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Completely Reducible p-compact Groups

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ABSTRACT. Rational automorphisms of products of simple p-compact groups are shown to be composites of products of rational automorphisms of the individual factors and permutation maps.

1. Introduction

Homotopy Lie groups, or p-compact groups, have come under intense scrutiny since their inaugural appearance in Dwyer and Wilkerson's foundational paper [**4**] and this note may be viewed as yet another piece of evidence in support of the prophecy of an "uncanny similarity" between compact Lie groups and p-compact groups.

The purpose of this note is to show that rational automorphisms of products of p-compact groups behave in a very rigid way.

A rational automorphism of a connected p-compact group Y is an endomorphism f of Y, i.e. a based self-map Bf of BY, inducing an automorphism $H^*(Bf;\mathbb{Z}_p)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p$ of the polynomial ring $H^*(BY;\mathbb{Z}_p)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p$. Conjugacy classes of rational automorphisms form a monoid $\varepsilon_{\mathbb{Q}}(Y)\subseteq[BY,BY]$.

As an illustration of the main result [Theorem 3.5], suppose that Y_1 and Y_2 are two *simple* [**8**, Definition 5] p-compact groups with nonisomorphic, nontrivial Weyl groups. Then the product map

$$\varepsilon_{\mathbb{Q}}(Y_1)\wr\Sigma_{n_1}\times\varepsilon_{\mathbb{Q}}(Y_2)\wr\Sigma_{n_2}\to\varepsilon_{\mathbb{Q}}(Y_1^{n_1}\times Y_2^{n_2})$$

is monoid isomorphism for any choice of exponents $n_1,n_2\geq 1$. In other words, any rational automorphism f of $Y=Y_1^{n_1}\times Y_2^{n_2}$ is, up to permutation of the n_1 factors equal to Y_1 and the n_2 factors equal to Y_2, conjugate to a product

$$f=\prod_{1\leq j\leq n_1}f_{1j}\times\prod_{1\leq j\leq n_2}f_{2j}$$

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of rational automorphisms f_{1j}, $1 \leq j \leq n_1$, of Y_1 and rational automorphisms f_{2j}, $1 \leq j \leq n_2$, of Y_2. Moreover, the corresponding product map of mapping spaces

$$\prod_{1\leq j\leq n_1} \operatorname{map}(BY_1, BY_1)_{Bf_{1j}} \times \prod_{1\leq j\leq n_2} \operatorname{map}(BY_2, BY_2)_{Bf_{2j}} \to \operatorname{map}(BY, BY)_{Bf}$$

is a homotopy equivalence. It follows [Corollary 3.9] that the precomposition map

$$\overline{Bf}\colon \operatorname{map}(BY, BY)_{B1} \to \operatorname{map}(BY, BY)_{Bf}$$

is a homotopy equivalence for each of the precomposition maps $\overline{Bf_{1j}}$ and $\overline{Bf_{2j}}$, involving simple p-compact groups, are known to be homotopy equivalences by an earlier result [**8**, Corollary 4.7]. Combined with Dwyer and Wilkerson's demonstration that the centralizer [**4**, 3.4] of the identity endomorphism is a p-compact group isomorphic [**10**, Theorem 1.3] to the center, we see that the centralizer $C_Y(fY)$ of any rational automorphism f of Y is a p-compact group isomorphic to the center [**10, 3**] $Z(Y)$ of Y.

The completely reducible p-compact groups of the title [Definition 3.10] constitute a class of connected p-compact groups to which the above computation of centralizers of rational automorphisms is extendable [Theorem 3.11] by covering group methods. The size of this class of p-compact groups has not yet been investigated.

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2. Representation theory

This section contains some preliminary remarks about representations of finite groups designed for use in the following sections.

Let R be an integral domain, k its field of fractions, and let E, E_1 and E_2 be R-modules. Suppose that the finite groups W, W_1 and W_2 act faithfully on the modules E, E_1 and E_2, respectively.

Definition 2.1. *The W_1-representation E_1 is* similar *to the W_2-representation E_2 if the there exists an R-module isomorphism $A\colon E_1 \to E_2$ and a group isomorphism $\alpha\colon W_1 \to W_2$ such that $A \circ w = \alpha(w) \circ A$ for all $w \in W_1$. The isomorphism A is called a* similarity *from E_1 to E_2.*

Let $\operatorname{Sim}(W) \subset \operatorname{Aut}_R(E)$ denote the group of self-similarities of the W-representation E. In other words, $\operatorname{Sim}(W)$ is the normalizer of the image of W in $\operatorname{Aut}_R(E)$. Note the exact sequence

$$1 \to \operatorname{Aut}_{R[W]}(E) \to \operatorname{Sim}(W) \to \operatorname{Aut}(W)$$

and that if the representations $W_1 \to \operatorname{Aut}_R(E_1)$ and $W_2 \to \operatorname{Aut}_R(W_2)$ are similar, conjugation with a similarity $A\colon E_1 \to E_2$ induces an isomorphism $\operatorname{Sim}(W_1) \cong \operatorname{Sim}(W_2)$ of self-similarity groups.

For any ring extension $R \subseteq S$, let $W \otimes_R S$ denote the induced representation of W in $E \otimes_R S$. In particular, $W \otimes_R k$ denotes the vector space representation of W in $E \otimes_R k$. An element $w \in W$ is called a *reflection* if the vector space endomorphism $1 - w \otimes_R k$ has rank 1 [**1**, V, §2, no 1, Definition 1].

Example 2.2. Let W_B denote the Weyl group of the Lie group $\mathrm{Spin}(2n+1)$ faithfully represented in the $\mathbb{Z}$-module E_B given by the fundamental group of a maximal torus. Similarly, let W_C denote the Weyl group of $\mathrm{Sp}(n)$ faithfully represented in the $\mathbb{Z}$-module given by the fundamental group E_C of a maximal torus in $\mathrm{Sp}(n)$, $n > 2$.

