

# TOMSK POLYTECHNIC UNIVERSITY

FACULTY OF NATURAL SCIENCES & MATHEMATICS

DEPT. OF HIGHER MATH. AND MATH. PHYSICS

ESTIMATION OF VOLATILITY AND DISTRIBUTION  
FUNCTION FOR THE STOCHASTIC PROCESSES  
CORRESPONDING TO STOCK PRICE DYNAMICS

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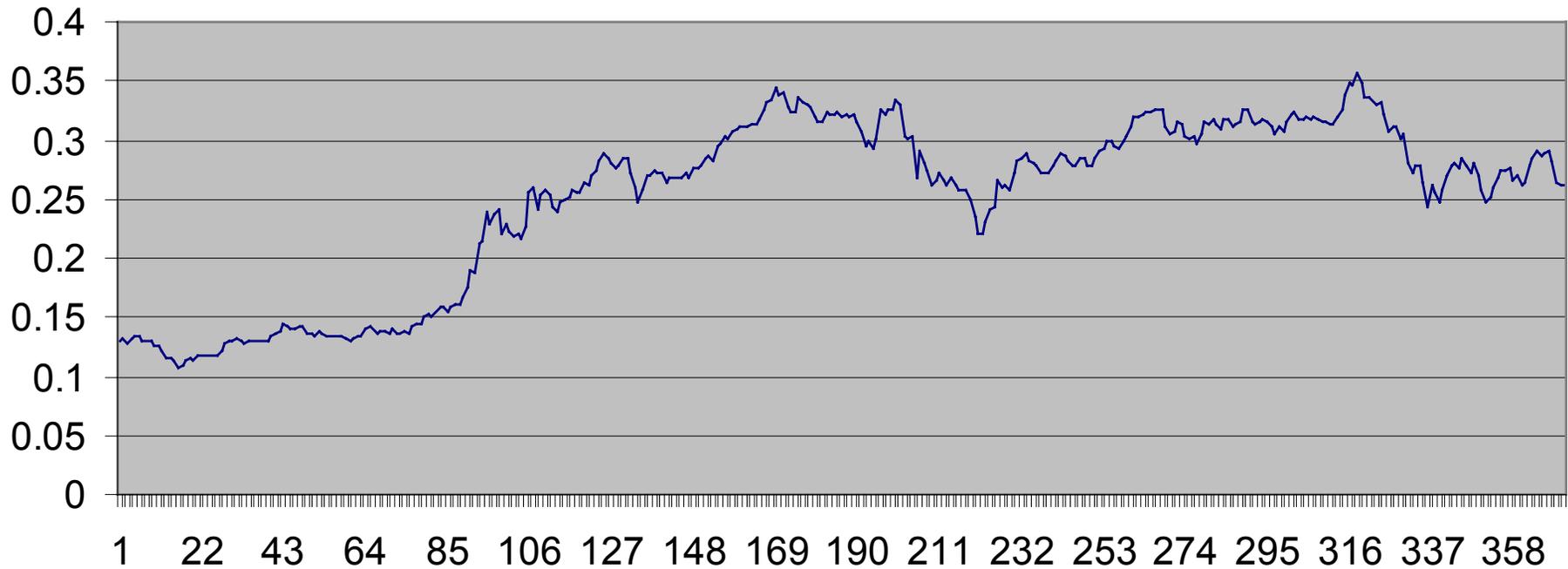
The statistical properties of the stochastic processes play a key role in the modeling of financial markets. For example, the knowledge of the stochastic nature of the price of a financial asset is crucial aspect for a sensible pricing of an option issued on it. And, of course, if the distribution function or conditional probability density of all orders are known, we can make a full characterization of stochastic process and forecast its stochastic behavior in any point of time.

The most common stochastic model of stock price dynamics assumes that stochastic processes to be used in the investigations are a diffusive process, and their increments have Gaussian distribution. Such model, known as Brownian motion, provides the easiest way to estimate the behaviour of observed empirical data. But, in practice, there are the systematic deviations from the model predictions because empirical distributions have more leptokurtic and skew distribution functions than Gaussian one. And we should estimate the probabilistic law corresponding to these data by applying some methods.

