CONTAGION VERSUS FLIGHT TO QUALITY IN FINANCIAL MARKETS

Jose Olmo
Department of Economics
City University, London

(joint work with Jesús Gonzalo, Universidad Carlos III de Madrid)
Outline

- Transmission of Risk between Economies
- Definitions of Interdependence and Contagion
- Statistical measures for dependence: Pitfalls of correlation
- Multivariate Extreme Value Theory: A new copula
- Measuring Interdependence and Contagion by tail dependence measures
- Causality in the Extremes
- Application: The flight to quality phenomenon
Transmission of Risk between Economies

Every economy is exposed to a series of factors that can culminate in what can be called crisis.

Types of crises: financial, liquidity, banking or currency crises.

Definition 1. A general definition of crisis in a market is given by a threshold level such that in case is exceeded, it results in the collapse of the system producing the triggering of negative effects in the rest of the markets.

In summary: A crisis in one market is characterized by the collapse not only of that market but by the negative effects produced on other markets.

Two ways of regarding dependence: (In particular in crises periods)

From the point of view of the direction (Causality in the Extremes).

From the point of view of the intensity: strength of the links in turmoil periods.
Interdependence and Contagion

- **Interdependence** due to rational links between the variables (markets).

- **Contagion** effects: abnormal links between the markets triggered by some phenomena (crisis).

- **Regarding the direction:**
  - Interdependence implies that both markets collapse because both are influenced by the same factors (Forbes and Rigobon (2001), Corsetti, Pericoli, Sbracia (2002)).
  - Contagion implies that the collapse in one market produces the fall of the other market.
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**Regarding the intensity:**
- Interdependence implies no significant change in cross market relationships.
- Contagion implies that cross market linkages are stronger after a shock to one market.
Transmission Channels connecting the markets

From an economic viewpoint:

- Economic fundamentals, market specific shocks, impact of bad news, psychological effects (herd behavior).

From a statistical viewpoint: Pearson correlation.

\[
Corr(X_1, X_2) = \frac{E(X_1 - E(X_1))(X_2 - E(X_2))}{\sqrt{V(X_1)}\sqrt{V(X_2)}},
\]

with \( X_1 \) and \( X_2 \) random variables.

Correlation is not sufficient to measure the dependence found in financial markets.

- It is only reliable when the random variables are jointly gaussian.

- Conditioning on extreme events can lead to misleading results.
Pitfalls of Correlation

These results are found in Embrechts, et al. (1999) and in Boyer et al. (1999).

- Correlation is an scalar measure (Not designed for the complete structure of dependence).
- A correlation of zero does not indicate independence between the variables.
- Correlation is not invariant under transformations of the risks.
- Correlation is only defined when the variances of the corresponding variables are finite.
- An increase in the correlation between two variables can be **JUST** due to an increase in the variance of one variable.

Ex.- Let $\rho$ be the correlation between two r.v.'s $X, Y$ and let us condition on $X \in A$. Then

$$\rho_A = \rho \left( \rho^2 + (1 - \rho^2) \frac{V(X)}{V(X|A)} \right)^{-1/2}$$

**SOLUTION:** A complete picture of the structure of dependence (Copula functions).
**Copula functions for dependence**

**Definition 2.** A function $C : [0, 1]^m \to [0, 1]$ is a $m$-dimensional copula if it satisfies the following properties:

(i) For all $u_i \in [0, 1]$, $C(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i$.

(ii) For all $u \in [0, 1]^m$, $C(u_1, \ldots, u_m) = 0$ if at least one of the coordinates is zero.

(iii) The volume of every box contained in $[0, 1]^m$ is non-negative, i.e., $V_C([u_1, \ldots, u_m] \times [v_1, \ldots, v_m])$ is non-negative. For $m = 2$, $V_C([u_1, u_2] \times [v_1, v_2]) = C(u_2, v_2) - C(u_1, v_2) - C(u_2, v_1) + C(u_1, v_1) \geq 0$ for $0 \leq u_i, v_i \leq 1$.

