\left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]_{+}^{-1/\xi} \right\}$$

  - $-\infty < \mu < \infty$; $\sigma > 0$; $-\infty < \xi < \infty$.

Extreme Value Modeling

- The traditional approach is based on the *Generalized Extreme Value* (GEV) distribution function:

$$H(z) = \exp\left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]_{+}^{-1/\xi} \right\}$$

  - $-\infty < \mu < \infty$; $\sigma > 0$; $-\infty < \xi < \infty$.
  - $+$ denotes the positive part of the argument.

Extreme Value Modeling

- The traditional approach is based on the *Generalized Extreme Value* (GEV) distribution function:

$$H(z) = \exp\left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]_+^{-1/\xi} \right\}$$

  - $-\infty < \mu < \infty$; $\sigma > 0$; $-\infty < \xi < \infty$.
  - $+$ denotes the positive part of the argument.
  - $\xi > 0$ *Fréchet* family; $\xi < 0$ the *Weibull* family; $\xi \to 0$ *Gumbel* family.

- The book by Coles (2001) presents a very clear account of statistical inference using the GEV.

- The book by Coles (2001) presents a very clear account of statistical inference using the GEV.
- A Bayesian analysis can be performed by imposing a prior on (μ, σ, ξ) as in Coles and Tawn (1996).

- The book by Coles (2001) presents a very clear account of statistical inference using the GEV.
- A Bayesian analysis can be performed by imposing a prior on (μ, σ, ξ) as in Coles and Tawn (1996).
- Alternatively, models for exceedances over a high treshold had been proposed.

- The book by Coles (2001) presents a very clear account of statistical inference using the GEV.
- A Bayesian analysis can be performed by imposing a prior on (μ, σ, ξ) as in Coles and Tawn (1996).
- Alternatively, models for exceedances over a high treshold had been proposed.
- This leads into the *Generalized Pareto Distributions* and *Point processes* approaches. (Pickands 1971 and 1975).

- The book by Coles (2001) presents a very clear account of statistical inference using the GEV.
- A Bayesian analysis can be performed by imposing a prior on (μ, σ, ξ) as in Coles and Tawn (1996).
- Alternatively, models for exceedances over a high treshold had been proposed.
- This leads into the *Generalized Pareto Distributions* and *Point processes* approaches. (Pickands 1971 and 1975).
- These ideas had been developed into a Bayesian hierarchical modeling framework. Smith et al. (1997); Assuncao et al. (2004); Casson and Coles (1999); Gilleland et. al. (2004).

Extremes for Non-Stationary Data

Extremes for Non-Stationary Data

- Coles (2001) mentions several possibilities.

Extremes for Non-Stationary Data

- Coles (2001) mentions several possibilities.
- Approach: $z_1, z_2, \ldots, z_m$; $z_t \sim GEV(\mu_t, \sigma, \xi)$.

Extremes for Non-Stationary Data

- Coles (2001) mentions several possibilities.
- Approach: $z_1, z_2, \ldots, z_m$; $z_t \sim GEV(\mu_t, \sigma, \xi)$.
- Deterministic functions: $\mu_t = \beta_0 + \beta_1 t$; $\mu_t = \beta_0 + \beta_1 + \beta_2 t + \beta_3 t^2$ or $\mu_t = \beta_0 + \beta_1 X_t$.

Extremes for Non-Stationary Data

- Coles (2001) mentions several possibilities.
- Approach: $z_1, z_2, \ldots, z_m$; $z_t \sim GEV(\mu_t, \sigma, \xi)$.
- Deterministic functions: $\mu_t = \beta_0 + \beta_1 t$; $\mu_t = \beta_0 + \beta_1 + \beta_2 t + \beta_3 t^2$ or $\mu_t = \beta_0 + \beta_1 X_t$.
- Non-stationarity can also be included for the shape and/or scale parameters: $\sigma_t = exp(\beta_0 + \beta_1 t)$; $\xi_t = \beta_0 + \beta_1 t$ or $\xi_t = \beta_0 + \beta_1 t + \beta_2 t^2$.

Extremes for Non-Stationary Data

- Coles (2001) mentions several possibilities.
- Approach: $z_1, z_2, \ldots, z_m$; $z_t \sim GEV(\mu_t, \sigma, \xi)$.
- Deterministic functions: $\mu_t = \beta_0 + \beta_1 t$; $\mu_t = \beta_0 + \beta_1 + \beta_2 t + \beta_3 t^2$ or $\mu_t = \beta_0 + \beta_1 X_t$.
- Non-stationarity can also be included for the shape and/or scale parameters: $\sigma_t = exp(\beta_0 + \beta_1 t)$; $\xi_t = \beta_0 + \beta_1 t$ or $\xi_t = \beta_0 + \beta_1 t + \beta_2 t^2$.
- We propose the use of Dynamic Linear Models (DLM) as in West and Harrison (1997) to model the parameter changes in time.

GEV distribution with DLM's

GEV distribution with DLM's

- For $z_1, z_2, \ldots, z_m,\ z_t \sim GEV(\mu_t, \sigma, \xi)$

$$
\begin{aligned}
H_t(z_t) &= \exp\left\{-[1+\xi(z_t-\mu_t)/\sigma]_+^{-1/\xi}\right\} \\
\mu_t &= \theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\
\theta_t &= \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V)
\end{aligned}
$$

GEV distribution with DLM's

- For $z_1, z_2, \ldots, z_m,\ z_t \sim GEV(\mu_t, \sigma, \xi)$

$$\begin{aligned} H_t(z_t) &= \exp\left\{-[1+\xi(z_t-\mu_t)/\sigma]_+^{-1/\xi}\right\} \\ \mu_t &= \theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\ \theta_t &= \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V) \end{aligned}$$

- Parameters $(t = 0)$ are assumed apriori independent.

GEV distribution with DLM's

- For $z_1, z_2, \ldots, z_m,\ z_t \sim GEV(\mu_t, \sigma, \xi)$

$$\begin{aligned} H_t(z_t) &= \exp\left\{-[1+\xi(z_t-\mu_t)/\sigma]_+^{-1/\xi}\right\} \\ \mu_t &= \theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\ \theta_t &= \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V) \end{aligned}$$

- Parameters $(t = 0)$ are assumed apriori independent.

- $\pi(\sigma) \sim LN(m_\sigma, s_\sigma)$; $\pi(\xi) \sim N(m_\xi, s_\xi)$.