## EXAMPLE OF THE STOCHASTICAL BEHAVIOUR

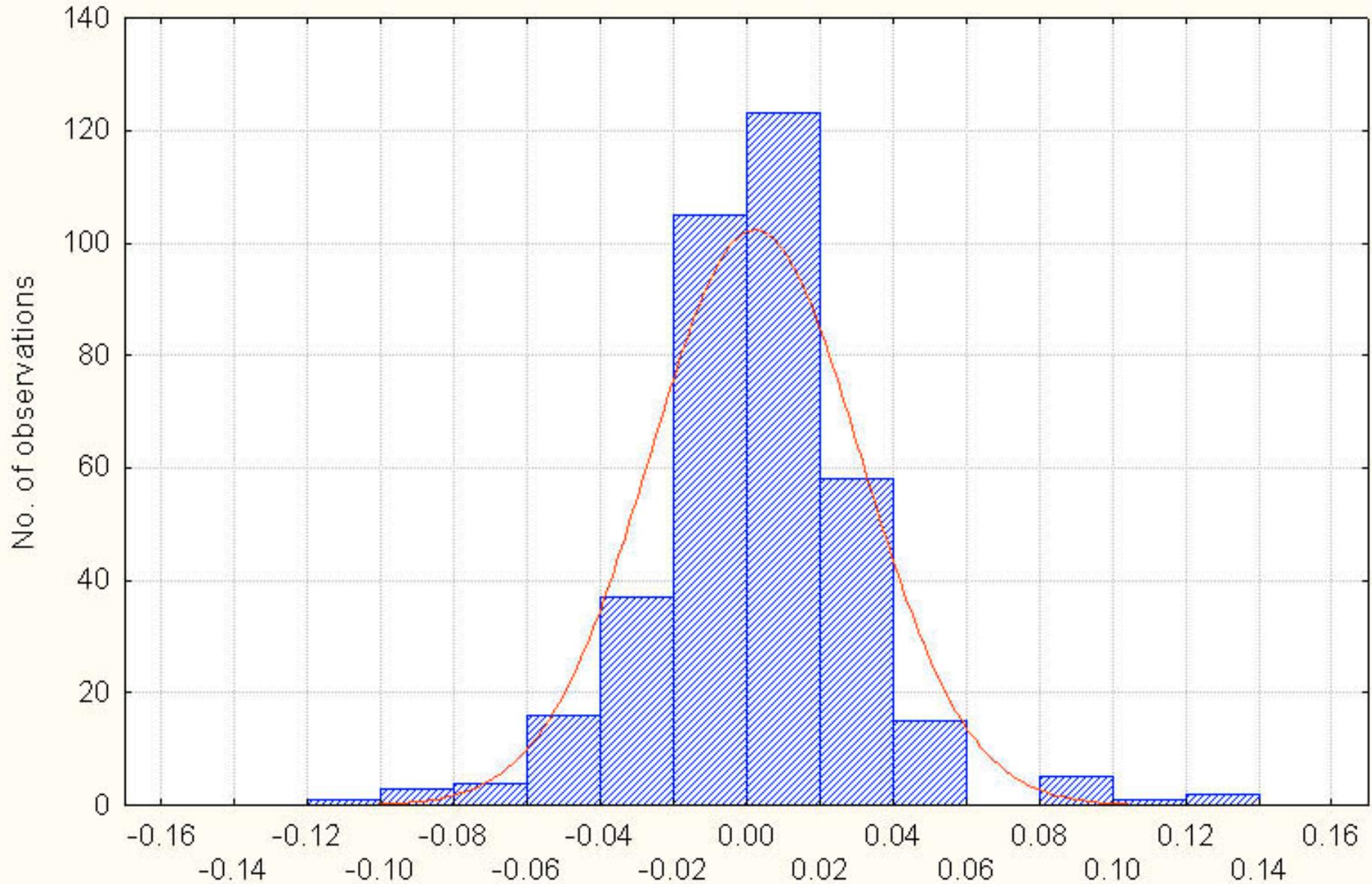
PRICE, US \$

**Price dynamics for RAO UES asset  
from 01.01.03 till 30.06.04**



TIME, days

Chi-Square test = 22.65767, df = 5 (adjusted) , p = 0.00039



Distribution of RAO UES shares and their daily price differences

It can be produced in many different ways. For example, for some time series  $S_i$  of daily stock prices we can compute a daily price differences by formula