By Sklar’s theorem (1959),

$$H(x_1, \ldots, x_m) = C(F_1(x_1), \ldots, F_m(x_m)),$$

with $H$ the multivariate distribution, and $F_i$ the margins.
Our Goal: Using dependence in the Extremes

Let \((M_{n1}, \ldots, M_{nm})\) be the vector of maxima, and denote its distribution by

\[
H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = P\{a_{ni}^{-1}(M_{ni} - b_{ni}) \leq x_i, i = 1, \ldots, m\}.
\]

The central result of EVT in the multivariate setting (mevt) is:

\[
\lim_{n \to \infty} H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = G(x_1, \ldots, x_m),
\]

with \(G\) a mevd.

**Theorem 1.** The class of mevd is precisely the class of max-stable distributions (Resnick (1987), proposition 5.9).

These distributions satisfy the following Invariance Property,

\[
G^t(tx_1, \ldots, tx_m) = G(\alpha_1x_1 + \beta_1, \ldots, \alpha_mx_m + \beta_m),
\]

for every \(t > 0\), and some \(\alpha_j > 0\) and \(\beta_j\).
By Sklar’s theorem,

\[
\lim_{n \to \infty} H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = C(G_1(x_1), \ldots, G_m(x_m)),
\]

with \( G_i \) univariate evd.

Under an appropriate transformation of the margins \((Z_i = 1/\log_{1/F_i(X)})\),

\[
\lim_{n \to \infty} H^*(nz_1, \ldots, nz_m) = C(\Psi_1(z_1), \ldots, \Psi_1(z_m)), \tag{1}
\]

with \( \Psi_1(z) = \exp(-\frac{1}{z}) \), standard Fréchet, and the invariance property for copulas reads

\[
C^n(\Psi_1(nz_1), \ldots, \Psi_1(nz_m)) = C(\Psi_1(z_1), \ldots, \Psi_1(z_m)).
\]

Taking logs in both sides of (1) and applying the invariance property we have

\[
\lim_{n \to \infty} \frac{H^*(nz_1, \ldots, nz_m)}{1 + \log C(\Psi_1(nz_1), \ldots, \Psi_1(nz_m))} = 1.
\]
Then, $H^*(z_1, \ldots, z_m) = C(\Psi_1(z_1), \ldots, \Psi(z_m))$, from some threshold vector $(z_1, \ldots, z_m)$ sufficiently high.

- The copula function $C$ is derived from the limiting distribution of the maximum.
- $C$ must be of exponential type (extension of the EVT for the univariate case).

The Gumbel copula is within this class. Its general expression is

$$C_G(u_1, \ldots, u_m; \theta) = \exp\left[-(\log u_1)^\theta + \ldots + (\log u_m)^\theta\right]^{1/\theta}, \quad \theta \geq 1,$$

with $u_1, \ldots, u_m \in [0, 1]$ and $\theta \geq 1$. 
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$$C_G(u_1, \ldots, u_m; \theta) = \exp^{-[(-\log u_1)^\theta + \ldots + (-\log u_m)^\theta]^{1/\theta}}, \quad \theta \geq 1,$$

with $u_1, \ldots, u_m \in [0, 1]$ and $\theta \geq 1$.

**Inconvenient:** This multivariate extreme value distribution describes the dependence between the variables for the *multivariate upper tail* $(z_1, \ldots, z_m)$ sufficiently high.
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**Intuition:** Analogous to the approximation of the upper tail of $F$ (conditional excess d.f. given a threshold) by the Generalized Pareto distribution in the univariate case.
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**Our aim:** Modelling the complete structure of dependence between the variables. Not just the relation in the extremes!
Our Contribution: A NEW Copula

WE PROPOSE instead (for \( m=2 \)):

\[
\tilde{C}_G(u_1, u_2; \Theta) = \exp -D(u_1, u_2; \gamma, \eta) \left[ (-\log u_1)^\theta + (-\log u_2)^\theta \right]^{1/\theta},
\]

(2)

with

\[
D(u_1, u_2; \gamma, \eta) = \exp \gamma (1-u_1)(1-u_2)^\eta, \quad \gamma \geq 0, \quad \eta > 0.
\]

(3)

The function \( D(u_1, u_2; \gamma, \eta) \) accommodates departures from the invariance property with \( \gamma > 0 \) and \( \eta \neq 1 \).

