Log-Scaling rainfall data: effects on GPD Bayesian goodness of fit.

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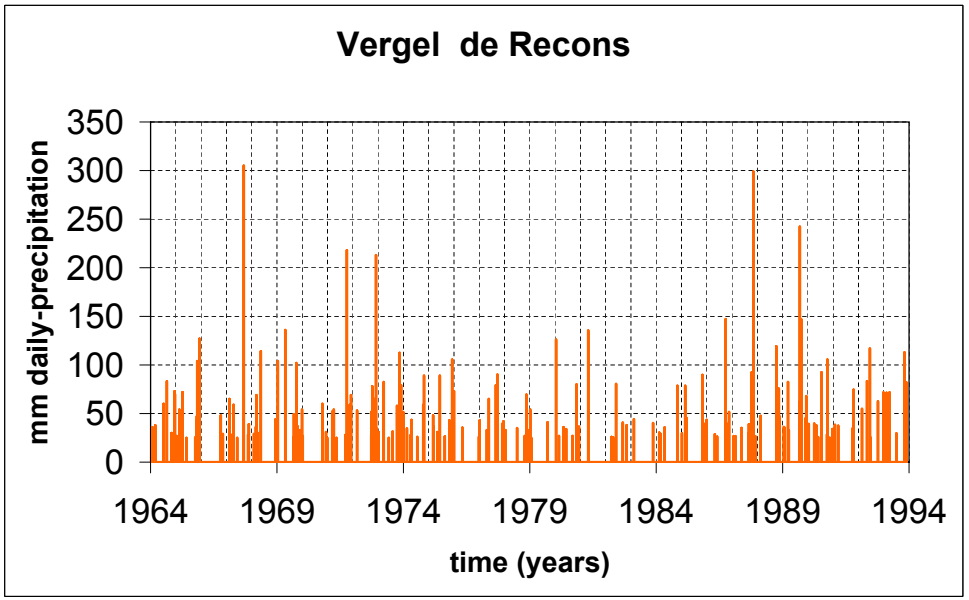
Outline

Vergel de Racons data.



Main goals:

- Finding suitable model.
- Hazard analysis.
- Occurrence probabilities; return periods.

For reference, see Romero et al. (1998), [4]

The model

- Scale of the reference variable, precipitation:
  - ◇ is a positive variable:
    (0 mm rainfall is not rainfall!)
  - ◇ has a relative scale:
    50 mm is double than 25mm daily rainfall, but
    500mm and 525 mm daily rainfall is nearly the same!

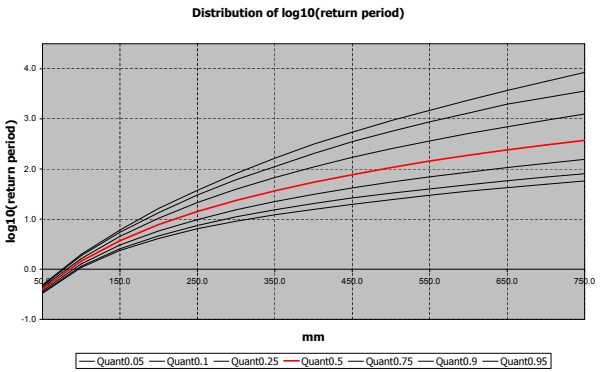
Logarithmic scale is needed!!!

The model

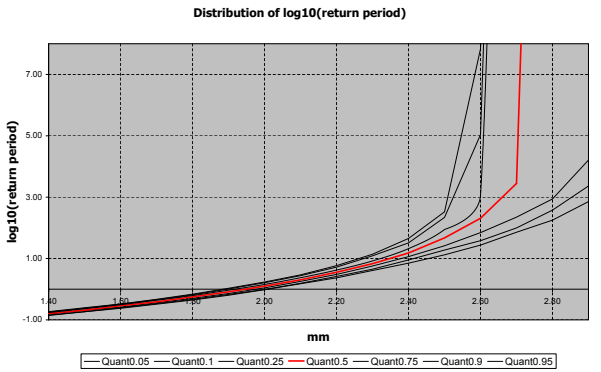
- Occurrence: Cramér-Lundberg model (Homogeneous Poisson process with intensity parameter λ).
- Magnitude: Excesses over threshold described by a Generalized Pareto Distribution (GPD).
- Bayesian parameter estimation.