Assume that the representations $W_B \otimes R$ and $W_C \otimes R$ are similar, i.e. that there exists an R-isomorphism $A\colon E_B \otimes R \to E_C \otimes R$ which is α-equivariant for some group automorphism $\alpha\colon W_B \to W_C$.

Let $\beta_1, \ldots, \beta_n$ be a basis for the root system (of type B_n) for $\mathrm{Spin}(2n+1)$ and σ_i^B the reflection of E_B corresponding to β_i, i.e. $\sigma_i^B(x) = x - \beta_i(x)\beta_i^\vee$ where $\beta_i^\vee \in E_B$ is the inverse root to β_i.

Note that, for any i, $A \circ \sigma_i^B \circ A^{-1}$ is a reflection of $E_C \otimes k$ so that [**1**, VI, §1, no 1, Remarque 3] $A \circ \sigma_i^B = \sigma_i^C \circ A$ for some reflection $\sigma_i^C \in W_C$. Let γ_i be the root and $\gamma_i^\vee$ the inverse root corresponding to the reflection σ_i^C. The vectors $A(\beta_i^\vee)$ and $\gamma_i^\vee$ are proportional; write $A(\beta_i^\vee) = \lambda_i \gamma_i^\vee$ with $\lambda_i \in k$. Since $(\beta_1^\vee, \ldots, \beta_n^\vee)$ is a $\mathbb{Z}$-basis for E_B [**2**, IX, §4, no 6, Proposition 11] and A is an R-automorphism, $(\lambda_1\gamma_1^\vee, \ldots, \lambda_n\gamma_n^\vee)$ is an R-basis for $E_C \otimes R$ and the linearly independent set $(\gamma_1^\vee, \ldots, \gamma_n^\vee)$ is a $\mathbb{Q}$-basis for $E_C \otimes \mathbb{Q}$ such that the corresponding set of reflections $\{\sigma_1^C, \cdots, \sigma_n^C\}$ generates [**1**, VI, §1, no 5, Remarque 1] the Weyl group W_C. This implies (since not all $\gamma_i^\vee$ can have the same length) that $(\gamma_1^\vee, \ldots, \gamma_n^\vee)$ is a basis for the inverse root system of $\mathrm{Sp}(n)$, hence a $\mathbb{Z}$-basis for E_C. As $A\colon E_B \otimes R \to E_C \otimes R$ is surjective, each coefficient λ_i must be a unit in the ring R.

The relation $A\sigma_i^B(\beta_j^\vee) = \sigma_i^C A(\beta_j^\vee)$ is equivalent to $\lambda_i\beta_i(\beta_j^\vee) = \lambda_j\gamma_i(\gamma_j^\vee)$ for all i and j. It follows that $\beta_i(\beta_j^\vee)\beta_j(\beta_i^\vee) = \gamma_i(\gamma_j^\vee)\gamma_j(\gamma_i^\vee)$, i.e. that the bijection $\beta_i \to \gamma_i$ of bases determines an isomorphism of the associated Coxeter graphs. Thus we may arrange the roots so that $\beta_{n-1}(\beta_n^\vee) = -1$ and $\gamma_{n-1}(\gamma_n^\vee) = -2$ leading to the relation $\lambda_{n-1} = 2\lambda_n$. Since both λ_{n-1} and λ_n are units in R, also 2 is a unit in R.

Conversely, if 2 is invertible in R, it is easy to write down a similarity between $W_B \otimes R$ and $W_C \otimes R$.

Let now S_i, $i \in I$, be a finite family of free R-modules of finite rank and W_i, $i \in I$, finite, nontrivial groups with W_i acting faithfully on S_i such that the corresponding vector space representation $W_i \otimes_R k$ is irreducible.

Let the product group $\prod_{i \in I} W_i$ act faithfully on $\prod_{i \in I} S_i$ in the obvious way.

The first lemma of this section computes the endomorphism ring of $\prod S_i$ as an $R[\prod W_i]$-module.

LEMMA 2.3. *Suppose that R is a principal ideal domain and that each group W_i contains a reflection of S_i. Then the obvious homomorphism*

$$\prod_{i\in I} R \to \mathrm{End}_{R[\prod W_i]}\left(\prod S_i\right)$$

is a ring isomorphism.

PROOF. Let A be an $R[\prod W_i]$-endomorphism of $\prod S_i$. Since the $\prod W_i$-representations $S_i \otimes_R k$, $i \in I$, are irreducible and pairwise nonisomorphic, $A = \prod A_i$ for some $A_i \in \mathrm{End}_{R[W_i]}(S_i)$ by Schur's lemma.

Let $\sigma_i \in W_i$ be a reflection of S_i. As the image $\mathrm{im}(1-\sigma_i)$ is a free R-module of rank 1 which is invariant under A_i, there exists a scalar $\lambda_i \in R$ such that $\mathrm{im}(1-\sigma_i) \subseteq \ker(A_i - \lambda_i)$ in S_i. Then also $\ker(A_i - \lambda_i) \neq 0$ in the irreducible W_i-representation $S_i \otimes_R k$; hence A_i is multiplication by λ_i on $S_i \otimes_R k$ and on S_i. □

The next two lemmas are concerned with groups of self-similarities of faithful representations.

Now pick a set of exponents $n_i \geq 1$, $i \in I$. The group $W := \prod_{i\in I} W_i^{n_i}$ is faithfully represented in the R-module $S := \prod_{i\in I} S_i^{n_i}$. Let $\Sigma_{n_i} < \mathrm{Sim}(W_i^{n_i}) < \mathrm{Sim}(W)$ denote the subgroup consisting of permutations of the n_i-factors of S_i in the product $S_i^{n_i}$ or in $S = \prod S_i^{n_i}$.

The proof of the following key lemma was kindly supplied by K. Uno.

LEMMA 2.4. *Assume that the W_i-representations S_i, $i \in I$, are pairwise non-similar. Then the canonical homomorphism*

$$\prod_{i\in I} \mathrm{Sim}(W_i) \wr \Sigma_{n_i} \to \mathrm{Sim}(W)$$

is a group isomorphism.