- $\theta_0 \sim N(m_0, C_0)$; $V \sim IG(\alpha_v, \beta_v)$; $\tau \sim IG(\alpha_\tau, \beta_\tau)$

GEV distribution with DLM's

- For $z_1, z_2, \ldots, z_m,\ z_t \sim GEV(\mu_t, \sigma, \xi)$

$$
\begin{aligned}
H_t(z_t) &= \exp\left\{-[1+\xi(z_t-\mu_t)/\sigma]_+^{-1/\xi}\right\} \\
\mu_t &= \theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\
\theta_t &= \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V)
\end{aligned}
$$

- Parameters $(t = 0)$ are assumed apriori independent.
- $\pi(\sigma) \sim LN(m_\sigma, s_\sigma)$; $\pi(\xi) \sim N(m_\xi, s_\xi)$.
- $\theta_0 \sim N(m_0, C_0)$; $V \sim IG(\alpha_v, \beta_v)$; $\tau \sim IG(\alpha_\tau, \beta_\tau)$.
- μ_t follows a *first order polynomial* DLM with state vector θ_t.

General DLM (F_t, V, G_t, W)

General DLM (F_t, V, G_t, W)

$$
\begin{aligned}
\mu_t &= F_t'\theta_t + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\
\theta_t &= G_t\theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, W)
\end{aligned}
$$

General DLM (F_t, V, G_t, W)

$$\begin{aligned} \mu_t &= F_t^{'}\theta_t + \epsilon_t; \quad \epsilon_t \sim N(0,V) \\ \theta_t &= G_t\theta_{t-1} + \omega_t; \quad \omega_t \sim N(0,W) \end{aligned}$$

- θ_t is a $k \times 1$ state vector;
- F_t is a $k \times 1$ regressor vector;
- G_t is a $k \times k$ evolution matrix;
- V is an observational variance and
- W is a $k \times k$ evolution covariance matrix.

Posterior Inference for DLM-GEV models

Posterior Inference for DLM-GEV models

- Define $Z = (z_1, z_2, \ldots, z_m)$; $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$ and $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$.

Posterior Inference for DLM-GEV models

- Define $Z = (z_1, z_2, \ldots, z_m)$; $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$ and $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$.

- $p(\mu_t | z_t, \sigma, \theta_t, V); t = 1, \ldots, m$ is sampled with a Metropolis-Hastings step.

Posterior Inference for DLM-GEV models

- Define $Z = (z_1, z_2, \ldots, z_m)$; $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$ and $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$.
- $p(\mu_t | z_t, \sigma, \theta_t, V); t = 1, \ldots, m$ is sampled with a Metropolis-Hastings step.
- $p(\sigma | Z, \mu, \xi)$ and $p(\xi | Z, \mu, \sigma)$ are also sampled via M-H.

Posterior Inference for DLM-GEV models

- Define $Z = (z_1, z_2, \ldots, z_m)$; $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$ and $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$.

- $p(\mu_t | z_t, \sigma, \theta_t, V); t = 1, \ldots, m$ is sampled with a Metropolis-Hastings step.

- $p(\sigma | Z, \mu, \xi)$ and $p(\xi | Z, \mu, \sigma)$ are also sampled via M-H.

- V and W are sampled from Inverse Gamma/Wishart distributions.

Posterior Inference for DLM-GEV models

- Define $Z = (z_1, z_2, \ldots, z_m)$; $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$ and $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$.
- $p(\mu_t | z_t, \sigma, \theta_t, V); t = 1, \ldots, m$ is sampled with a Metropolis-Hastings step.
- $p(\sigma | Z, \mu, \xi)$ and $p(\xi | Z, \mu, \sigma)$ are also sampled via M-H.
- V and W are sampled from Inverse Gamma/Wishart distributions.
- For θ_t, we apply *Forward Filtering Backward Simulation* (FFBS) as in Carter and Kohn or Frühwirth-Schnatter (1994).

– Forward in time, we obtain $p(\theta_t | D_t, V, W); t = 1, 2, \ldots, m$

- Forward in time, we obtain $p(\theta_t|D_t, V, W); t = 1, 2, \ldots, m$
- Backwards in time, we sample $p(\theta_m|D_m, V, W)$ and then, recursively we sample from $p(\theta_t|\theta_{t+1}, D_m, V, W); t = m-1, \ldots, 1$

- Forward in time, we obtain $p(\theta_t | D_t, V, W); t = 1, 2, \ldots, m$
- Backwards in time, we sample $p(\theta_m | D_m, V, W)$ and then, recursively we sample from $p(\theta_t | \theta_{t+1}, D_m, V, W); t = m - 1, \ldots, 1$

- A similar approach is discussed by Gaetan and Grigoletto (2004) *Extremes*.

- Forward in time, we obtain $p(\theta_t|D_t, V, W); t = 1, 2, \ldots, m$
- Backwards in time, we sample $p(\theta_m|D_m, V, W)$ and then, recursively we sample from $p(\theta_t|\theta_{t+1}, D_m, V, W); t = m - 1, \ldots, 1$

- A similar approach is discussed by Gaetan and Grigoletto (2004) *Extremes*.
  - Dynamics for scale/shape parameters. Sequential updating with *Particle Filters*.

- Forward in time, we obtain $p(\theta_t|D_t, V, W); t = 1, 2, \ldots, m$
- Backwards in time, we sample $p(\theta_m|D_m, V, W)$ and then, recursively we sample from $p(\theta_t|\theta_{t+1}, D_m, V, W); t = m - 1, \ldots, 1$

