\[ g = 12.24 \]

The En of the is

\[ \sum x \left( \frac{C_x(G)}{C_G(G)} \right) = \chi_G(A, G) \]

\[ \sigma_G \]

The En of the is

\[ k(G) \geq k_p(G) \geq z_p(G) \]

\[ (G, p) \]

\[ k(G) = \#(\text{conjugacy class in } G) = \#(\text{irr. } \overline{PG}\text{-mod}) \]

\[ k_p(G) = \#(\text{irr. } \overline{PG}\text{-mod}) \]

\[ z_p(G) = \#(\text{irr. } \overline{PG}\text{-mod}) \]

\[ PG = \text{post of } G \]
Example: \( G = SL_2(F) \), \( n = 2 \), \( |G| = \frac{1}{6} \cdot 8 = 8.2 \)

\[ \rho_{\text{red}}^* \frac{|G|}{G} = G_2 \times G_2 \times G_2 \times G_2 \times G_2 \times D_8 \]

\( N_G(P) \Delta G \)

\( \sum_{\Delta G} (-1)^{d(\phi)} \text{rk}(G(\sigma)) = \text{rk}(D_8) + 2 \text{rk}(S_4) = -5 + 2 \times 5 = 5 \)

\[ \chi(G, \rho_{\text{red}}^*) \frac{|G|}{G} = \chi(D_8, \rho_{\text{red}}^*) - \chi(S_4, \rho_{\text{red}}^*) = -5 + 2 \times 5 = 5 \]

Also:

\[ \chi(G, \rho_{\text{red}}^*) \frac{|G|}{G} = -|G : D_8| \cdot 2|G : S_4| = -\frac{1}{2} \cdot 2 \cdot \frac{1}{2} = -\frac{1}{2} \]

\[ \chi(G, \rho_{\text{red}}^*) \frac{|G|}{G} = -\frac{1}{2} \cdot 2 \cdot \frac{1}{2} = -\frac{1}{2} \]
\[ \sum \chi(g) = \chi(1) = 1 \]

\[ \chi(g) = \chi(1) = 1 \]

\[ \text{Believed to be a cosmic coincidence} \]

\[ \chi(\tau P) = \chi(1) = 1 \]

\[ \text{Some of us are reminded about } H^1(G, \mathbb{Z}) \rightarrow H^1(G, \mathbb{Z}) \]

\[ \text{Webb Thm: } \chi(\langle g \rangle / G) = 1 \]

\[ \text{Brown Thm: } 1 \chi(g) \chi(g^{p_1 p_2 \ldots p_n}) \]
Equivalence Edun: Lemma

\[ \Delta \mathcal{C} = \Delta \mathcal{C} \text{-derived complex} \]

\[ \chi_r(\Delta, C) = \frac{1}{n!} \sum_{x \in Hm(\mathcal{Z}, C)} \chi(\zeta(x)) \]

\[ \chi_1(\Delta, C) = \frac{1}{\mathfrak{g}!} \sum_{\mathfrak{g} \in \mathcal{G}} \chi(\mathfrak{g}) = \chi(\mathfrak{g}/C) \]

\[ \chi_2(\Delta, C) = \frac{1}{\mathfrak{g}!} \sum_{\mathfrak{g} \in \mathcal{G}} \chi_1(\zeta(\mathfrak{g})/\zeta(\mathfrak{g})) = \frac{1}{\mathfrak{g}!} \sum_{\mathfrak{g} \in \mathcal{G}} \chi(\zeta(\mathfrak{g})/\zeta(\mathfrak{g})) \]

\[ = \dim K^0_C(\Delta) \otimes \mathbb{C} - \dim_K K^r_C(\Delta) \otimes \mathbb{C} \]

\[ K^*(x) \otimes \mathbb{C} = \bigoplus_{g \in \mathcal{G}} K^*(x^g) \otimes \mathbb{C} = \bigoplus_{g \in \mathcal{G}} K^*(x^g) \otimes \mathbb{C} \]

In general:

\[ \chi_r(\Delta, C) = \frac{1}{\mathfrak{g}!} \sum_{x \in Hm(\mathcal{Z}, C)} \sum_{\sigma \in C} (-1)^{d(\sigma)} \]

\[ = \ldots \]

\[ = \sum_{x \in Hm(\mathcal{Z}, C) \cap \mathcal{A}} (-1)^{d(\sigma)} \dim \left( \mathcal{H}^r(\mathcal{Z}, \zeta(x)^g) / \mathcal{H}^r(\mathcal{Z}, \zeta(x)) \right) \]