PROOF. Write $W = \prod_i W_i^{n_i} = \prod_{i,j} W_{ij}$ and $S = \prod_i S_i^{n_i} = \prod_{i,j} S_{ij}$ where $W_{ij} = W_i$ and $S_{ij} = S_i$ for $1 \leq j \leq n_i$.

Let A be an R-automorphism of S and α a group automorphism of W such that $A \circ w = \alpha(w) \circ A$ for all $w \in W$. Note that the submodule AS_{ij} of S is W-invariant as

$$\alpha(w)AS_{ij} = AwS_{ij} = AS_{ij}$$

for all $w \in W$. Let now v be a nonzero element of AS_{ij}. Write v on the form $v = \sum_{i,j} v_{ij}$ with $v_{ij} \in S_{ij}$. Pick k and l such that v_{kl} is nonzero. Since AS_{ij} is invariant under the action of the subgroup $W_{kl} < W$,

$$\sum_{(i,j)\neq(k,l)} v_{ij} + W_{kl}v_{kl} \subseteq AS_{ij}$$

and hence also

$$W_{kl}v_{kl} - W_{kl}v_{kl} \subseteq AS_{ij}.$$

The left hand side generates a nontrivial W_{kl}-invariant submodule of S_{kl}, so

$$S_{kl} \otimes_R k \subseteq A(S_{ij} \otimes_R k)$$

by irreducibility of $S_{kl} \otimes_R k$. In fact, since A is an automorphism of $S \otimes_R k$, $S_{\sigma(i,j)} \otimes_R k = A(S_{ij} \otimes_R k)$ for some permutation σ of the index set. Then also $S_{\sigma(i,j)} = AS_{ij}$ since A is an automorphism of $S = \prod S_{ij}$ and each S_{ij} is a free R-module.

If $w \in W_{ij}$, $\alpha(w) \in W_{\sigma(i,j)}$ for $\alpha(w) = AwA^{-1}$ fixes pointwise $\prod_{(u,v)\neq\sigma(i,j)} S_{uv}$. In fact, $\alpha(W_{ij}) = W_{\sigma(i,j)}$ since α is an automorphism of $W = \prod W_{ij}$. Thus A restricts to a similarity from S_{ij} to $S_{\sigma(i,j)}$ and therefore, by the nonsimilarity assumption, $\sigma(i,j) = (i, \sigma_i(j))$ for some permutation $\sigma_i \in \Sigma_{n_i}$.

We conclude that $A = \prod_{i\in I}(\prod_j A_{ij}) \circ \sigma_i$ where $A_{ij} \in \mathrm{Sim}(W_{ij}) = \mathrm{Sim}(W_i)$ for $1 \leq j \leq n_i$. $\square$

For instance

$$\mathrm{Sim}(W_B^{n_1} \otimes \mathbb{Z}_2 \times W_C^{n_2} \otimes \mathbb{Z}_2) \cong \mathrm{Sim}(W_B \otimes \mathbb{Z}_2) \wr \Sigma_{n_1} \times \mathrm{Sim}(W_C \otimes \mathbb{Z}_2) \wr \Sigma_{n_2}$$

where W_B and W_C are the representations over $\mathbb{Z}$ as in Example 2.2 and $\mathbb{Z}_2$ is the ring of 2-adic integers.

In Lemma 2.4 the representations W_i, $i \in I$, are assumed to be pairwise nonsimilar. We now omit this assumption and instead group together similar representations.

More specifically, declare $i_1, i_2 \in I$ to be equivalent indices if the representations $W_{i_1} \to \mathrm{Aut}_R(S_{i_1})$ and $W_{i_2} \to \mathrm{Aut}_R(S_{i_2})$ are similar. Write the index set $I = \bigcup_{j\in J} I(j)$ as a disjoint union of equivalence classes $I(j)$, $j \in J$. Then $W(j) := \prod_{i\in I(j)} W_i^{n_i}$ is faithfully represented in $S(j) := \prod_{i\in I(j)} S_i^{n_i}$ and $W = \prod_{j\in J} W(j)$ faithfully represented in $S = \prod_{j\in J} S(j)$.

Corollary 2.5. *Assume that the index set $I = \bigcup_{j\in J} I(j)$ is divided into equivalence classes such that S_{i_1} and S_{i_2} are similar if and only if $i_1, i_2 \in I(j)$ for some $j \in J$. Then the canonical homomorphism*

$$\prod_{j\in J} \mathrm{Sim}(W(j)) \to \mathrm{Sim}(W)$$

is a group isomorphism.

Proof. Choose an element $i(j) \in I(j)$ and let $n(j) := \sum_{i\in I(j)} n_i$. Because $W(j) = \prod_{i\in I(j)} W_i^{n_i}$ is similar to $W_{i(j)}^{n(j)}$,

$$\mathrm{Sim}(W) \cong \mathrm{Sim}\left(\prod_{i\in I} W_i^{n_i}\right) = Sim\left(\prod_{j\in J}\prod_{i\in I(j)} W_i^{n_i}\right) \cong \mathrm{Sim}\left(\prod_{j\in J} W_{i(j)}^{n(j)}\right)$$
$$\cong \prod_{j\in J} \mathrm{Sim}(W_{i(j)}) \wr \Sigma_{n(j)}$$

and

$$\mathrm{Sim}(W(j)) \cong \mathrm{Sim}\left(\prod_{i\in I(j)} W_i^{n_i}\right) \cong \mathrm{Sim}\left(W_{i(j)}^{n(j)}\right) \cong \mathrm{Sim}(W_{i(j)}) \wr \Sigma_{n(j)}$$

for all $j \in J$. □

In the applications to be discussed the next section, W_i will be the Weyl group and S_i the free $\mathbb{Z}_p$-module given by the fundamental group of the maximal torus of a p-compact group.

3. Rational automorphisms

This section introduces a class of p-compact groups, called completely reducible p-compact groups, and investigates their rational automorphisms.