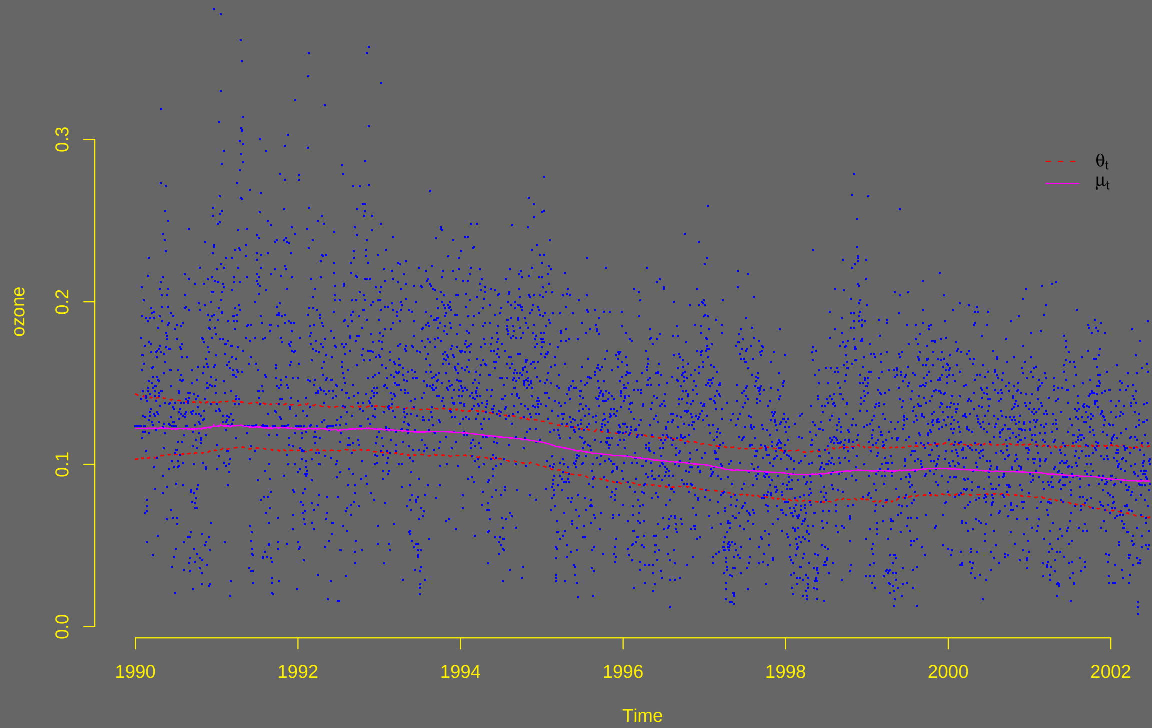
- A similar approach is discussed by Gaetan and Grigoletto (2004) *Extremes*.
  - Dynamics for scale/shape parameters. Sequential updating with *Particle Filters*.
  - Observation variance equals zero.

- Forward in time, we obtain $p(\theta_t|D_t, V, W); t = 1, 2, \ldots, m$
- Backwards in time, we sample $p(\theta_m|D_m, V, W)$ and then, recursively we sample from $p(\theta_t|\theta_{t+1}, D_m, V, W); t = m-1, \ldots, 1$

• A similar approach is discussed by Gaetan and Grigoletto (2004) *Extremes*.

- Dynamics for scale/shape parameters. Sequential updating with *Particle Filters*.
- Observation variance equals zero.
- No space or space-time structure.

Figure 2: Posterior mean for μ_t, and 90% probability interval for θ_t; 1990-2002 data



Detecting Trends

Detecting Trends

- Is there an overall decreasing trend in the maxima of the previous figure?

Detecting Trends

- Is there an overall decreasing trend in the maxima of the previous figure?
- One possibility is to add an extra model parameter and estimate incremental growth.

Detecting Trends

- Is there an overall decreasing trend in the maxima of the previous figure?
- One possibility is to add an extra model parameter and estimate incremental growth.
- Alternatively, we considered a *regression* model on μ_t with time-varying intercept but a constant slope:

Detecting Trends

- Is there an overall decreasing trend in the maxima of the previous figure?

- One possibility is to add an extra model parameter and estimate incremental growth.

- Alternatively, we considered a *regression* model on μ_t with time-varying intercept but a constant slope:

$$
\begin{aligned}
z_t &\sim GEV(\mu_t, \sigma, \xi) \\
\mu_t &= \theta_t + \beta(t - \bar{t}) + \epsilon_t; \quad \epsilon_t \sim N(0, V) \\
\theta_t &= \theta_{t-1} + \omega_t; \quad \omega_t \sim N(0, \tau V)
\end{aligned}
$$

where $\bar{t} = (1/T)(\sum_{t=1}^{T} t)$ and β represents change of level per unit of time.

where $\bar{t} = (1/T)(\sum_{t=1}^{T} t)$ and β represents change of level per unit of time.

- For model fitting, notice that:

where $\bar{t} = (1/T)(\sum_{t=1}^{T} t)$ and β represents change of level per unit of time.

- For model fitting, notice that:
  - Conditional on θ_t, the difference $\mu_t - \theta_t$ follows a regression model.
  - Conditional on β, $\mu_t - \beta(t - \bar{t})$ follows a first order polynomial DLM.

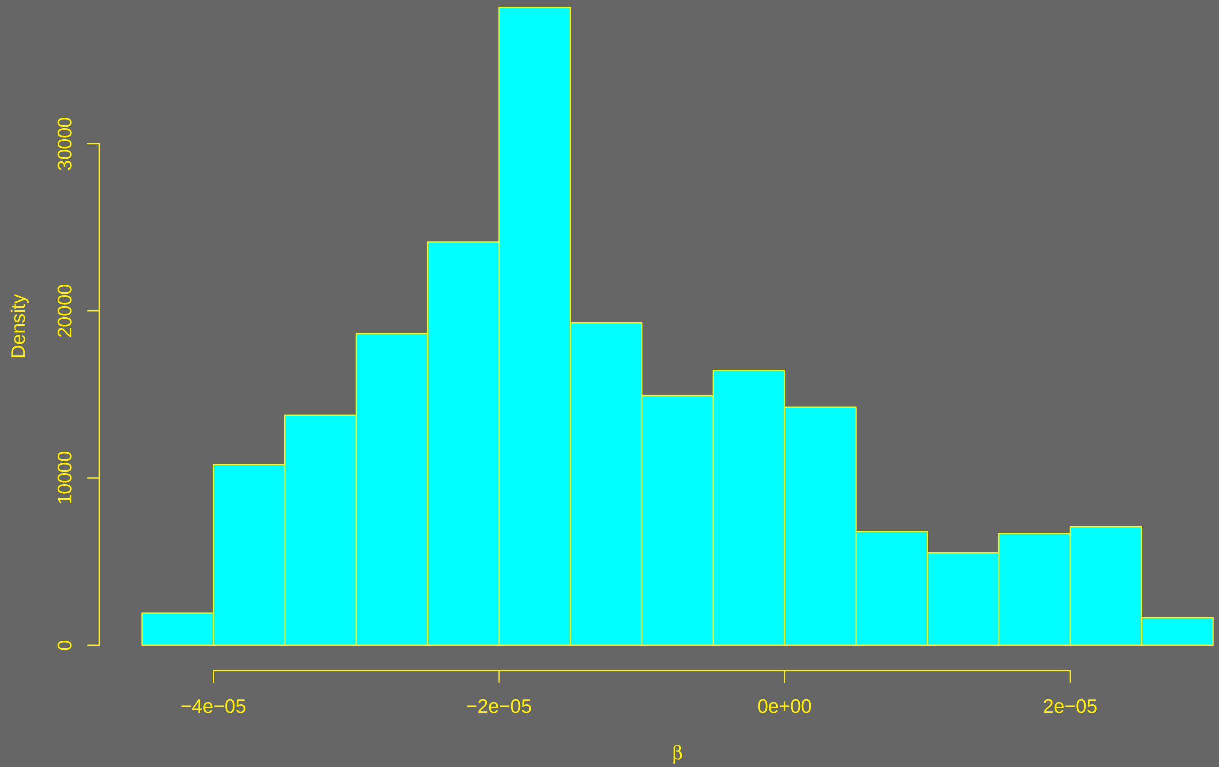
where $\bar{t} = (1/T)(\sum_{t=1}^{T} t)$ and β represents change of level per unit of time.