**Theorem 2.** The function \( \tilde{C}_G : [0, 1] \times [0, 1] \to [0, 1] \) defined in (2) and (3) is a copula function if the parameters in \( \Theta \) satisfy that \( \tilde{c}_G(u_1, u_2; \Theta) > 0, \forall (u_1, u_2) \in [0, 1] \times [0, 1], \) with \( \tilde{c}_G(u_1, u_2; \Theta) = \frac{\delta^2 \tilde{C}_G(u_1, u_2; \Theta)}{du_1 du_2} \) the density function of the copula \( \tilde{C}_G. \)
Advantages of this NEW Copula

• This copula function is derived from the multivariate extreme value theory, in contrast to ad-hoc models for the dependence structure.

• The function $D(u_1, u_2; \gamma, \eta)$ and in particular the parameter $\gamma$ extend the multivariate extreme value theory results to the entire range of the random variables.

• $\tilde{C}_G$ is able to explain asymmetric effects of the variables for $\eta \neq 1$, and $\gamma > 0$.

• This copula is sufficiently flexible to describe different forms of dependence,
  • Dependence: $\theta \neq 1$ or $\theta = 1$ and $\gamma > 0$.
  • Independence: $\gamma = 0$, $\theta = 1$.
  • Asymptotic dependence: $\theta > 1$.
  • Asymptotic independence: $\theta = 1$. 
Our Contribution: Tail Dependence Measures

- Alternatives to the standard $\aleph$,

\[ \aleph = \lim_{t \to \infty} P\{Z_2 > t | Z_1 > t\}, \]


- Definitions of Interdependence and Contagion by means of tail dependence measures.

- The translation of these definitions to mathematical expressions by using copula functions.

- The distinction between types of contagion: In Intensity and In the direction.
Interdependence

Lehman (1966) defined two random variables $Z_1, Z_2$ as positively quadrant dependent ($PQD$) if for all $(z_1, z_2) \in \mathbb{R}^2$,

$$P\{Z_1 > z_1, Z_2 > z_2\} \geq P\{Z_1 > z_1\}P\{Z_2 > z_2\},$$

or equivalently if

$$P\{Z_1 \leq z_1, Z_2 \leq z_2\} \geq P\{Z_1 \leq z_1\}P\{Z_2 \leq z_2\}.$$

**Definition 3.** Two random variables are Interdependent if they are PQD. Interdependence is characterized by joint movements in the same direction (co-movements).

In terms of the copula Interdependence amounts to see that $g(u_1, u_2) > 0$, with

$$g(u_1, u_2) = \tilde{C}_G(u_1, u_2) - u_1u_2.$$
Contagion in Intensity

A stronger condition is required: **Tail Monotonicity**.

**Definition 4.** Suppose $Z_1, Z_2$ with common $\Psi_1$ and consider $z$ a threshold that determines the extremes in the upper tail of both random variables. There exists a contagion effect between $Z_1$ and $Z_2$ if $g(u_1, u_2)$ is an increasing function for both random variables, and for $u_1, u_2 \geq u$ with $u = \Psi_1(z)$.

For the lower tails contagion in intensity is characterized by **decreasing tail monotonicity** in

$$P\{Z_1 \leq z_1, Z_2 \leq z_2\} - P\{Z_1 \leq z_1\}P\{Z_2 \leq z_2\}.$$

In terms of copulas contagion in the upper tails amounts to

$$h_1(u_1, u_2) = \frac{\delta \tilde{C}_G(u_1, u_2)}{du_1} - u_2 > 0, \quad h_2(u_1, u_2) = \frac{\delta \tilde{C}_G(u_1, u_2)}{du_2} - u_1 > 0.$$
Directional Contagion: Causality in the Extremes

The conditional probability is interpreted as a causality relationship.

Let $z$ be a threshold determining the extremes for both random variables.

