\[ \alpha : B \to B \]
\[ B^\alpha \to B \]
\[ \phi \]
\[ B \to B \times B \]

Joint with Carlos Bustamante

\[ B = \text{homody fixed point of } \alpha \]

\[ \text{Fixed points and homody fixed points} \]

\[ \text{B has } \text{homody fixed points } B = \text{fixed points of } \alpha \]

\[ [B \times B] = \text{Out}(X) = \text{Aut}(\tau(X)) / \langle \tau(X) \rangle \]

Note: \[ \mathbb{Z}[\tau] = \text{Out}(\mathbb{Z}[\tau]) \to \text{Out}(X) \]

\[ \text{Maximal torus} \]

\[ \text{Weyl group} \]

\[ B^\alpha \to B \]

\[ \text{p-compact groups and their automorphisms in which Adams groups} \]

\[ B \times B \approx (\mathbb{Z}/p)^2 \]

\[ \text{Maximal torus} \]

\[ \text{Weyl group} \]

\[ B^\alpha \to B \times B \]

\[ \text{Aut}(\tau(X)) = \text{GL}(\mathbb{Z}/p) \]

\[ B \times B \approx (\mathbb{Z}/p)^2 \]

\[ \text{p-compact groups} \]

\[ \text{p-adic reductive groups} \]

\[ X \to (\tau(X), \langle \tau(X) \rangle) \]

\[ [B \times B] = \text{Out}(X) = \text{Aut}(\tau(X)) / \langle \tau(X) \rangle \]

\[ \mathbb{Z}[\tau] = \text{Out}(\mathbb{Z}[\tau]) \to \text{Out}(X) \]

\[ \text{Weyl group} \]

\[ \tau \]

\[ \phi \]

\[ \text{homody fixed point of } \alpha \]

\[ B = \text{homody fixed point of } \alpha \]

\[ \mathbb{Z}[\tau] = \text{Out}(\mathbb{Z}[\tau]) \to \text{Out}(X) \]

\[ \text{Weyl group} \]

\[ \tau \]

\[ \phi \]
\[ BX \text{ include } \Rightarrow \text{ out}(X) \in \mathbb{Z}^x / \mathbb{Z}^w + \text{ a little more} \]

Any \( p \)-coefficient gap has the form

\[ BX = B(\mathbb{Y}^x T / (\mathbb{Z}, p)) \]

\[ BY = \prod BX_i \]

Shephard-Todd

Clow-Evans classification

Shephard-Todd

Clow-Evans classification

Family 1

\[ W(SU(n+1)) = \sum_n \leq GL_n(\mathbb{Z}_p) \]

\[ G(m, r, n) \quad (m, r, n) \neq (m, m, 2) \]

\[ r \mid m \perp 1 \quad \text{permutations} \]

\[ G(m, m, 2) \quad m \geq 3 \]

Family 2

\[ C_m \leq GL_1(\mathbb{Z}_p) = \text{Aut}(\mathbb{Z}/p) \] \( m \perp n - 1 \)

Symmetric

\[ G_i \quad 4 \leq i \leq 37 \]

Observation

Any sheaf is pure \( \Rightarrow \) \( H^*(BX; \mathbb{Z}_p) = H^*(BG; \mathbb{Z}_p) \)

An inductive project gap \( BX \) is \( BG \) for some \( \text{Theorem} \) (plus more)

The following are constructed as very simple homology + BG
Example

\[ BX = B(\Pi \times W) \cong \text{hood}(B\widetilde{T}, W) \]

\[ H^*(BX) = H^*(B\widetilde{T}) \]