- For model fitting, notice that:
  - Conditional on θ_t, the difference $\mu_t - \theta_t$ follows a regression model.
  - Conditional on β, $\mu_t - \beta(t - \bar{t})$ follows a first order polynomial DLM.
  - This defines a *Gibbs sampler* scheme that produces posterior samples for β and θ_t.

where $\bar{t} = (1/T)(\sum_{t=1}^{T} t)$ and β represents change of level per unit of time.

- For model fitting, notice that:
  - Conditional on θ_t, the difference $\mu_t - \theta_t$ follows a regression model.
  - Conditional on β, $\mu_t - \beta(t - \bar{t})$ follows a first order polynomial DLM.
  - This defines a *Gibbs sampler* scheme that produces posterior samples for β and θ_t.
- In fact, $Pr(\beta < 0|Z) \approx 0.79$ indication of a decreasing trend.

Figure 3: Posterior distribution for β; ozone data 1990-2002.



Maxima monthly rainfall values in Venezuela

Maxima monthly rainfall values in Venezuela

- Measurements were taken at the Maiquetía station located at the Simon Bolivar Airport near Caracas (Y_t).

Maxima monthly rainfall values in Venezuela

- Measurements were taken at the Maiquetía station located at the Simon Bolivar Airport near Caracas (Y_t).

- The monthly North Atlantic Oscillation (NAO) index is considered as a covariate (X_t).

Maxima monthly rainfall values in Venezuela

- Measurements were taken at the Maiquetía station located at the Simon Bolivar Airport near Caracas (Y_t).

- The monthly North Atlantic Oscillation (NAO) index is considered as a covariate (X_t).

- The model is:

$$
\begin{aligned}
Y_t &\sim GEV(\mu_t, \sigma, \xi) \\
\mu_t &= \theta_t + \beta_t X_t + \epsilon_t \\
\theta_t &= \theta_{t-1} + \omega_{1t} \\
\beta_t &= \beta_{t-1} + \omega_{2t}
\end{aligned}
$$

Figure 4: Maxima monthly rainfall values in Venezuela and NAO index

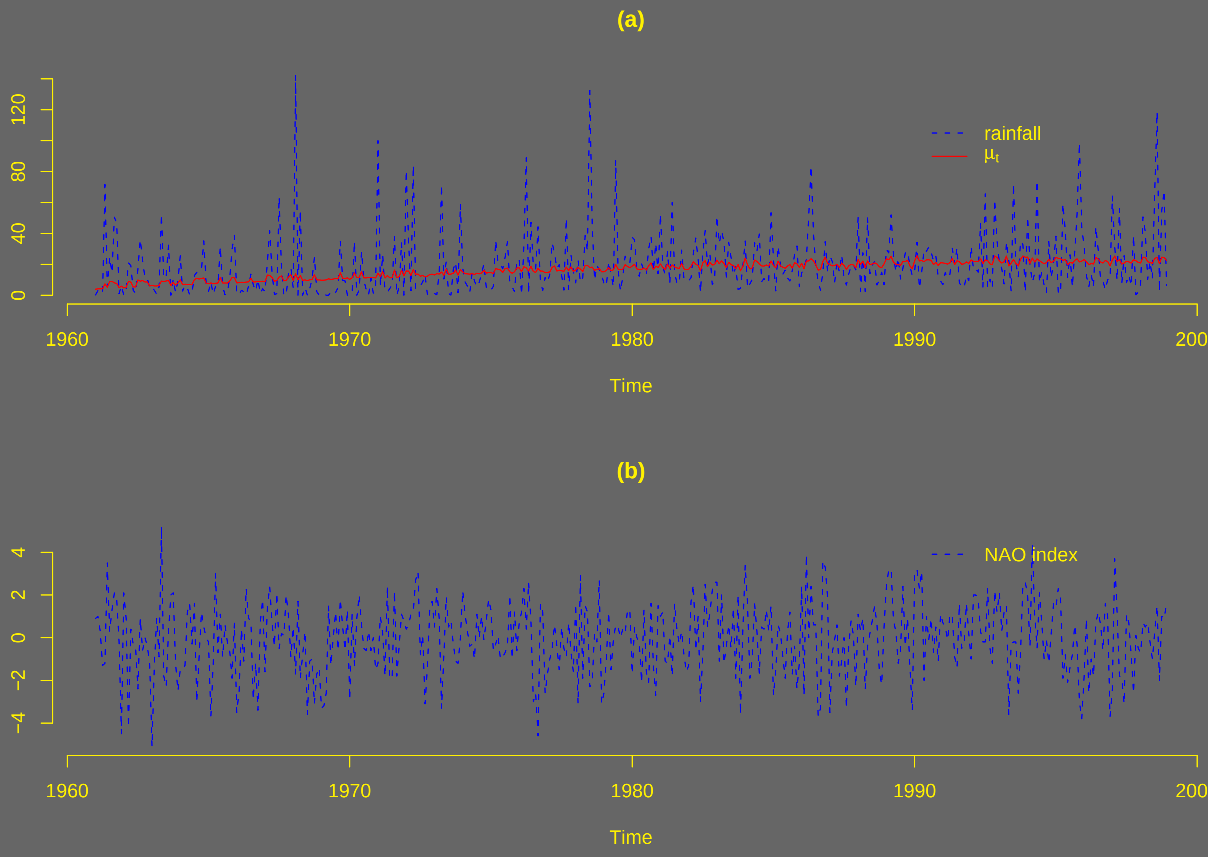


Figure 5: Posterior median and 90% probability intervals for β_t

Space-time model with Process Convolutions

Space-time model with Process Convolutions

- Consider data $y_t = (y_{1,t}, \ldots, y_{n_t,t})'$ which is recorded at sites $s_1, \ldots, s_{n_t}$.