**Motivation:** $P\{Z_2 > z'|Z_1 > z\} > P\{Z_2 > z'\}$ $\equiv Z_1 \Rightarrow Z_2$, with $z' > z$.

($Z_1$ taking on extreme values is causing that $Z_2$ takes on extreme values).

However, This is not true!

**False Intuition:**

$$P\{Z_2 > z'|Z_1 > z\} > P\{Z_2 > z'\} \equiv P\{Z_2 > z', Z_1 > z\} > P\{Z_2 > z'\}P\{Z_1 > z\}$$

This condition determines Contagion in Intensity NOT in the direction (No causality).
Assuming a common marginal \textit{d.f.} \(\Psi_1\), and a threshold \(z\) determining the extremes for both random variables, we find contagion spill-over from \(Z_1\) to \(Z_2\) if

\[
P\{Z_2 > z' | Z_1 > z\} > P\{Z_1 > z' | Z_2 > z\},
\]

or equivalently if

\[
P\{Z_2 \leq z' | Z_1 \leq z\} > P\{Z_1 \leq z' | Z_2 \leq z\}.
\]

These conditions boil down to see

\[
\tilde{C}_G(u, v) > \tilde{C}_G(v, u) \quad \text{for} \quad Z_1 \Rightarrow Z_2 \quad (\text{Causality in the extremes}),
\]

with \(u = \Psi_1(z)\), and \(v = \Psi_1(z')\).

Define \(gd_v(u) = \tilde{C}_G(u, v) - \tilde{C}_G(v, u)\). Then

\textbf{Definition 5.} \(Z_1\) is influencing \(Z_2\) in the extreme values (contagion effect) if \(gd_v(u)\) is strictly positive for all \(v > u\) for the upper tail, and for all \(v < u\) for the lower tail, with \(u = \Psi_1(z)\).
Intuition

$\tilde{Z}_G(y,v) > \tilde{Z}_G(v,u) \Rightarrow Z_1 \neq Z_2$

(In the upper tail)

$\tilde{Z}_G(u,v) > \tilde{Z}_G(v,u) \Rightarrow Z_1 \Rightarrow Z_2$

(In the lower tail)
Application: Flight to quality versus Contagion

Definition 6. Outflows of capital from the stock markets ($Z_2$) to the bonds markets ($Z_1$) in crises periods.

This is represented by

\[ P\{Z_1 > z | Z_2 < z'\} - P\{Z_1 > z\} > 0, \]

with $z$ defining the extreme values in the upper tail, and $z'$ in the lower tail.

Experiment: Dow Jones Corporate 02 Years Bond Index (DJBI02) vs Dow Jones Industrial Average: Dow 30 Industrial Stock Price Index (DJSI).

General Model:

\[
\begin{align*}
X_{1,t} &= g_1(X_{1,t-1}, X_{2,t-1}) + \varepsilon_{1,t} \\
X_{2,t} &= g_2(X_{1,t-1}, X_{2,t-1}) + \varepsilon_{2,t}
\end{align*}
\]

with $(\varepsilon_{1,t}, \varepsilon_{2,t}) \sim \tilde{C}_G$.

Financial Sequence: $X_{i,t} = 100 (logP_{i,t} - logP_{i,t-1})$, $i = 1, 2$, with $P_{i,t}$ the corresponding prices.
Modelling Rational Dependence

*DJBI02* index is well modelled by an AR(1)-GARCH(1,1) model as follows,

\[ X_{1,t} = 0.00025 + 0.089X_{1,t-1} + \sigma_{1,t}\varepsilon_{1,t}, \text{ with } \varepsilon_{1,t} \text{ i.i.d. } (0,1), \]

and \( \sigma_{1,t}^2 = 6.194 \cdot 10^{-8} + 0.071\varepsilon_{1,t-1}^2 + 0.903\sigma_{1,t-1}^2. \)

*DJSI* Index is modelled by the following pure GARCH(1,1) model,

\[ X_{2,t} = \sigma_{2,t}\varepsilon_{2,t}, \text{ with } \varepsilon_{2,t} \text{ i.i.d. } (0,1), \]

and \( \sigma_{2,t}^2 = 3.0012 \cdot 10^{-6} + 0.096\varepsilon_{2,t-1}^2 + 0.887\sigma_{2,t-1}^2. \)

The evolution of prices in one market is independent of the other.