Is it a suitable model?

- Hazard Estimates: At high levels, great uncertainty of estimates due to scarcity of data.
- Estimates of typical hazard parameters (e.g. Return period) vary *dramatically* depending on the selected model:

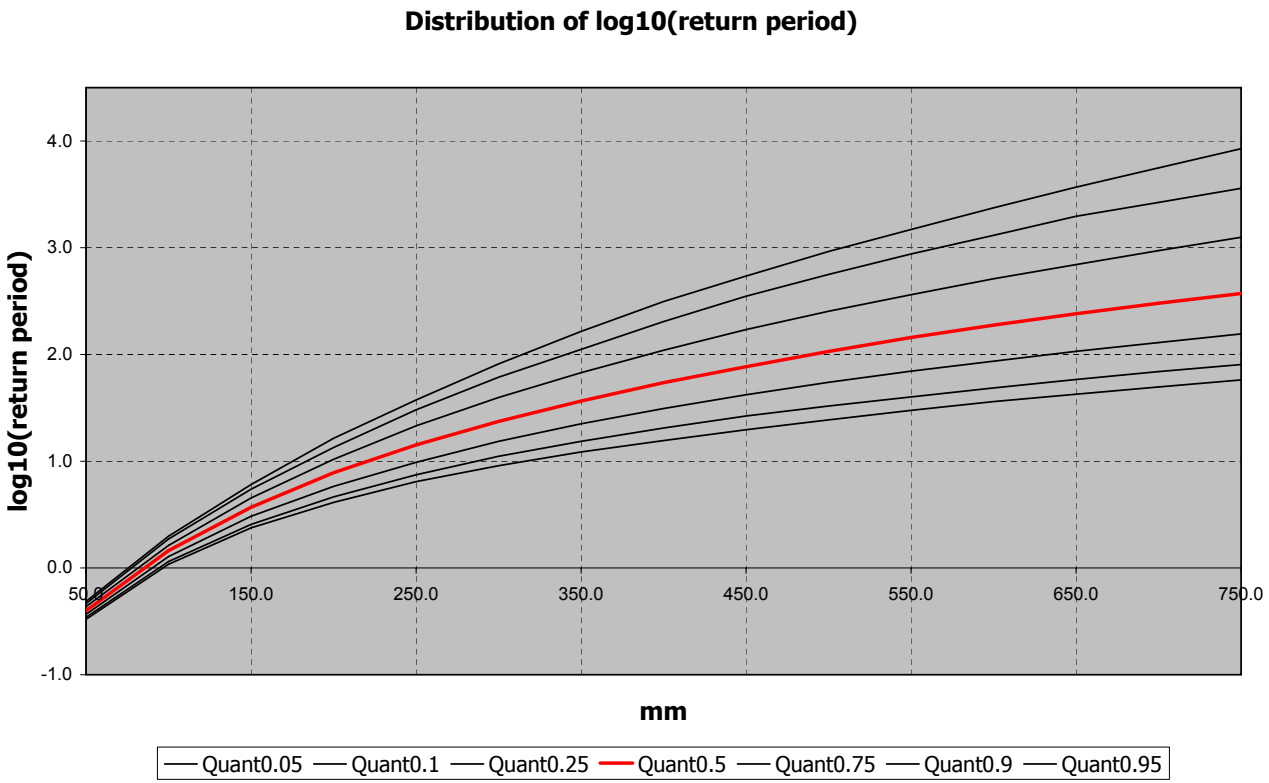


Return period of raw data

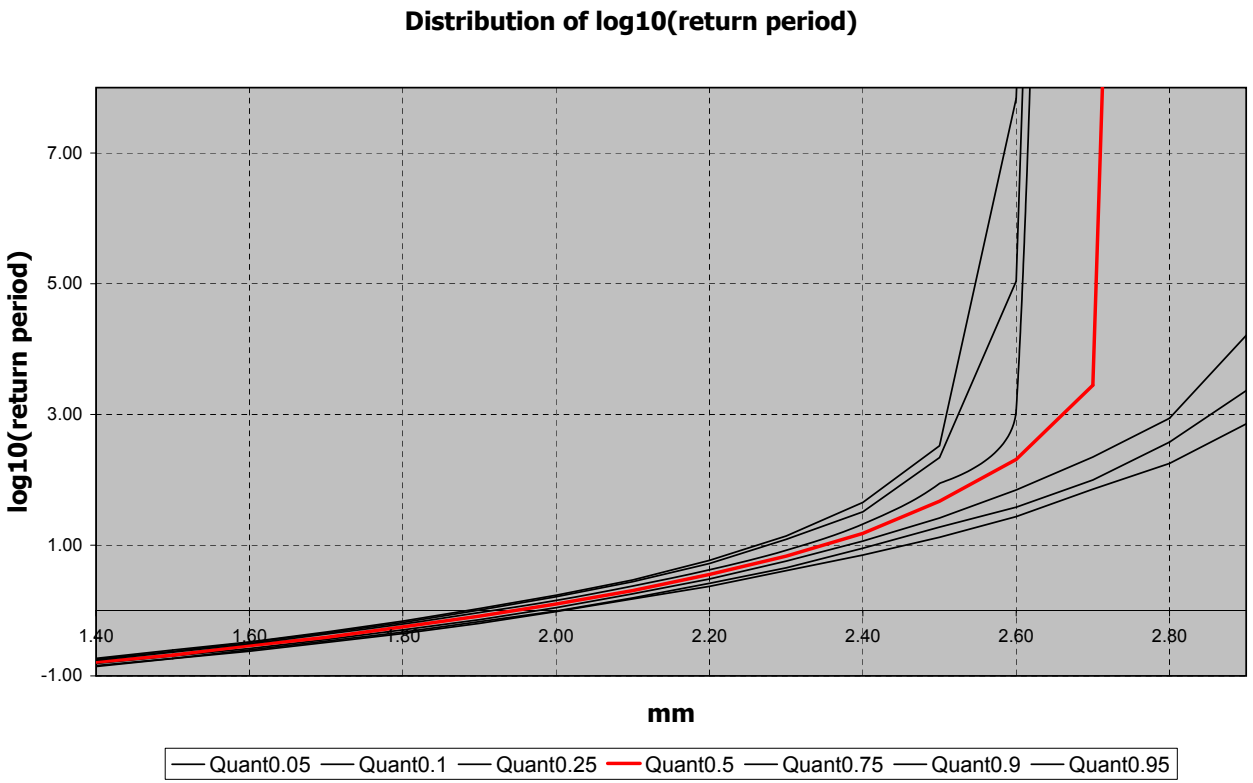


Return period of log data

Return period of raw data



Return period of log data



p-values

Whole model checking

(prior +likelihood +GPD) For reference, see Gelman et al. 1995, 1996, [6]
Several ways of checking it.

Goodness-of-fit checking

$GPD(\xi, \beta)$ goodness-of-fit assessing.
Several ways of checking it

p-values

A first approach:

plug-in-p-value (p_{plug})

$$p_{plug} = \mathrm{P}^{gpd(\cdot|\widehat{\xi},\widehat{\beta})}[t(\mathbf{X}) \geq t(\mathbf{x}_{obs})] \ ,$$

$gpd(\mathbf{x}|\xi,\beta)$ is replaced by $gpd(\cdot|\widehat{\xi},\widehat{\beta})$, where $\widehat{\xi},\widehat{\beta}$ is the maximum likelihood estimate of the parameters.

p-values

Bayesian *p*-values:

posterior predictive p-value (p_{post}) Guttman (1967) and Rubin (1984)

$$p_{post} = \mathrm{P}^{m_{post}(\cdot|\mathbf{x}_{obs})}[t(\mathbf{X}) \geq t(\mathbf{x}_{obs})] \ ,$$

where $m_{post}(\mathbf{x}|\mathbf{x}_{obs})$ is the *posterior predictive distribution*,

$$m_{post}(\mathbf{x}|\mathbf{x}_{obs}) = \int gpd(\mathbf{x}|\xi,\beta)\pi(\xi,\beta|\mathbf{x_{obs}})d(\xi,\beta) \ ,$$

and $\pi(\xi,\beta|\mathbf{x_{obs}})$ is the *posterior density* for ξ,β.