A homomorphism $f\colon X_1 \to X_2$ between two connected p-compact groups, X_1 and X_2, i.e. a based map $Bf\colon BX_1 \to BX_2$ between the classifying spaces, is a *rational isomorphism* if [**8**, Definition 4]

$$H^*(Bf;\mathbb{Z}_p)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p\colon H^*(BX_2;\mathbb{Z}_p)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p \to H^*(BX_1;\mathbb{Z}_p)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p$$

is an isomorphism. An endomorphism $f\colon Y \to Y$ of a p-compact group Y which is a rational equivalence will be called a *rational automorphism.*

All homomorphisms between p-compact groups restrict to homomorphisms between the maximal tori [**8**, Theorem 2.4]. For endomorphisms, the precise formulation of this assertion is the following version for p-compact groups of the classical Adams-Mahmud theorem.

THEOREM 3.1. [**8**, Theorem 2.5] *Let Y be a connected p-compact group with maximal torus $T \to Y$ and let $f\colon Y \to Y$ be an endomorphism of Y. There exists an endomorphism φ of T such that the diagram*

$$\begin{array}{ccc} T & \xrightarrow{\varphi} & T \\ \downarrow & & \downarrow \\ Y & \xrightarrow[f]{} & Y \end{array}$$

commutes and

(i) *The endomorphism φ is a (rational) automorphism if and only if f is a (rational) automorphism.*
(ii) *The homotopy class of $B\varphi$ in $[BT, BT]$ is unique up to left action by the Weyl group $W_T(Y)$.*

The Weyl group $W_T(Y)$ is faithfully represented [**4**, Theorem 9.7] in the free $\mathbb{Z}_p$-module $\pi_2(BT)$. The uniqueness of the lift φ has the consequence that for any $w \in W_T(Y) \subseteq [BT, BT]$, $B\varphi\circ w = \alpha(w)\circ B\varphi$ for some $\alpha(w) \in W_T(Y)$. Since

$\pi_2(B\varphi)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p$ is an automorphism of $\pi_2(BT)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p$, α is an automorphism of $W_T(Y)$ and

$$\pi_2(B\varphi)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p \in \mathrm{Sim}(W_T(Y)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p)$$

is a self-similarity of the vector space representation $W_T(Y)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p$. If f is an (honest) automorphism of Y, $\pi_2(B\varphi)$ is an automorphism of $\pi_2(BT)$ and the uniqueness clause implies that

$$\pi_2(B\varphi)\in \mathrm{Sim}(W_T(Y))$$

is a self-similarity of the $\mathbb{Z}_p$-representation $W_T(Y)$.

Denote by $\varepsilon_{\mathbb{Q}}(Y)\subseteq [BY,BY]$ the monoid of conjugacy classes of rational automorphism of Y. The invertible elements in this monoid is the group $\mathrm{Out}(Y)$ of conjugacy classes of automorphisms of Y.

Now consider a finite collection Y_i, $i\in I$, of connected p-compact groups. Let W_i be the Weyl group of Y_i with respect to some maximal torus $T_i\to Y_i$.

The following lemma, based on the vanishing theorem of [**8**, Theorem 5.1], will be applied repeatedly in the following.

LEMMA 3.2. *Let f be an endomorphism of the product p-compact group $\prod_{i\in I} Y_i$. Suppose that there exist endomorphisms $\varphi_i\colon T_i\to T_i$, $i\in I$, such that*

$$\begin{array}{ccc} \prod_i T_i & \xrightarrow{\prod_i \varphi_i} & \prod_i T_i \\ \downarrow & & \downarrow \\ \prod_i Y_i & \xrightarrow[f]{} & \prod_i Y_i \end{array}$$

commutes up to conjugacy. Then

(i) *There exist endomorphisms f_i of Y_i such that f is conjugate to the product endomorphism $\prod_i f_i$ of $\prod_i Y_i$.*
(ii) *If f is a (rational) automorphism, each f_i is a (rational) automorphism.*

PROOF. Let, for each $i\in I$, $\iota_i\colon Y_i\to\prod_i Y_i$ and $\pi_i\colon \prod_i Y_i\to Y_i$ be the canonical injection and projection homomorphisms. Define $f_i\colon Y_i\to Y_i$ as the composite $\pi_i\circ f\circ\iota_i$.

The assumption that f lifts to a product endomorphism of the product maximal torus $\prod_i T_i$ implies that the restriction of $\pi_i\circ f$ to $\prod_{j\neq i}T_j$ is trivial and hence also [**8**, Corollary 5.7] that the restriction of $\pi_i\circ f$ to $\prod_{j\neq i}Y_j$ is trivial. Consequently, $\pi_i\circ f$ admits a factorization [**8**, Corollary 1.8]

$$\begin{array}{ccc} \prod_i X_i & \xrightarrow{\pi_i\circ f} & X_i \\ {\scriptstyle \pi_i}\downarrow & \nearrow & \\ X_i & & \end{array}$$

through X_i. The only possible factorization is the endomorphism f_i. Of course, $[BY, BY] = \prod_i [BY, BY_i]$, so $\pi_i \circ f = f_i \circ \pi_i = \pi_i \circ (\prod_i f_i)$ shows that f and $\prod_i f_i$ are conjugate.

If f is a (rational) automorphism, the lift $\prod_i \varphi_i$ is a (rational) automorphism too [Theorem 3.1] and hence each φ_i, $i \in I$, is a (rational) automorphism. Since $\varphi_i \colon T_i \to T_i$ covers $f_i \colon X_i \to X_i$ it follows, Theorem 3.1 again, that f_i is a (rational) automorphism. □

Assume now that the faithful representation $W_i \to \mathrm{Aut}_{\mathbb{Q}_p}(\pi_2(BT_i) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$ is irreducible for each $i \in I$, i.e. that each Y_i is a *simple* [**8**, Definition 5] p-compact group.

Lemma 3.2 leads in the first place to a slight generalization of [**8**, Theorem 4.5] or, in other words, to a p-compact group version of [**5**, Proposition 1.3] from where the proof is copied.

Proposition 3.3. *Suppose that the prime p divides the order of each of the Weyl groups W_i of the connected, simple p-compact groups Y_i, $i \in I$. Then* $\mathrm{Out}(\prod_i Y_i) = \varepsilon_{\mathbb{Q}}(\prod_i Y_i)$.