Space-time model with Process Convolutions

- Consider data $y_t = (y_{1,t}, \ldots, y_{n_t,t})'$ which is recorded at sites $s_1, \ldots, s_{n_t}$.
- A possible model (Higdon 2002) is

$$
\begin{aligned}
y_t &= K^t x_t + \epsilon_t \\
x_t &= x_t + \nu_t
\end{aligned}
$$

Space-time model with Process Convolutions

- Consider data $y_t = (y_{1,t}, \dots, y_{n_t,t})'$ which is recorded at sites $s_1, \dots, s_{n_t}$.
- A possible model (Higdon 2002) is

$$\begin{aligned} y_t &= K^t x_t + \epsilon_t \\ x_t &= x_t + \nu_t \end{aligned}$$

K^t is a $n_t \times \kappa$ matrix given by

$$\begin{aligned} K^t_{ij} &= k(s_i - \omega_j),\ t = 1, \dots, m \\ \epsilon_t &\sim N(0, \sigma^2_\epsilon),\ t = 1, \dots, m \\ \nu_t &\sim N(0, \sigma^2_\nu),\ t = 1, \dots, m \\ x_1 &\sim N(0, \sigma^2_x \boldsymbol{I}_\kappa) \end{aligned}$$

- $k(\cdot - \omega_j)$ defines a smoothing kernel.
- $\omega_1, \ldots, \omega_\kappa$ are spatial sites where kernels are centered.
- x_t is interpreted as a *latent process*

$$y(s,t) = \sum_j k(s - \omega_j) x_{jt}$$

- $k(\cdot - \omega_j)$ defines a smoothing kernel.
- $\omega_1, \ldots, \omega_\kappa$ are spatial sites where kernels are centered.
- x_t is interpreted as a *latent process*

$$y(s,t) = \sum_j k(s - \omega_j) x_{jt}$$

- Possible kernels are:
  - *Gaussian*: $k(s) \propto \exp\left\{-||s||^2/2\eta\right\}$ $(\eta > 0)$.
  - *Exponential*: $k(s) \propto \exp\left\{-||s||/\eta\right\}$ $(\eta > 0)$.
  - *Spherical*: $k(s) \propto \left(1 - \frac{||s||^3}{r^3}\right)^3 I[s \leq r]$.

A Spatio-Temporal Model for the GEV distribution

A Spatio-Temporal Model for the GEV distribution

- Assume $y_{s,t} \sim GEV(\mu_{s,t}, \sigma, \xi)$; $s = 1, \ldots, S$, $t = 1, \ldots, m$

$$H_{s,t}(y_{s,t}; \mu_{s,t}, \xi, \sigma) = \exp\left\{ -\left[1 + \xi\left(\frac{y_{s,t} - \mu_{s,t}}{\sigma}\right)\right]_+^{-1/\xi} \right\}$$

A Spatio-Temporal Model for the GEV distribution

- Assume $y_{s,t} \sim GEV(\mu_{s,t}, \sigma, \xi)$; $s = 1, \ldots, S$, $t = 1, \ldots, m$

$$H_{s,t}(y_{s,t}; \mu_{s,t}, \xi, \sigma) = \exp\left\{ - \left[1 + \xi \left(\frac{y_{s,t} - \mu_{s,t}}{\sigma} \right) \right]_+^{-1/\xi} \right\}$$

- For each t, $\mu_t = (\mu_{1,t}, \mu_{2,t}, \ldots, \mu_{S,t})'$. (σ, ξ) constant in time.

A Spatio-Temporal Model for the GEV distribution

- Assume $y_{s,t} \sim GEV(\mu_{s,t}, \sigma, \xi)$; $s = 1, \ldots, S$, $t = 1, \ldots, m$

$$H_{s,t}(y_{s,t}; \mu_{s,t}, \xi, \sigma) = \exp\left\{ - \left[1 + \xi \left(\frac{y_{s,t} - \mu_{s,t}}{\sigma} \right) \right]_{+}^{-1/\xi} \right\}$$

- For each t, $\mu_t = (\mu_{1,t}, \mu_{2,t}, \ldots, \mu_{S,t})'$. (σ, ξ) constant in time.

- We define a DLM on μ_t:

$$\begin{aligned} \mu_t &= K'\theta_t + \epsilon_t; \\ \theta_t &= \theta_{t-1} + \nu_t \end{aligned}$$

- $\theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})'$, $\epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})'$, $\nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})'$.

- $\theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})'$, $\epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})'$, $\nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})'$.
- $\epsilon_t \sim N(0, \sigma_\epsilon^2 I_{\kappa \times \kappa})$; $\nu_t \sim N(0, \sigma_\nu^2 I_{\kappa \times \kappa})$

- $\theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})'$, $\epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})'$, $\nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})'$.
- $\epsilon_t \sim N(0, \sigma_\epsilon^2 I_{\kappa\times\kappa})$; $\nu_t \sim N(0, \sigma_\nu^2 I_{\kappa\times\kappa})$
- With a Gaussian kernel, K' is an $S \times \kappa$ matrix with entries:

$$K'_{ij} = K(s_i - \omega_j);$$
$$K(s_i - \omega_j) \propto exp(-d||s_i - \omega_j||^2/2)$$

- s_i is the position of station i.
- ω_j is the position of the *kernel* $j = 1, \ldots, \kappa$.