$$R_i = \frac{S_{i+1} - S_i}{S_i}, \quad i = 0, 1, 2, \dots$$
$$\gamma_n = \sum_{j=1}^s \frac{(v_j - np_j(\Theta))^2}{np_j(\Theta)}$$

where  $n$  – total amount of points of the sample,  $s = \log_2 n + 1$  – quantity of classification intervals,  $\Theta$  – consistent estimate of evaluating parameters (in our case it is the vector consists of empirical estimation of average and dispersion

$$p_j(\Theta) = F_{\text{mod}}(c_j, \Theta) - F_{\text{mod}}(c_{j-1}, \Theta)$$

When the statistical parameters of time series considered as stochastic process will be found it is very important to analyze behaviour of this series and forecast its parameters on the nearest future period of time. So, the next objective to be investigated is the estimating of volatility of time series and its forecasting.

In recent years there is some support for the hypothesis that the information provided by implied volatilities from daily option prices is more relevant in forecasting volatility than the volatility information provided by historical returns.

Although the idea of option-implied volatility estimates is relatively simple, there is not one straightforward method to extract the information. Every proposed method relies on a number of assumptions regarding the model underlying option prices.

This is why we will concentrate on a relatively recent option–pricing model that applies the time series generalized autoregressive conditionally heteroskedastic (GARCH) methodology (i.e., model is driven by a skewed leptokurtic distribution) to predicting the volatility.

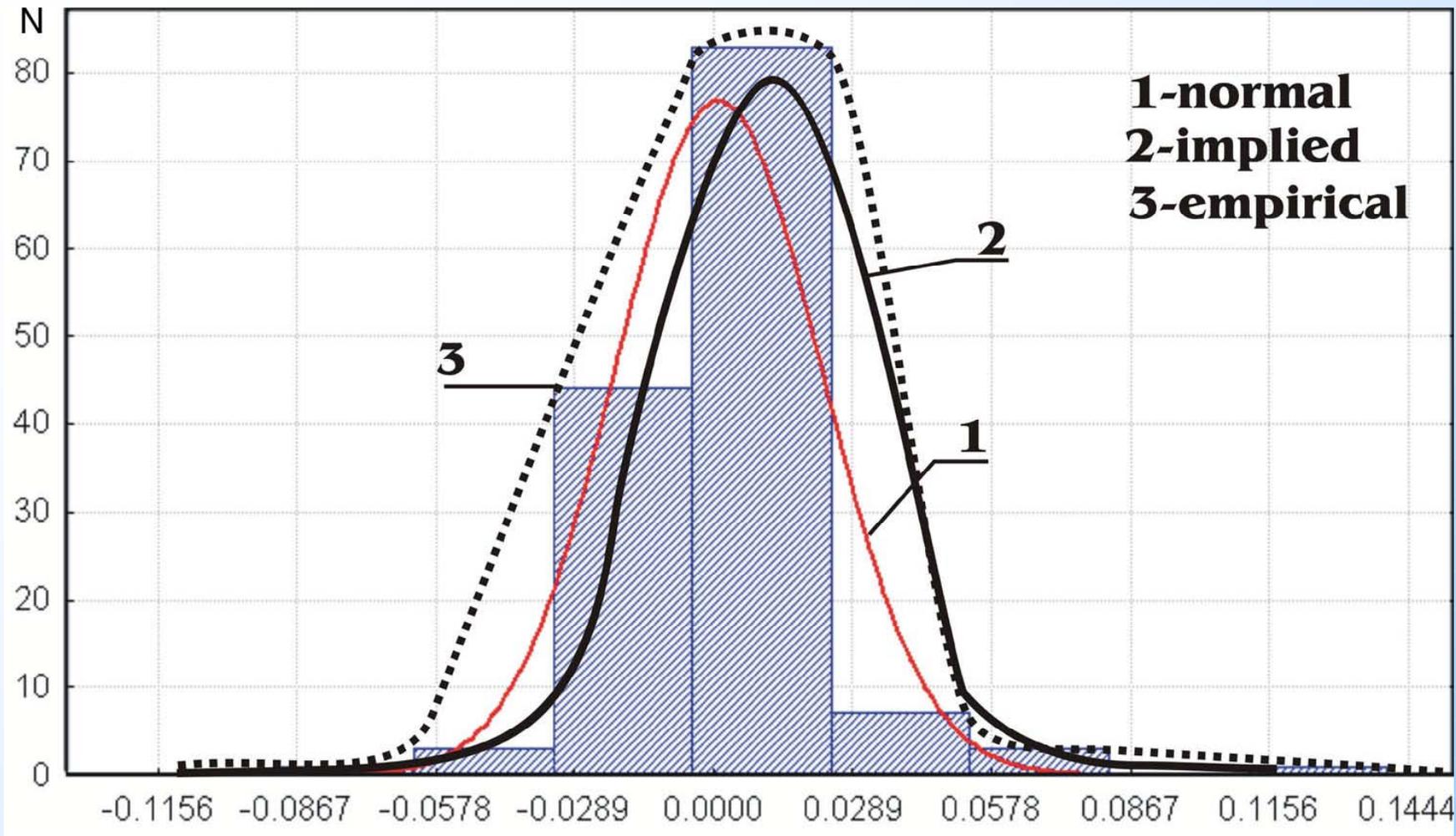
This model was chosen because of its flexibility and recent empirical successes. Furthermore, GARCH and its various extensions allow to observe the situation when negative shocks affect volatility quite differently than positive shocks of equal size.

