This paper discusses goodness-of-fit tests for linear covariance stationary processes based on the empirical spectral distribution function. We show that the limits of the tests are functionals of a Gaussian process, say, $\tilde{B}(\vartheta)$ with $\vartheta \in [0, 1]$. Since, in general, it is not easy, if at all possible, to find a time deformation $g(\vartheta)$ such that $\tilde{B}(g(\vartheta))$ is a Brownian (bridge) process, tests based on $\tilde{B}(\vartheta)$ will have limited value for the purpose of statistical inference. To circumvent the mentioned problem, we propose to bootstrap the test showing its validity. We also provide a Monte-Carlo experiment to examine the finite sample behaviour of the bootstrap.