- $\theta_t = (\theta_{t,1}, \ldots, \theta_{t,\kappa})'$, $\epsilon_t = (\epsilon_{t,1}, \ldots, \epsilon_{t,\kappa})'$, $\nu_t = (\nu_{t,1}, \ldots, \nu_{t,\kappa})'$.
- $\epsilon_t \sim N(0, \sigma_\epsilon^2 I_{\kappa\times\kappa})$; $\nu_t \sim N(0, \sigma_\nu^2 I_{\kappa\times\kappa})$
- With a Gaussian kernel, K' is an $S \times \kappa$ matrix with entries:

$$K'_{ij} = K(s_i - \omega_j);$$
$$K(s_i - \omega_j) \propto exp(-d||s_i - \omega_j||^2/2)$$

- s_i is the position of station i.
- ω_j is the position of the *kernel* $j = 1, \ldots, \kappa$.
- d is a range parameter; $d = c\phi$; $1/2 < c < 2$; $\phi =$ *knot* distance.

- 1st stage priors: $\pi(\sigma) \sim LN(\mu_\sigma, s_\sigma)$ and $\pi(\xi) \sim N(\mu_\xi, s_\xi)$ are th

- 1st stage priors: $\pi(\sigma) \sim LN(\mu_\sigma, s_\sigma)$ and $\pi(\xi) \sim N(\mu_\xi, s_\xi)$ are th
- 2nd stage priors: $\theta_0 \sim N(0, \sigma_\theta^2 I_{\kappa\times\kappa})$; $1/\sigma_\epsilon^2 \sim Gamma(\alpha_\epsilon, \beta_\epsilon)$; $1/\sigma_\nu^2 \sim Gamma(\alpha_\nu, \beta_\nu)$ and $1/\sigma_\theta^2 \sim Gamma(\alpha_\theta, \beta_\theta)$.

- 1st stage priors: $\pi(\sigma) \sim LN(\mu_\sigma, s_\sigma)$ and $\pi(\xi) \sim N(\mu_\xi, s_\xi)$ are th
- 2nd stage priors: $\theta_0 \sim N(0, \sigma_\theta^2 I_{\kappa\times\kappa})$; $1/\sigma_\epsilon^2 \sim Gamma(\alpha_\epsilon, \beta_\epsilon)$; $1/\sigma_\nu^2 \sim Gamma(\alpha_\nu, \beta_\nu)$ and $1/\sigma_\theta^2 \sim Gamma(\alpha_\theta, \beta_\theta)$.
- The log-likelihood is equal to

$$\begin{aligned} l(\boldsymbol{\theta}) &= -mS\log\sigma - \sum_{t=1}^{m}\sum_{s=1}^{S}\left[1+\xi\left(\frac{z_{s,t}-\mu_{s,t}}{\sigma}\right)\right]_+^{-1/\xi} \\ &- \left(1+\frac{1}{\sigma}\right)\sum_{t=1}^{m}\sum_{s=1}^{S}\log\left[1+\xi\left(\frac{y_{s,t}-\mu_{s,t}}{\sigma}\right)\right]_+ \end{aligned}$$

Posterior Inference and Simulation for Space-Time Model

Posterior Inference and Simulation for Space-Time Model

- Follows similar lines as for the GEV time model.

Posterior Inference and Simulation for Space-Time Model

- Follows similar lines as for the GEV time model.
- The full conditional for $\mu_{t,s}$, σ and ξ are sampled through a Metropolis-Hastings step.

Posterior Inference and Simulation for Space-Time Model

- Follows similar lines as for the GEV time model.
- The full conditional for $\mu_{t,s}$, σ and ξ are sampled through a Metropolis-Hastings step.
- θ_t is sampled with Forward Filtering Backward Simulation.

Posterior Inference and Simulation for Space-Time Model

- Follows similar lines as for the GEV time model.
- The full conditional for $\mu_{t,s}$, σ and ξ are sampled through a Metropolis-Hastings step.
- θ_t is sampled with Forward Filtering Backward Simulation.
- The full conditionals of σ_ϵ^2; σ_ν^2 and σ_θ^2 are sampled with Inverse Gamma distributions.

Posterior Inference and Simulation for Space-Time Model

- Follows similar lines as for the GEV time model.
- The full conditional for $\mu_{t,s}$, σ and ξ are sampled through a Metropolis-Hastings step.
- θ_t is sampled with Forward Filtering Backward Simulation.
- The full conditionals of σ^2_ϵ; σ^2_ν and σ^2_θ are sampled with Inverse Gamma distributions.
- The range parameter d is assumed fixed, $d = c\phi$.

Figure 6: RAMA stations, kernel and interpolation grid positions.