p-values

discrepancy p-value(p_{dis}) (Gelman et al., 1995)

The test statistic $t(\mathbf{X})$ is replaced by a discrepancy $t(\mathbf{X}, \xi, \beta)$

$$p_{dis} = \mathrm{P}^{m_{dis}(\cdot)}[t(\mathbf{X}, \xi, \beta) \geq t(\mathbf{x}_{obs}, \xi, \beta)] \ ,$$

where $m_{dis}(\mathbf{x}, \xi, \beta | \mathbf{x}_{obs})$ is ,

$$m_{dis}(\mathbf{x}, \xi, \beta | \mathbf{x}_{obs}) = gpd(\mathbf{x} | \xi, \beta) \pi(\xi, \beta | \mathbf{x_{obs}}) \ ,$$

and $\pi(\xi, \beta | \mathbf{x_{obs}})$ is the *posterior density* for ξ, β.

p-values: pros and cons

Desirable characteristics:

- Uniform distribution.
- Easy to compute.

Other useful characteristics:

- Known distribution of used statistic (even asymptotically).
- Easiness of interpretation.

Pros and cons:

- *plug-in p-value*: Easy to compute. Uncertainty ignored.
- *posterior predictive p-value* is not uniform. Easy to compute.
- *discrepancy p-value* is not uniform.

Bayesian GP Estimation (BGPE)

- Three parameters to estimate in the model:
  Poisson rate, λ, of $Poisson(\lambda)$ and ξ, β of the magnitude, modelled by $GPD(\xi, \beta)$:

$$GPD_X(x|\xi,\beta) = 1 - \left(1 + \frac{\xi}{\beta}x\right)^{\frac{-1}{\xi}}$$

- A suitable joint prior distribution for λ, ξ, β is set.
  Prior distributions for λ and ξ, β are independent $\rightarrow$ the joint prior factorizes:

$$\pi_{\lambda,\xi,\beta}(\lambda,\xi,\beta) = \pi_{\lambda}(\lambda) \cdot \pi_{\xi,\beta}(\xi,\beta)$$

- The joint likelihood of parameters, $L(\lambda, \xi, \beta | x_{obs})$, splits into two terms:

$$L(\lambda, \xi, \beta | x_{obs}) = L(\lambda | x_{obs}) \cdot L(\xi, \beta | x_{obs})$$

- Finally, the Posterior distribution of λ, ξ, β, $\pi_{\lambda,\xi,\beta}(\lambda, \xi, \beta | x_{obs})$, is obtained:

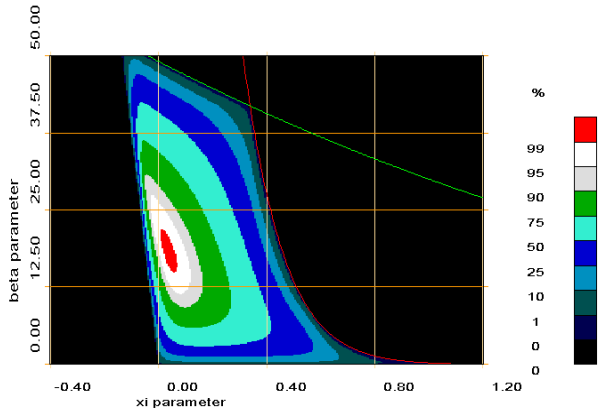
$$\pi_{\lambda,\xi,\beta}(\lambda, \xi, \beta | x_{obs}) = L(\lambda, \xi, \beta | x_{obs}) \cdot \pi_{\lambda}(\lambda) \cdot \pi_{\xi,\beta}(\xi, \beta)$$

Attention is set to marginal posterior distribution of ξ, β:

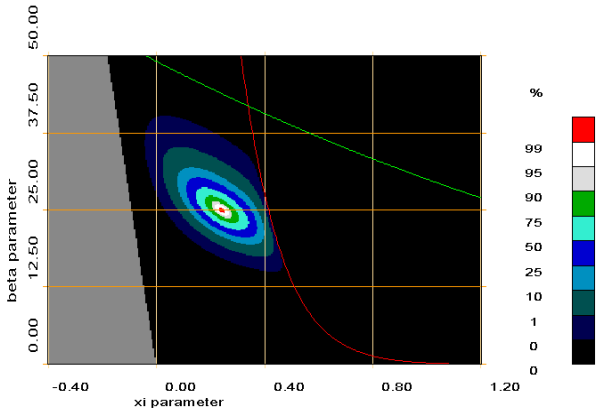
$$\pi_{\xi,\beta}(\xi, \beta | x_{obs}) = L(\xi, \beta | x_{obs}) \cdot \pi_{\xi,\beta}(\xi, \beta)$$

For reference, see Egozcue and Ramis (2001), [1], and Egozcue and Tolosana (2002), [2] .

Prior and posterior distributions : Raw data (I)



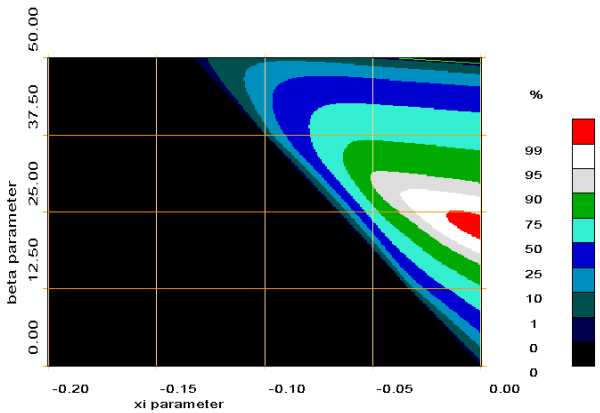
Prior density



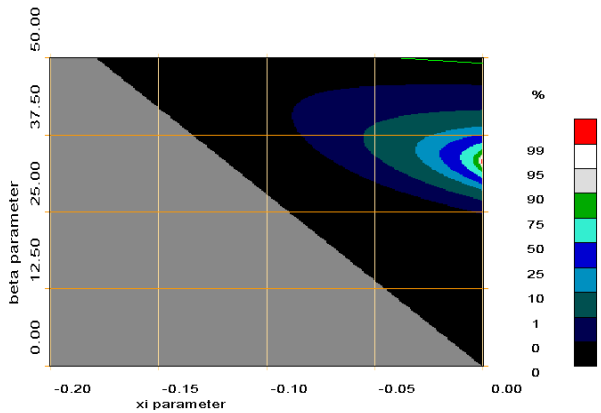
Posterior density

Something is lost!!

Prior and posterior distributions : Raw data (II)

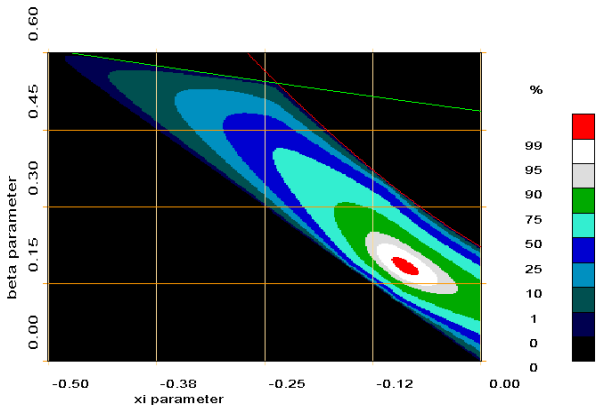


Prior density, $\xi < 0$

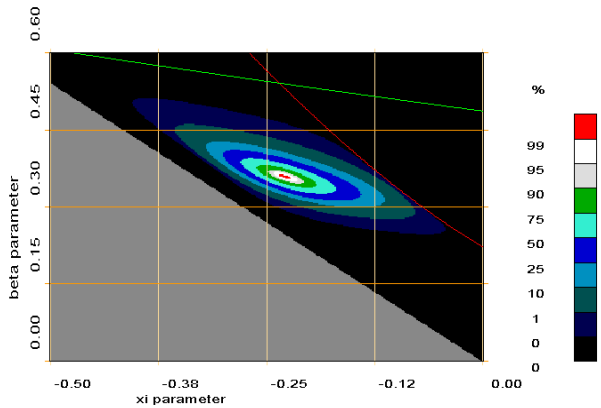


Posterior density, $\xi < 0$

Prior and posterior distributions: log data



Prior density



Posterior density

p-values: Our alternative

First approach

$$pp = \sum_i \psi_i p_i, \quad \textit{predictive KS } p - \textit{value} \text{ , where}$$