Let us see the GARCH(1,1) model for predicting the future volatility as follows:

$$\sigma_n^2 = \gamma V + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (1)$$

where  $\gamma > 0$ ,  $\alpha > 0$ ,  $\beta > 0$  - some coefficients to be estimated,

$u_n = \frac{h_n - h_{n-1}}{h_n}$  - daily increments of time series,  $\sigma_n$  - volatility in  $n^{\text{th}}$  day



Distribution functions for call option on futures contract of RAO UES RTS shares (143 quotations)

1 - normal distribution with  $\mu=10^{-3}$ ,  $\sigma=4,5 \times 10^{-4}$ ; 2 – implied distribution with the same parameters; 3 - empirical distribution with  $\mu=10^{-3}$ ,  $\sigma=4,5 \times 10^{-4}$

The GARCH(1,1) coefficients was estimated from minimax problem

$$L_k = \max_{n,k} \{ \sigma_n^{GARCH} - \sigma_n^{(k)} \} \rightarrow \min,$$

The GARCH model appears to have done a good job in forecasting the volatility if

$$\gamma < \chi_{1-a}^2(m)$$

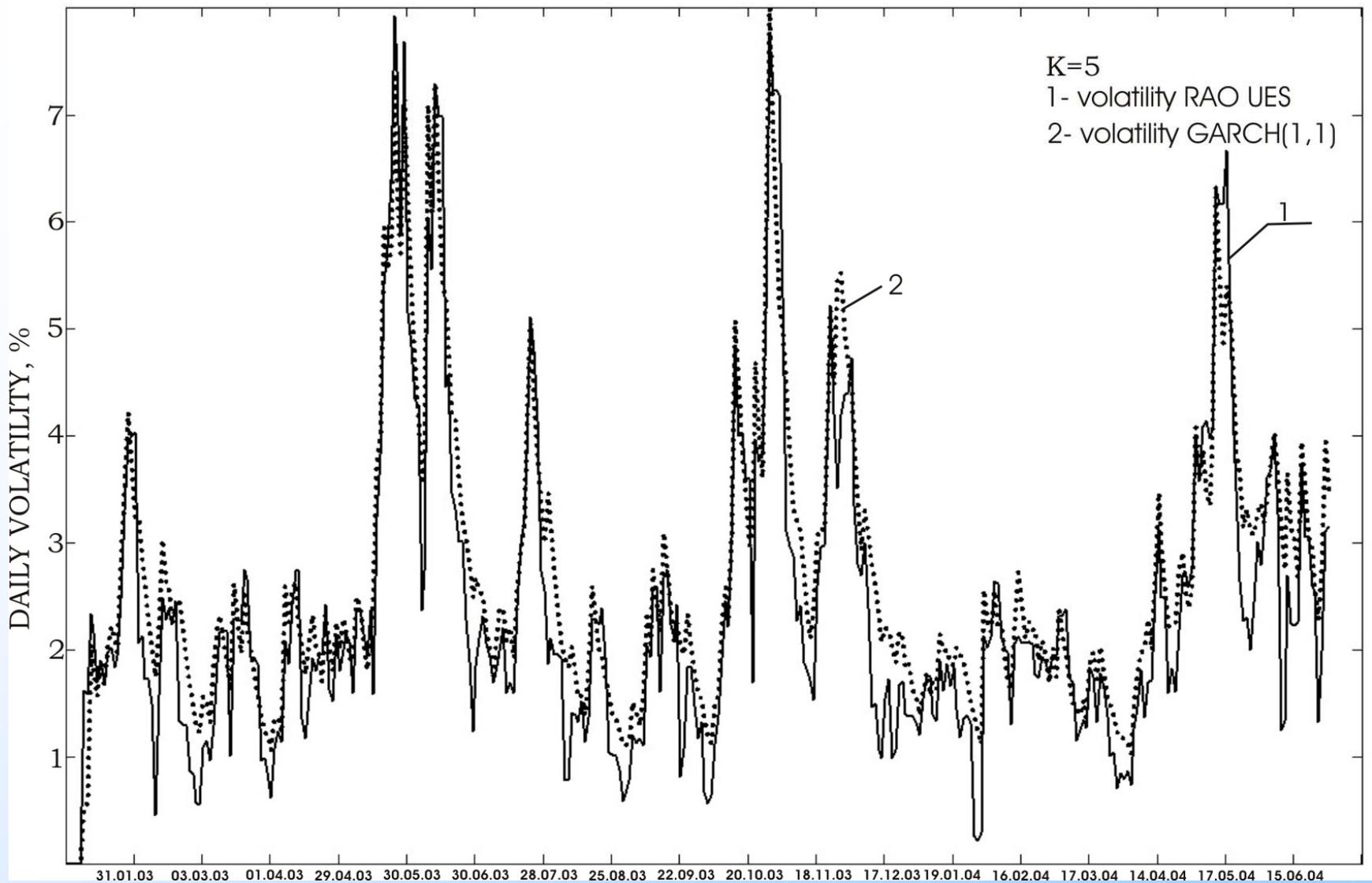
where  $\gamma = n(n-2) \sum_{k=1}^m \frac{\gamma_k^2}{n-k}$  - Ljung-Box statistics, a - significance

level (e.g., a=0,05%), and autocorrelation with lag k is

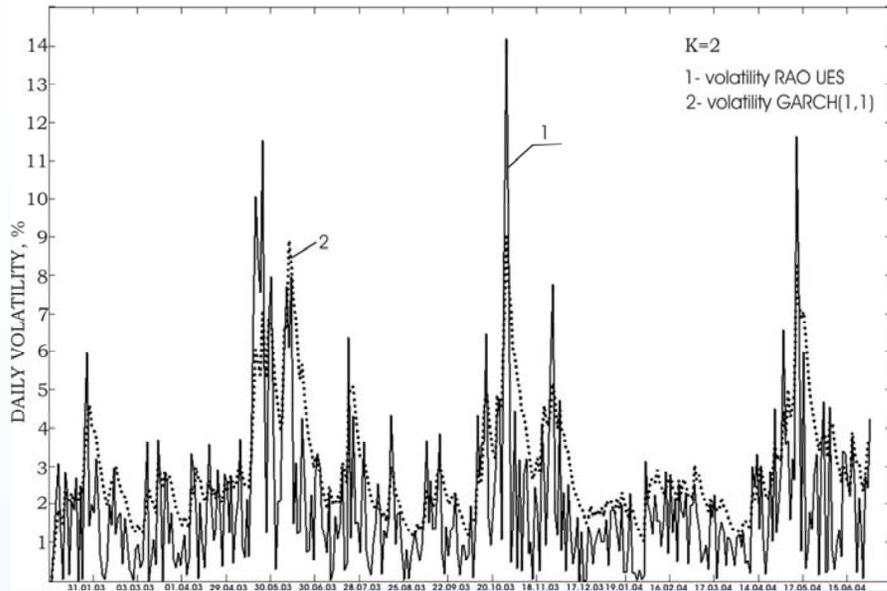
$$\gamma_k = \text{corr}(u_i, u_{i+k}) = E[(u_i - \bar{u}_i)(u_{i+k} - \bar{u}_{i+k})] \left[ \sqrt{Du_i} \sqrt{Du_{i+k}} \right]^{-1}, i = \overline{1, (n-k)}.$$