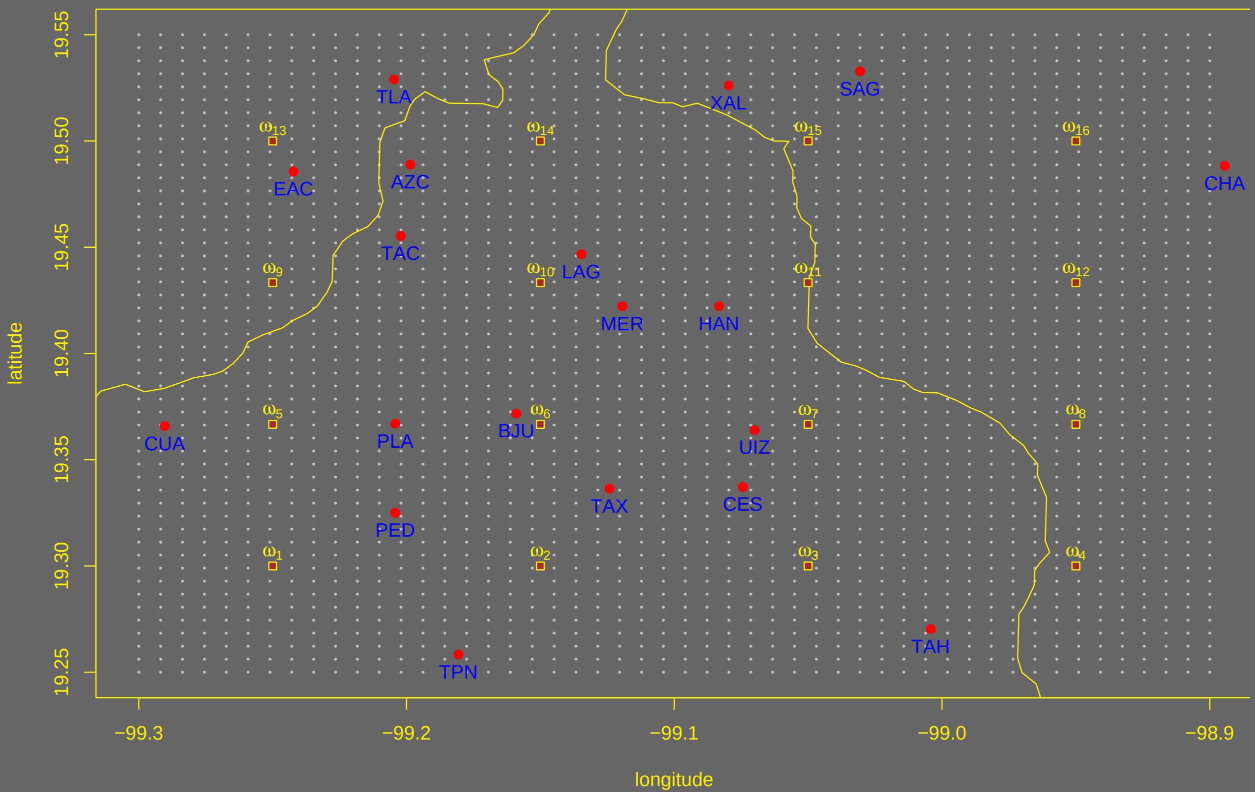


Figure 7: Daily maxima for 1999 and posterior estimates of 0.5 quantile.

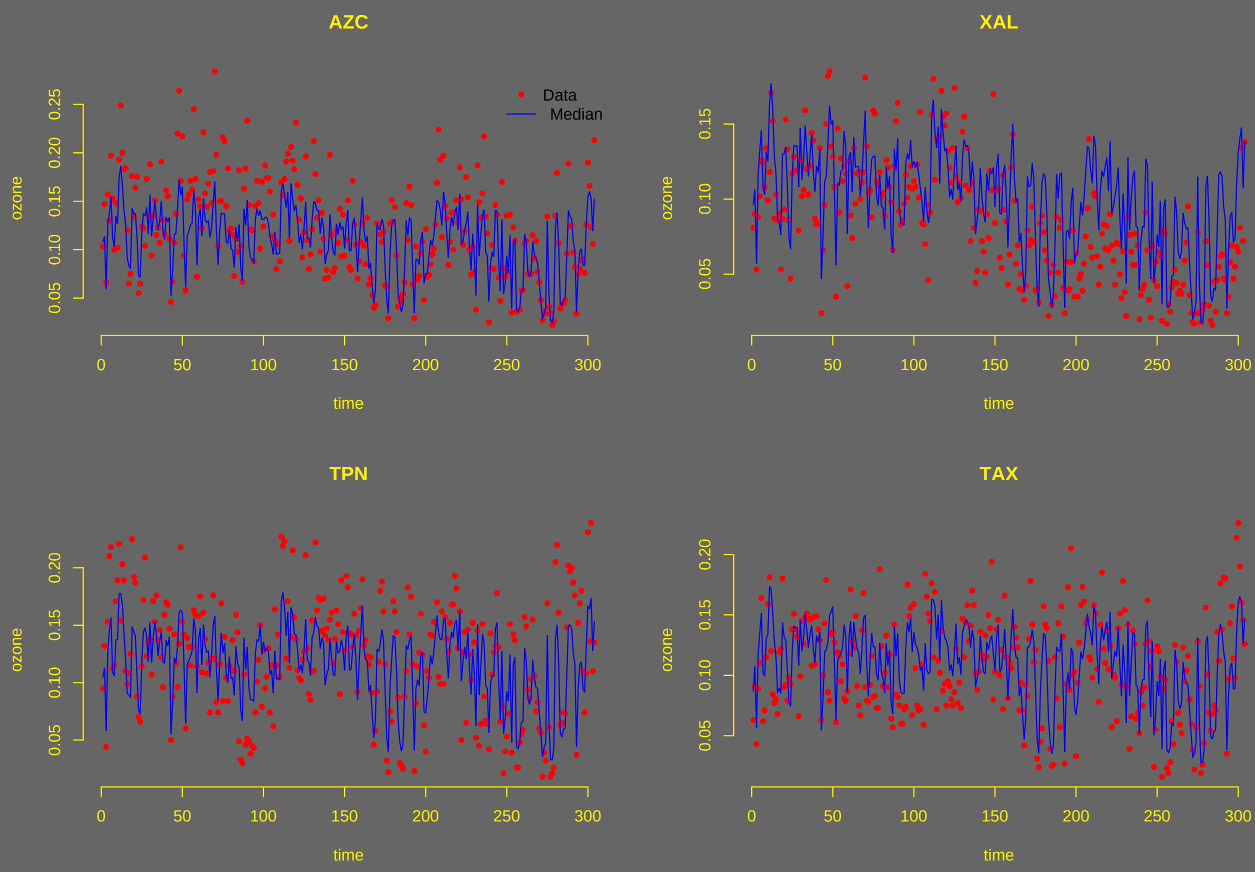


Figure 8: Posterior estimate of the 0.5 quantile of the space-time GEV distribution for a 50×50 resolution grid.