$p_i = KSGOF(\xi_i, \beta_i)$, for fixed (ξ_i, β_i) and $\psi_i = \pi(\xi_i, \beta_i | \mathbf{x_{obs}})$

Our alternative

$$pp = \Phi\left(\frac{\sum_{i=1}^{n} \psi_i \ \Phi^{-1}(p_i)}{\sqrt{\sum_i \psi_i^{\delta}}}\right) \ , \quad 1 \leq \delta \leq 2 \ , \delta \simeq 1 \ .$$

$p_i = KSGOF(\xi_i, \beta_i)$, for fixed (ξ_i, β_i); $\psi_i = \pi(\xi_i, \beta_i | \mathbf{x_{obs}})$

GPD Goodness-of-fit

	RAW DATA	LOG DATA
K-S posterior predictive Gof	$4.984 * 10^{-2}$	0.7028
K-S discrepancy *p*-value	$4.781 * 10^{-2}$	0.7549
Our approach	$\lesssim 2.075 * 10^{-2}$	$\gtrsim 0.9605$

(other statistics can be used)

Whole model assessing

Slope discrepancy

Estimation of the slope of the expected excesses regression line

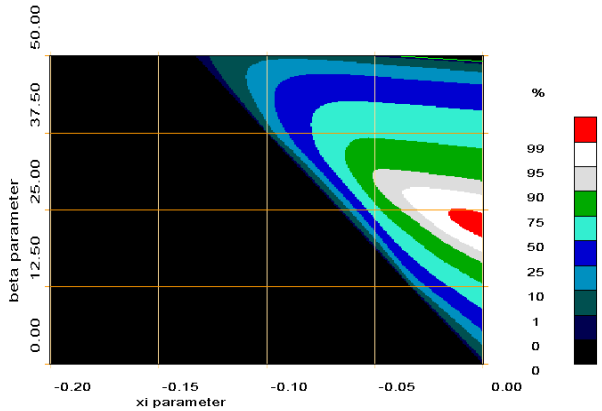
$$\mathrm{E}[X - u | X > u; \xi, \beta] = \frac{\beta + \xi u}{1 - \xi} \quad ,$$

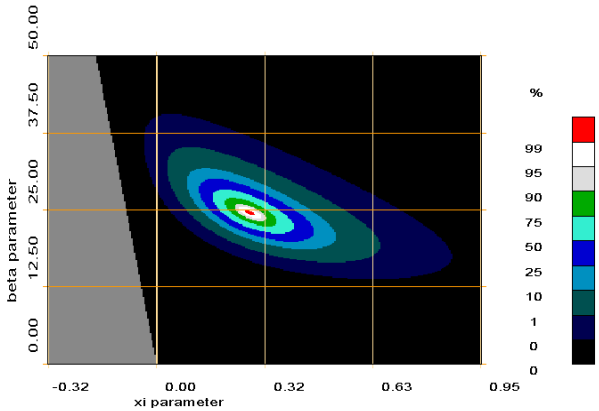
an estimator of ξ, $\xi < 1$.

Results:

	RAW DATA	LOG DATA
Slope discrepancy p-value	$8.765 * 10^{-2}$	0.4902

Priori vs. likelihood: raw data





Prior density, $\xi < 0$

Raw data likelihood

For raw data, likelihood is out of priori domain!

Conclussions

- Model fits better log-scaled data (i.e. Weibull d.a. prior, GPD)
  Checked by:
  - predictive goodness of fit ;
  - discrepancy bayesian *p*-value.
- Are Gumbel/Frechet d.a. admisible models for natural (finite) phenomena?

For Further Reading I

Egozcue , J.J. and Ramis, C.
Bayesian hazard analysis of heavy precipitation in Eastern Spain.
Int. J. Climatol., **21**: 1263-1279,2001.

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For Further Reading II


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