Found coefficients of model (1) with different lags k

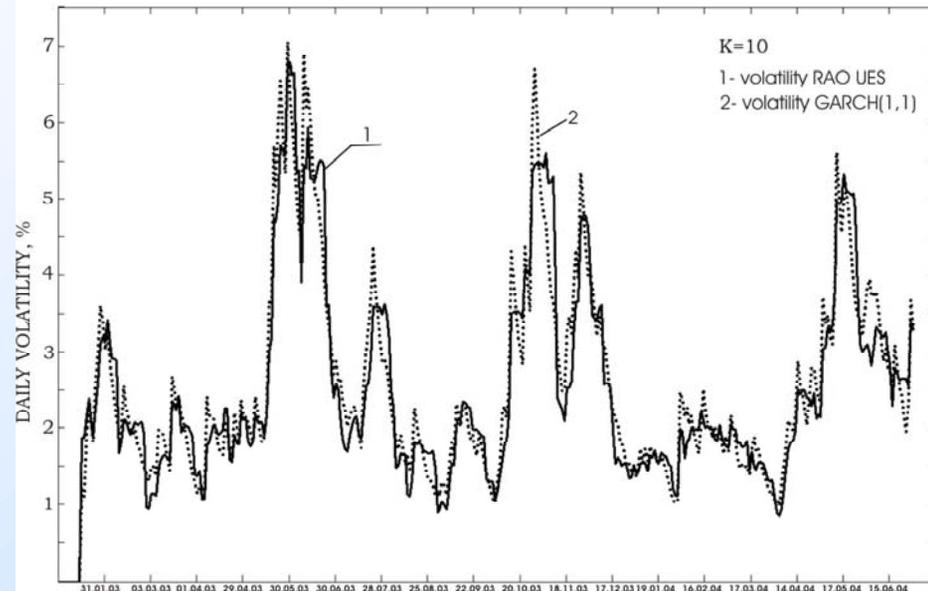
k	$\omega$	$\alpha$	$\beta$	$\gamma$	V
2	$10^{-5}$	0,24	0,73	0,03	$3,3 \times 10^{-4}$
5	$10^{-5}$	0,22	0,77	$10^{-2}$	$10^{-3}$
10	$10^{-7}$	0,18	0,8	0,02	$5 \times 10^{-6}$



Daily volatility of RAO UES RTS shares (371 quotations) from 01.01.2003 till 30.06.2004. Lag  $K=5$ . Curve 1 is historical volatility, curve 2 - volatility, forecasted by GARCH(1,1)



K=2



K=10

Daily volatility of RAO UES RTS shares (371 quotations) from 01.01.2003 till 30.06.2004. Curve 1 is historical volatility, curve 2 - volatility, forecasted by GARCH(1,1)

AUTOCORRELATION FOR TIME SERIES  $u_n$  AND  $u_n/\sigma_n$   
COMPUTED WITH K=10

$k$	1	2	3	4	5	6	7	8	9	10
before	0.106	0.289	0.09	0.19	0.05	0.14	0.03	-0.016	0.05	0.055
after	-0.031	0.0056	0.0068	0.0063	-0.039	0.0306	-0.087	-0.079	-0.04	-0.015

Thank you for your attention

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