Sullivan spheres \( m/\pi^2 \)

\[ BX = B\left( \mathbb{Z}/\infty \times C_m \right) \]

\[ H^*(BX) = H^*(\mathbb{Z}/\infty) \cong H^*[u^m], \quad |u|^m = 2^m, \quad |u^m| = 2 \]

\[ X = \partial BX \quad H^*(X) = H^*(S^{2m-1}) \]

\[ BX = BS^{2m-1} \quad S^{2m-1} \text{ is a project space} \]

\[ \text{Agol's project space} \quad \left\{ \begin{array}{c}
\text{Agol's project space} \\
\end{array} \right\} \]

\[ BX = \text{hood}(B(\mathbb{Z}/\infty), \mathbb{Z}/\infty) \]

\[ H^*(BG_{12}) = H^2[\mathbb{Z}_{12}, \mathbb{Z}_{16}] \text{ rank 2 Deligne abelian} \]

\( G_{12} \quad \pi = 3 \)

\( G_{23} \quad \pi = 5 \)

\( G_{34} \quad \pi = 7 \)

\( G_{24} \quad \pi = 7 \)
Friedlander's Thm. From compact Lie groups to Lie type.

\[ \eta^a : BG \to BG \quad u \in \mathbb{Z}_p \]

\[ \chi : BG \to BG \quad \text{representation isosing of finite order} \]

\[ \chi \eta^a : BG \to BG \quad \text{what are the homotopy fixed points?} \]

\[ B(T^G(u)) \to mp(I, BS) \]

\[ \downarrow \quad \downarrow \]

\[ BG \to BG \times BG \]

BM: What happens if we replace $BG$ by $BX$? (David Benson on bad south of Wales)

Then $A \times l$-connected $p - 1$-group, if prime power $p^f$, then $1$-an abelian

\[ \chi_1 \times \chi_2 \quad \text{of order prime by} \]

\[ BX(T_1^f) \to BX \]

\[ \downarrow \quad \downarrow \]

\[ BX \to BX \times BX \]

\[ (p, T_1^f) \]

is to simplify yet a $p$-local finite group.

Then in a consequence of

The proof splits into 2 cases.
Action of $G$ on $BX$ is a fibration. Port splits into two cases.

\[ BX \to BX \to BG \]

The fixed point of this action $BX$ is the space of unmarked.

Hence fixed point $f$ acts on $\ast$.

(1) $f$ lifts to an automorphism of $G$ on $BX$.

(2) $BX$ is an $H$-fibre and

\[ H^*(BX; \mathbb{Q}) = \mathbb{Q}[Q, H^*(BX; \mathbb{Q})] \]

(3) $X = X \times H$ and $X \times H$ is an $H$-fibre.

(9) If $H^*(BX; \mathbb{Q})$ is cyclic so is $H^*(BX; \mathbb{Q})$. $F = X \times H_{12}$ at $p = 3$.

Thus $C \cong I_{mod}^1$, $g \neq 1$.

$B(K)$ is the sphere of a spherical fibrations.
Theorem E \ X \ 1-Oxistml 1 has order descn in Thm B.

(1) If \( q = p \) and \( p \) is prime \( h \in \mathbb{Z} \),

\[
BX(T^q) = BX(T^q)
\]

(2) If \( q \neq q' \) have the same multiplicative order \( q \) and \( q' \),

\[
BX(q) = BX(q')
\]

\[\text{Theorem D} \quad q = p \text{ and } q \neq p. \text{ Exotic } p \text{-beal finitgaps}\]

- \( BX_{29}(q), BX_{34}(q) \) at \( p = 5, 7 \)
- \( BX(m, r, n)(q) \) for \( n \geq p, r > 2 \)

Example The p-beal finit gaps \( BX_{29}(q) \) and \( BX_{31}(q) \) are not exotic.

\( p = 3 \)

\[
BX_{29}(q) = B(E_4(2^{3m+1}))
\]

\( p = 5 \)

\[
BX_{31}(q) = B(E_8(2^3))
\]

\[
BX_{12} = \text{homolim } B\left( F(2^3) \right)^n
\]

\[
BX_{31} = \text{homolim } B(E_8(2^3))
\]

\[
BX(m, r, n)(q) = B_{GL_{1m}}(q)
\]