Proof. Let f be any rational automorphism of the product p-compact group $\prod_i Y_i$. The task is to show that f is invertible.

Choose [Theorem 3.1] a rational automorphism φ such that

$$\begin{array}{ccc} \prod_i T_i & \xrightarrow{\varphi} & \prod_i T_i \\ \downarrow & & \downarrow \\ \prod_i Y_i & \xrightarrow[f]{} & \prod_i Y_i \end{array}$$

commutes up to conjugacy and $\pi_2(B\varphi) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p \in \mathrm{Sim}(\prod_i W_i \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)$. In particular, φ determines an automorphism of the Weyl group $\prod_i W_i$ of $\prod_i Y_i$ and by replacing f by an iterate of itself, if necessary, we may assume that this automorphism is in fact the identity. Then the induced homomorphism

$$\pi_2(B\varphi) \in \mathrm{End}_{\mathbb{Z}_p[\prod W_i]}\left(\prod \pi_2(BT_i)\right) \cong \prod \mathbb{Z}_p$$

is a product homomorphism by Lemma 2.3 meaning that φ itself is conjugate to a product $\prod_i \varphi_i$ of endomorphisms φ_i of T_i. That also f is conjugate to a product $\prod_i f_i$ of rational automorphisms $f_i \in \varepsilon_{\mathbb{Q}}(Y_i)$ now follows from Proposition 3.2. However, since p divides the order of the Weyl group W_i and Y_i is simple, any rational automorphism of Y_i is invertible by [**8**, Theorem 4.5]. □

Rational automorphisms of the product p-compact group $\prod_i Y_i$ can be analyzed in detail in case the factors are distinct in the sense defined below.

Definition 3.4. *A finite family Y_i, $i \in I$, of connected, simple p-compact groups is* similarity free *if each Weyl group W_i is nontrivial and*

(i) *The representations* $W_i \to \mathrm{Aut}(\pi_2(BT_i))$ *of the Weyl groups in the maximal tori are pairwise nonsimilar.*
(ii) *If, for some pair* $i_1, i_2 \in I$, *the representations* $W_{i_1} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ *and* $W_{i_2} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ *are similar, then* p *divides the Weyl group order* $|W_{i_1}| = |W_{i_2}|$.

Any family G_i, $i \in I$, of pairwise nonisomorphic, simply connected, simple compact Lie groups (none of which is equal to $\mathrm{Sp}(n)$, $n > 2$, if $p > 2$) is similarity free; see Example 2.2 when $p = 2$.

The main motivation for the introduction of this concept is the following main result; cfr. [**9**, Theorem 3.1].

THEOREM 3.5. *Let* Y_i, $i \in I$, *be a similarity free family of connected, simple* p*-compact groups and let* $Y = \prod_{i \in I} Y_i^{n_i}$, $n_i \geq 1$.
(i) *The canonical homomorphisms*

$$\prod_{i \in I} \varepsilon_{\mathbb{Q}}(Y_i) \wr \Sigma_{n_i} \to \varepsilon_{\mathbb{Q}}(Y)$$
$$\prod_{i \in I} \mathrm{Out}(Y_i) \wr \Sigma_{n_i} \to \mathrm{Out}(Y)$$

are isomorphisms.
(ii) *If* $f = (\prod_{i \in I} \prod_{1 \leq j \leq n_i} f_{ij}) \circ \sigma$ *for some* $f_{ij} \in \varepsilon_{\mathbb{Q}}(X_i)$, $1 \leq j \leq n_i$, *and some permutation map* $\sigma \in \prod_{i \in I} \Sigma_{n_i}$, *then the product map*

$$\prod_{i \in I} \prod_{1 \leq j \leq n_i} \mathrm{map}(BY_i, BY_i)_{Bf_{ij}} \to \mathrm{map}(BY, BY)_{B(f \circ \sigma^{-1})}$$

is a homotopy equivalence.

The proof is divided into four parts.

By assumption, the index set I can be divided into equivalence classes $I = \bigcup_{j \in J} I(j)$ such that $i_1, i_2 \in I(j)$ for some $j \in J$ if and only if the representations $W_{i_1} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ and $W_{i_2} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ are similar. Note that if the equivalence class $I(j)$ contains more than one element then $p \mid |W_i|$ for all $i \in I(j)$.

Put $Y(j) = \prod_{i \in I(j)} Y_i^{n_i}$, $j \in J$, so that $Y = \prod_{j \in J} Y(j)$.

LEMMA 3.6. *The product maps*

$$\prod_{j \in J} \varepsilon_{\mathbb{Q}}(Y(j)) \to \varepsilon_{\mathbb{Q}}(Y)$$
$$\prod_{j \in J} \mathrm{Out}(Y(j)) \to \mathrm{Out}(Y)$$

are monoid isomorphisms.

PROOF. It suffices to show surjectivity as the above maps are monoid monomorphisms by general and elementary principles. Let $f \in \varepsilon_{\mathbb{Q}}(Y)$ be a (rational) automorphism of Y. Let $T(j) := \prod_{i\in I(j)} T_i$ be the maximal torus and $W(j) := \prod_{i\in I(j)} W_i$ the Weyl group of $Y(j)$. Choose [Theorem 3.1] a (rational) automorphism $\varphi\colon \prod_{j\in J} T(j) \to \prod_{j\in J} T(j)$ that covers f. Then

$$\pi_2(B\varphi)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p \in \mathrm{Sim}\left(\prod_{j\in J} W(j)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p\right) \cong \prod_{j\in J}\mathrm{Sim}(W(j)\otimes_{\mathbb{Z}_p}\mathbb{Q}_p)$$

by Lemma 2.5. It follows in particular, that the induced homomorphism $\pi_2(B\varphi)$ preserves the factors of $\prod_{j\in J}\pi_2(BT(j))$ and hence that also $\varphi = \prod_{j\in J}\varphi(j)$ is a product of (rational) automorphisms $\varphi(j) \in \varepsilon_{\mathbb{Q}}(T(j))$, $j \in J$. Now apply Lemma 3.2. □

In the next lemma, $\Sigma_{n_i} < \mathrm{Out}(Y(j))$, $i \in I(j)$, denotes the subgroup consisting of permutations of the n_i factors equal to Y_i in $Y(j) = \prod_{i\in I(j)} Y_i^{n_i}$.