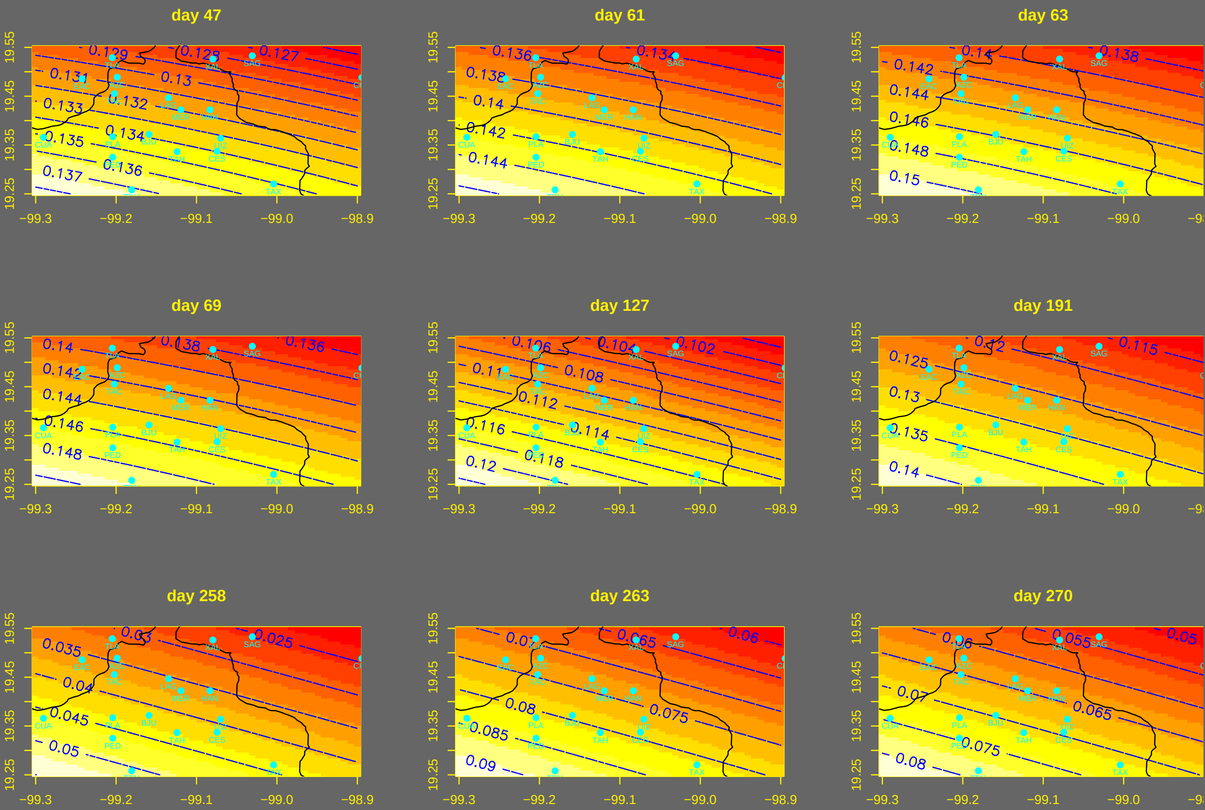
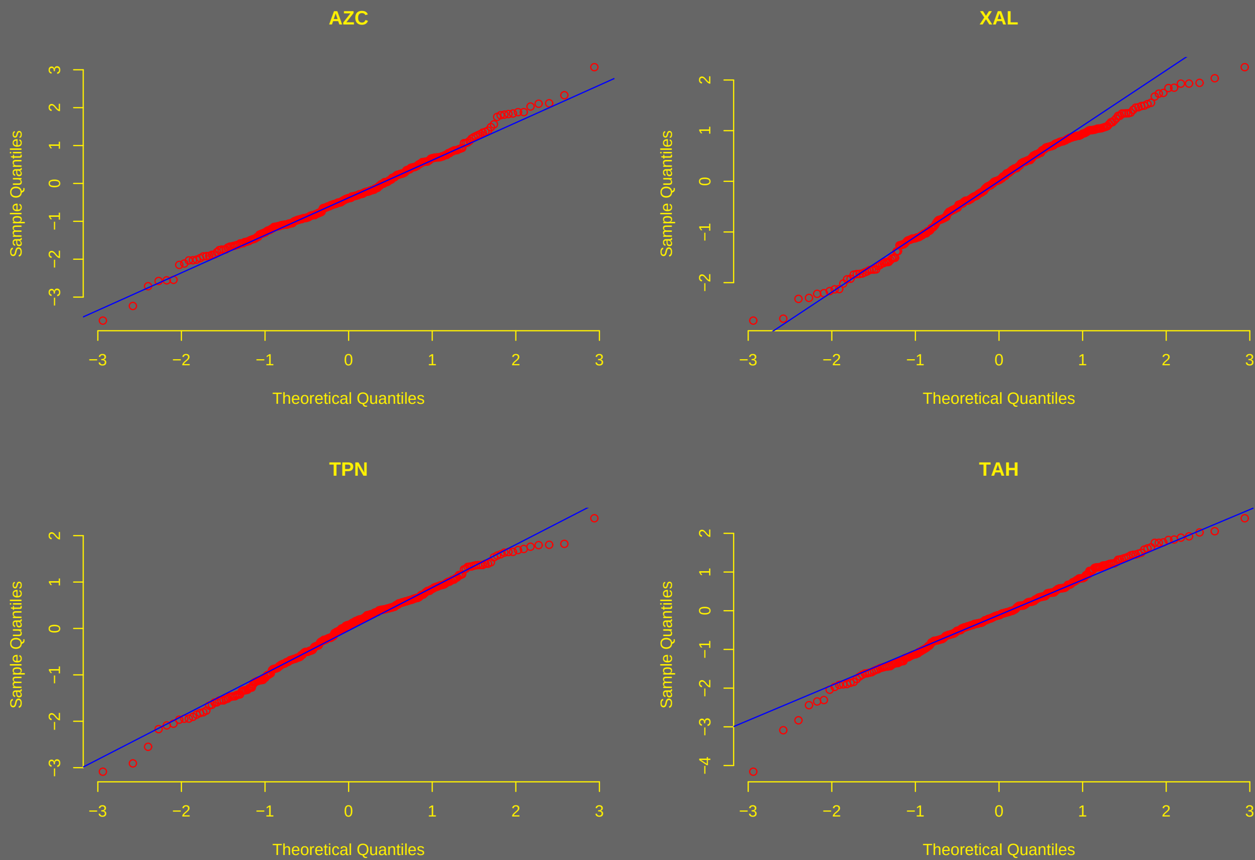


Figure 9: $u_t = G(y_t)$ diagnostics based on leaving one station out



Discussion

Discussion

- GEV distribution with a location parameter that varies in time and space.

Discussion

- GEV distribution with a location parameter that varies in time and space.
- Inference for space-time quantiles is straightforward.

Discussion

- GEV distribution with a location parameter that varies in time and space.
- Inference for space-time quantiles is straightforward.
- Extension: space-time changes for scale and shape parameters.

Discussion

- GEV distribution with a location parameter that varies in time and space.
- Inference for space-time quantiles is straightforward.
- Extension: space-time changes for scale and shape parameters.
  - Station by station analysis of Mexico City data provided no grounds for such extension.

Discussion

- GEV distribution with a location parameter that varies in time and space.
- Inference for space-time quantiles is straightforward.
- Extension: space-time changes for scale and shape parameters.
  - Station by station analysis of Mexico City data provided no grounds for such extension.
- Extension: consider time-varying or spatial dependent thresholds.

Discussion

- GEV distribution with a location parameter that varies in time and space.
- Inference for space-time quantiles is straightforward.
- Extension: space-time changes for scale and shape parameters.
  - Station by station analysis of Mexico City data provided no grounds for such extension.
- Extension: consider time-varying or spatial dependent thresholds.