The irrational dependence (dependence in the innovations) is measured by the links between the vectors \((\varepsilon_{1,t}, \varepsilon_{2,t})\) and \(\tilde{C}_G.\)

**Estimate of \(\tilde{C}_G: \)** \(\hat{\theta}_n = 1.031, \hat{\eta}_n = 1\) and \(\hat{\gamma}_n = 0.175. \) (\(\downarrow\))
IGARCH Effect

Consider

\[ X_t = \sigma_t \varepsilon_t, \text{ with } \varepsilon_t \ i.i.d. \ (0, 1), \]

and

\[ \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \]

with \( \alpha + \beta = 1. \)

Features of the model:

• \( V(X_t) = \infty. \)

• In the same way that \( I(1) \) represents \textbf{persistence} in linear models, \textit{IGARCH}(1,1) describes \textbf{persistence} in the square and absolute observations.

• Persistence, \textbf{NOT Long Range Dependence}, because the latter implies finite marginal variances.
However, the IGARCH effect may show up by (Mikosch and Stărică):

- Persistence in the squares (true IGARCH).

- Non-stationarity due to different regimes (different means, different stationary GARCH, etc.)

Regarding Contagion:

- **For true IGARCH**: Study the contagion effect for the vector of innovations \((\varepsilon_1, \varepsilon_2)\) obtained from the IGARCH model.

- **Non-stationarity**: Consider the univariate sequence \(\{X_t\}\) and filter it by the corresponding regimes to obtain a sequence of innovations \(\varepsilon_t\) that is \(I(0)\) and serially independent.
Modelling Irrational Dependence

Cross correlation for different lags of the bivariate innovation sequence, spanning the period 02/01/1997 – 24/09/2004, $n = 1942$ observations.
Empirical (o--) and theoretical (+--) margins. The upper panel for the vertical sections and the lower panel for the horizontal section. 0.05 quantile, 0.50 quantile and 0.95 quantile respectively.
The upper panel depicts the function $g(u, v)$ plotted against the innovations of $DJSI$. The lower panel $g(u, v)$ plotted against the innovations of $DJBI02$. 
The upper panel depicts $h_1(u, v)$ against $DJBI02$ and the lower panel depicts $h_2(u, v)$ against $DJSI$. (+−) for 0.05 quantile, (o−) the 0.50 quantile and (⋄−) the 0.95 quantile.
The upper panel depicts $gd_v(u)$ for $v \leq u$. $(+-)$ for $u = 0.50$, $(o-)$ for $u = 0.30$ and $(\diamond -$) for $u = 0.10$. The lower panel depicts $gd_v(u)$ for $v > u$. $(+-)$ for the $u = 0.50$, $(o-)$ for $u = 0.70$ and $(\diamond -)$ for $u = 0.90$. 

4th Conference on Extreme Value Analysis
Flight to Quality: \[ P\{Z_1 > u \mid Z_2 < v \} - P\{Z_1 > u \} > 0 \]

In the upper panel (+−) for \( u = 0.60 \), (o−) for \( u = 0.80 \) and (⋄−) for \( u = 0.95 \). In the lower panel (+−) for \( v = 0.60 \), (o−) for \( v = 0.80 \) and (⋄−) for \( v = 0.95 \).
Some Interesting Facts

- Negative interdependence in the left tail, that turns positive in the right tail.

- Absence of directional contagion (Symmetric effects between the variables).

- Strong opposite movements in the middle of the domain (negative interdependence) that decrease when the variables take on extreme values. (Intensity Contagion without Interdependence).

- Evidence of Flight to Quality in both tails. This phenomenon can be interpreted as a substitution effect between bonds ($DJBI02$) and stocks ($DJSI$) when either of the sequences are in crises periods.

- $DJBI02$ depends on its past and in the volatility dynamics.

- $DJSI$ depends only on its volatility dynamics.