LEMMA 3.7. *The canonical homomorphism*

$$\prod_{i\in I(j)} \mathrm{Out}(Y_i)\wr\Sigma_{n_i} \to \mathrm{Out}(Y(j))$$

is an isomorphism for each $j \in J$.

PROOF. Let f be any automorphism of $Y(j)$. Choose [Theorem 3.5] an automorphism φ of maximal torus $T(j)$ that covers f. Then

$$\pi_2(B\varphi) \in \mathrm{Sim}(W(j)) \cong \prod_{i\in I(j)} \mathrm{Sim}(W_i)\wr\Sigma_{n_i}$$

by Lemma 2.4. It follows in particular that $B(\varphi\circ\sigma^{-1}) = \prod_{i\in I(j)}\varphi_i$ for some automorphisms $\varphi_i \in \mathrm{Out}(T_i)$, $i \in I(j)$, and some permutation map $\sigma \in \prod_{i\in I(j)}\Sigma_{n_i}$. Now apply Lemma 3.2 to $f\circ\sigma^{-1}$. □

In case the index set $I(j)$ contains more than one element, then the formula

$$\prod_{j\in I(j)} \varepsilon_{\mathbb{Q}}(Y_i)\wr\Sigma_{n_i} \cong \varepsilon_{\mathbb{Q}}(Y(j))$$

is, by Proposition 3.3, just an alternative formulation of Lemma 3.7. The computation of the rational equivalences in case the index set $I(j)$ does consist of just a single element is handled by the following lemma.

LEMMA 3.8. *The canonical homomorphism*

$$\varepsilon_{\mathbb{Q}}(Y_i)\wr\Sigma_{n_i} \to \varepsilon_{\mathbb{Q}}(Y_i^{n_i})$$

is a monoid isomorphism for all $i \in I$.

Proof. Proceed as in the proofs of Lemma 3.6 and Lemma 3.7 using the isomorphism $\mathrm{Sim}(W_i^{n_i} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p) \cong \mathrm{Sim}(W_i \otimes_{\mathbb{Z}_p} \mathbb{Q}_p) \wr \Sigma_{n_i}$ from Lemma 2.4 together with Lemma 3.2. $\square$

After these three lemmas it is time for

Proof of Theorem 3.5. The preceding three lemmas imply that

$$\prod_{i\in I} \varepsilon_{\mathbb{Q}}(Y_i) \wr \Sigma_{n_i} = \prod_{j\in J} \prod_{i\in I(j)} \varepsilon_{\mathbb{Q}}(Y_i) \wr \Sigma_{n_i} \cong \prod_{j\in J} \varepsilon_{\mathbb{Q}}(Y(j)) \cong \varepsilon_{\mathbb{Q}}(Y)$$

and a similar computation applies to the case of (genuine) automorphisms.

As to point (ii), write $Y = \prod_i Y_i^{n_i} = \prod_{i,j} Y_{ij}$ where $Y_{ij} = Y_i$ for $1 \le j \le n_i$. The projection homomorphism π_{ij} of Y to Y_{ij} is part of a short exact sequence

$$\prod_{(k,l)\neq(i,j)} Y_{kl} \to Y \xrightarrow{\pi_{ij}} Y_{ij}$$

of p-compact groups. In this situation, precomposition with $B\pi_{ij}$ induces a homotopy equivalence of mapping spaces [**8**, Lemma 1.7] which, as $\pi_{ij} \circ f \circ \sigma^{-1} = f_{ij} \circ \pi_{ij}$, restricts to a homotopy equivalence

$$\overline{B\pi_{ij}}\colon\ \mathrm{map}(BY_{ij}, BY_{ij})_{Bf_{ij}} \to \mathrm{map}(BY, BY_{ij})_{B(\pi_{ij}\circ f\circ\sigma^{-1})}$$

of connected components. With the map of point (ii) as the top horizontal map, the diagram

$$\begin{array}{ccc} \prod_{i,j} \mathrm{map}(BY_{ij}, BY_{ij})_{Bf_{ij}} & \longrightarrow & \mathrm{map}(BY, BY)_{B(f\circ\sigma^{-1})} \\ & \searrow^{\simeq}_{\prod \overline{B\pi_{ij}}} & \Big\| \\ & & \prod_{i,j} \mathrm{map}(BY, BY_{ij})_{B(\pi_{ij}\circ f\circ\sigma^{-1})} \end{array}$$

commutes and point (ii) follows. $\square$

Any rational automorphism f of the connected p-compact group Y induces (restricts to) a rational automorphism $Z(f)$ of the center $Z(Y)$ characterized as the unique endomorphism that makes

$$\begin{array}{ccc} Z(Y) & \xrightarrow{Z(f)} & Z(Y) \\ \downarrow & & \downarrow \\ Y & \xrightarrow[f]{} & Y \end{array}$$

commute up to conjugacy [**8**, Corollary 3.2].

Thanks to the homotopy equivalence $BZ(Y) \to \mathrm{map}(BY, BY)_{B1}$ from Dwyer and Wilkerson [**3**, Theorem 1.3], $Z(f)$ can also be computed as a map between mapping spaces.

Corollary 3.9. *Let f be a rational automorphism of the p-compact group $Y=\prod Y_i^{n_i}$ from Theorem* 3.5. *Then the post- and precomposition maps*

$$\mathrm{map}(BY,BY)_{B1}\xrightarrow{\underline{Bf}}\mathrm{map}(BY,BY)_{Bf}\xleftarrow{\overline{Bf}}\mathrm{map}(BY,BY)_{B1}$$

are homotopy equivalences and the endomorphism $Z(f)$ is an automorphism of $Z(Y)$.