In particular, \( r = 2 \):

\[ \chi_2(\Delta, C) = \sum_{\sigma \in C} (-1)^{d(\sigma)} k(\zeta(\sigma)) \]

\[ \chi_2(\mathfrak{g}) = \frac{1}{\mathfrak{g}!} \sum_{\mathfrak{g} \in \mathcal{G}} \chi(\mathfrak{g}) \]

\[ \chi_2(\mathfrak{g}) = \frac{1}{\mathfrak{g}!} \sum_{\mathfrak{g} \in \mathcal{G}} \chi(\mathfrak{g}) \]
Both to \( KRC_{p}(G) \): \( \Delta = \Delta(G) \) and we set \( \mu(KRC_{p}(G)) \)

\[
\gamma_{2}(p^{i+1}\mathbb{Z}/G) = k(G) - 2 \gamma_{p}(G)
\]

Can replace \( p^{i+1}\mathbb{Z}/G \) by \( p^{i+1}\mathbb{Z}/G \) (look back at \( G = SL_{2}(\mathbb{F}) \))

\[
\sum_{\alpha \in \Delta_{0}(G)_{c}} k(G, \alpha) = \sum_{\alpha \in \Delta_{0}(G)_{c}} k(G, \alpha) = 5
\]

\( KRC_{p}(G) \neq KRC_{p}(G) \neq 1 \):

\[
\sum_{G} \gamma_{2}(p^{i+1}\mathbb{Z}/G) = \gamma_{2}(\mathbb{Z}/G) = 1\text{Hom}(\mathbb{Z}, G)/kG = 1\text{Hom}(\mathbb{Z}, G)/kG
\]

This then implies that \( \gamma_{p}(G) = 0 \), so \( KRC_{p}(G) \) is true.

**FALSE proof for \( AWG_{p}(G) \):

\[
KRC_{p}(G \times \mathbb{F}) \iff AWG_{p}(G \times \mathbb{F}) \Rightarrow AWG_{p}(G)
\]

Then maybe I went off a tangent.

\[
\gamma_{2}(p^{i+1}\mathbb{Z}/G) = 1
\]

\[
\gamma_{2}(p^{i+1}\mathbb{Z}/G) = k(G) - 2 \gamma_{p}(G)
\]

\[
\gamma_{2}(p^{i+1}\mathbb{Z}/G) = ?
\]

Maybe \( KRC_{p}(G) \) is just fine.
Examples of $\tilde{\chi}_r(G)$, $G$

\[ \frac{s}{\text{char}(G)} \]

\[ 1 + \sum \tilde{\chi}_r \left( G^{\alpha_1 \cdots \alpha_r}, G \right) \frac{x^n}{n} = \prod (1 - (1 + \text{tr}(i)) \frac{x}{q^n})^{(-1)^i(j)} \]

\[ 0 \leq i, j \leq r \]

\[ 1 - \sum \tilde{\chi}_r \left( G^{\alpha_1 \cdots \alpha_r}, G \right) \frac{x^n}{n} = \prod (1 - \text{tr}(i) x)^{(-1)^i(j)} \]

\[ 0 \leq i, j \leq r \]

\[ j \neq r \mod 2 \]

For $G_n \left( \mathbb{F}_q \right)$ and $r = 3$

\[ E_{G_n^+} \prod_{0 \leq i \leq 3} \frac{1 - 2 \frac{x}{q^i}}{(1 - q^{i+1} x)^3 (1 - q^i x)} \]

Some of these have occurred before in maths as Hasse-Weil zeta functions.

Remark equiv. Eqn

\[ \tilde{\chi}_r(\Delta, G) = \chi_r(\Delta, G) - \chi_r(\mathbb{F}_q, G) = \chi_r(\Delta, G) - \frac{|\text{Hom}(\mathbb{Z}^r, G)|}{|G|} \]
\[\chi^p_\mu(\Delta, G) = \frac{\Lambda}{G} \sum_{x \in H_1'(\mathbb{Z}/p, \mathbb{Z})} \chi_G^\mu(\bar{c}(x))\]

This \(\chi^p_\mu(\Delta, G)\) in the Euc of \(\Delta_{\text{Euc}} = (\Delta \times \mathbb{E}_2)/G\) computed in Norava \(K(r)\)-theory at \(p\).

\[\sum_{n=1}^{\infty} \left( \frac{-\chi^p_\mu(\bar{c}(x^n))}{n^2} \right) \exp\left( -\sum_{n=1}^{\infty} \frac{\phi_n(-1)^n}{n} x^n \right)\]