Munkres §20

Ex. 20.5. Consider $\mathbb{R}^\omega$ with the uniform topology and let $d$ be the uniform metric. Let $C \subset \mathbb{R}^\omega$ be the set of sequences that converge to 0. Then

$$\mathbb{R}^\omega = C.$$  

⊂: Since clearly $\mathbb{R}^\omega \subset C$ it is enough to show that $C$ is closed. Let $(x_n) \in \mathbb{R}^\omega - C$ be a sequence that does not converge to 0. This means that there is some $1 > \varepsilon > 0$ such that $|x_n| > \varepsilon$ for infinitely many $n$. Then $B_d((x_n), \frac{1}{2}\varepsilon) \subset \mathbb{R}^\omega - C$.

⊃: Let $(x_n) \in C$. For any $1 > \varepsilon > 0$ we have $|x_n| < \varepsilon$ for all but finitely many $n$. Thus $B_d((x_n), 2\varepsilon) \cap \mathbb{R}^\omega \neq \emptyset$.

References