Proof. According to Theorem 3.5 we may assume that $f=\prod_i\prod_j f_{ij}$ for some rational automorphisms f_{ij} of $Y_{ij}=Y_i$, $i\in I$, $1\le j\le n_i$. The postcomposition and precomposition maps fit into the commutative diagram

$$\begin{array}{ccccc}
\mathrm{map}(BY,BY)_{B1} & \xrightarrow{\underline{Bf}} & \mathrm{map}(BY,BY)_{Bf} & \xleftarrow{\overline{Bf}} & \mathrm{map}(BY,BY)_{B1}\\
\simeq\uparrow & & \uparrow\simeq & & \simeq\uparrow\\
\prod\mathrm{map}(BY_{ij},BY_{ij})_{B1} & \xrightarrow[\prod\underline{Bf_{ij}}]{} & \prod\mathrm{map}(BY_{ij},BY_{ij})_{Bf_{ij}} & \xleftarrow[\prod\overline{Bf_{ij}}]{} & \prod\mathrm{map}(BY_{ij},BY_{ij})_{B1}
\end{array}$$

where the vertical maps are the homotopy equivalences of Theorem 3.5. Since each of the p-compact groups Y_{ij} is simple with nontrivial Weyl group, the postcomoposition maps $\underline{Bf_{ij}}$ as well as the precompostion maps $\overline{Bf_{ij}}$ are homotopy equivalences by [**8**, Corollary 4.7].

Evaluation at the base point determines a commutative diagram

$$\begin{array}{ccccc}
\mathrm{map}(BY,BY)_{B1} & \xrightarrow[\simeq]{\underline{Bf}} & \mathrm{map}(BY,BY)_{Bf} & \xleftarrow[\simeq]{\overline{Bf}} & \mathrm{map}(BY,BY)_{B1}\\
\downarrow & & \downarrow & & \downarrow\\
BY & \xrightarrow{Bf} & BY & = & BY
\end{array}$$

showing that $Z(f)$ identifies to the homotopy equivalence $(\overline{Bf})^{-1}\circ\underline{Bf}$. □

In particular, we see that Z may be regarded as a homomorphism $Z\colon\varepsilon_{\mathbb{Q}}(Y)\to\mathrm{Aut}(Z(Y))$.

The following definition was introduced as an attempt to extend Corollary 3.9 to a larger class of p-compact groups by applying Notbohm's [**11**, Theorem].

Definition 3.10. *A connected p-compact group is* completely reducible *if its universal covering p-compact group* [**10**, Lemma 3.2] *is isomorphic to a product p-compact group of the form $\prod_{i\in I}Y_i^{n_i}$ where $(Y_i)_{i\in I}$ is a finite, similarity free family of simply connected, simple p-compact groups.*

This definition should be regarded as provisional since, with a little luck, any connected p-compact group will turn out to be completely reducible.

For any completely reducible p-compact group X there exists [**10**, Theorem 5.4] a short exact sequence of the form

$$K\xrightarrow{(\mathrm{incl},\varphi)}\left(\prod Y_i^{n_i}\right)\times S\xrightarrow{q}X \tag{3.1}$$

where K is a subgroup of the finite [**10**, Theorem 5.3] center $Z\left(\prod Y_i^{n_i}\right) \cong \prod Z(Y_i)^{n_i}$, S is a p-compact torus (equal to the connected component of the center of X), and $\varphi\colon K \to S$ a homomorphism.

Suppose now that X_1 and X_2 are two connected, *locally isomorphic* [**8**, Definition 3], completely reducible p-compact groups, i.e. [**8**, Proposition 1.4] that there exist short exact sequences

$$K_j \xrightarrow{(\mathrm{incl},\varphi_j)} \left(\prod Y_i^{n_i}\right) \times S \xrightarrow{q_j} X_j \tag{3.2}$$

for $j = 1, 2$. Let $\varepsilon_{\mathbb{Q}}(X_1, X_2) \subseteq [BX_1, BX_2]$ denote the set of conjugacy classes of rational equivalences from X_1 to X_2.

By Theorem 3.5 and [**8**, Theorem 3.2] there exists an injection

$$\Lambda\colon \varepsilon_{\mathbb{Q}}(X_1, X_2) \to \left(\prod \varepsilon_{\mathbb{Q}}(Y_i) \wr \Sigma_{n_i}\right) \times \varepsilon_{\mathbb{Q}}(S)$$

associating to each $f \in \varepsilon_{\mathbb{Q}}(X_1, X_2)$, rational automorphisms $g_{ij} \in \varepsilon_{\mathbb{Q}}(Y_i)$, $i \in I$, $1 \le j \le n_i$, $h \in \varepsilon_{\mathbb{Q}}(S)$, and a permutation map $\sigma \in \prod \Sigma_{n_i}$ such that

$$\begin{array}{ccc} \left(\prod_{i\in I} Y_i^{n_i}\right) \times S & \xrightarrow{g\times h} & \left(\prod_{i\in I} Y_i^{n_i}\right) \times S \\ {\scriptstyle q_1}\downarrow & & \downarrow{\scriptstyle q_2} \\ X_1 & \xrightarrow[f]{} & X_2 \end{array}$$

commutes up to conjugacy with $g = \prod_{i\in I}(\prod_j g_{ij}) \circ \sigma_i$. The image of Λ consists of those pairs (g, h) for which $Z(g)(K_1) \subseteq K_2$ and the diagram

$$\begin{array}{ccc} K_1 & \xrightarrow{\varphi_1} & S \\ {\scriptstyle Z(g)}\downarrow & & \downarrow{\scriptstyle h} \\ K_2 & \xrightarrow[\varphi_2]{} & S \end{array}$$

commutes up to conjugacy.

In particular, the sets $\varepsilon_{\mathbb{Q}}(X_1, X_2)$ will be completely known for all pairs (X_1, X_2) of locally isomorphic, completely reducible p-compact groups once the the monoids $\varepsilon_{\mathbb{Q}}(Y)$ together with the homomorphisms $Z\colon \varepsilon_{\mathbb{Q}}(Y) \to \mathrm{Aut}(Z(Y))$ are known for all simply connected, simple p-compact groups Y.

Here is a consequence that already now is obtainable. In point (i), $Z\colon \varepsilon_{\mathbb{Q}}(X_1, X_2) \to \varepsilon_{\mathbb{Q}}(Z(X_1), Z(X_2))$ is the homomorphism of [**8**, Corollary 3.2] (considered above in case of rational automorphisms).

THEOREM 3.11. *Suppose that X_1 and X_2 are the two locally isomorphic, completely reducible p-compact groups given by the short exact sequences (*3.2*).*

(i) *For any rational equivalence $f\colon X_1 \to X_2$, the precomposition map*

$$\overline{Bf}\colon\ \mathrm{map}(BX_2, BX_2)_{B1} \to \mathrm{map}(BX_1, BX_2)_{Bf}$$

is a homotopy equivalence (i.e. the centralizer of f is isomorphic to the center of X_2) and the diagram

$$\begin{array}{ccccc} Z(X_1) & & \xrightarrow{\quad Z(f) \quad} & & Z(X_2) \\ \simeq \downarrow & & & & \downarrow \simeq \\ \mathrm{map}(BX_1, BX_1)_{B1} & \xrightarrow[\underline{Bf}]{} & \mathrm{map}(BX_1, BX_2)_{Bf} & \xleftarrow[\overline{Bf}]{\simeq} & \mathrm{map}(BX_2, BX_2)_{B1} \end{array}$$

commutes up to conjugacy.

(ii) *If* $|K_1| > |K_2|$*, then* $\varepsilon_{\mathbb{Q}}(X_1, X_2) = \emptyset$.

Proof. Let $g \in \varepsilon_{\mathbb{Q}}(Y)$, where $Y = \prod Y_i^{n_i}$ is the universal covering p-compact group of X, and $h \in \varepsilon_{\mathbb{Q}}(S)$ be rational automorphisms such that $g \times h$ covers f up to conjugacy. Precomposition with these maps induces a fibre map

$$\begin{array}{ccccc} BK_2 & \longrightarrow & \mathrm{map}(BY, BY)_{B1} \times \mathrm{map}(BS, BS)_{B1} & \longrightarrow & \mathrm{map}(BX_2, BX_2)_{B1} \\ \Vert & & \simeq \downarrow \overline{Bg} \times \overline{Bh} & & \downarrow \overline{Bf} \\ BK_2 & \longrightarrow & \mathrm{map}(BY, BY)_{Bg} \times \mathrm{map}(BS, BS)_{Bh} & \longrightarrow & \mathrm{map}(BX_1, BX_2)_{Bf} \end{array}$$

of fibrations as in [**8**, Proposition 3.4]. The precomposition map $\overline{Bg}$ is a homotopy equivalence by Corollary 3.9 and $\overline{Bh}$ is a homotopy equivalence since S is an abelian p-compact group. Hence also $\overline{Bf}$ is a homotopy equivalence.

The computation of $Z(f)$ now proceeds as in Corollary 3.9.

The second statement of the theorem follows because $Z(g)$ is an automorphism of $Z(Y)$ by Corollary 3.9. □

The postcomposition map $\underline{Bf}$ is in general *not* a homotopy equivalence.

Another consequence is a version of Proposition 3.3 concerning invertibility of rational automorphisms of semisimple p-compact groups, i.e. p-compact groups with finite fundamental groups.

Corollary 3.12. *Let X be a semisimple, completely reducible p-compact group given as in (*3.1*) with S trivial. Assume that the prime p divides the order $|W_i|$ of the Weyl group of Y_i for all $i \in I$. Then $\varepsilon_{\mathbb{Q}}(X) = \mathrm{Out}(X)$.*

Proof. By Proposition 3.3, any rational automorphism of Y has a homotopy inverse, so the monoid monomorphism $\Lambda\colon \varepsilon_{\mathbb{Q}}(X) \to \varepsilon_{\mathbb{Q}}(Y)$ shows that the same is true for X. □

Finally, a few examples to illustrate the use of Theorem 3.5.

Example 3.13. (1) According to [**5**, Proposition 1.3] or Proposition 3.3 and [**6**]

$$\varepsilon_{\mathbb{Q}}(\mathrm{Spin}(2n+1)^\wedge_2) = \mathrm{Out}(\mathrm{Spin}(2n+1)^\wedge_2) = \{\psi^u \mid u \in \mathbb{Z}_2^*\}$$

and $Z(\psi^u)$ is the identity map of $Z(\mathrm{Spin}(2n+1)^\wedge_2) = \mathbb{Z}/2$. Hence [**8**, Theorem 3.2]

$$\mathrm{Out}(\mathrm{SO}(2n+1)^\wedge_2) = \mathrm{Out}(\mathrm{Spin}(2n+1)^\wedge_2)$$

and the product map

$$\mathrm{Out}(\mathrm{Spin}(2n+1)^\wedge_2) \times \mathrm{Out}(\mathrm{SO}(2n+1)^\wedge_2) \to \mathrm{Out}(\mathrm{Spin}(2n+1)^\wedge_2 \times \mathrm{SO}(2n+1)^\wedge_2)$$

is an isomorphism.

(2) The monomorphism $\Lambda\colon\ \mathrm{Out}(\mathrm{SO}(2n)^\wedge_2) \to \mathrm{Out}(\mathrm{Spin}(2n)^\wedge_2)$ is an isomorphism for $n > 4$ but not for $n = 4$; see [**7**, Example 2.2].

(3) For any finite collection $(G_i)_{i\in I}$ of pairwise nonisomorphic simply connected, simple, compact Lie groups

$$\prod_{i\in I} \varepsilon_{\mathbb{Q}}((G_i)^\wedge_2) \wr \Sigma_{n_i} \cong \varepsilon_{\mathbb{Q}}(\textstyle\prod_{i\in I}(G_i^{n_i})^\wedge_2)$$

and the similar formula holds for odd primes too if $G_i \neq \mathrm{Sp}(n)$, $n > 2$, for all $i \in I$.

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