PROJECTS AND IDEAS

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proj-form

1. Preliminary project formulation

The purpose of this project is to investigate properties of catgories of subgroups. First we want to compute homology or cohomology of the Brown poset of p-subgroups of a given finite group, at least in some concrete cases. The second equivariant Euler characteristic, which is a K-theory Euler characteristic, is of special interest because of its connection to the Knorr-Robinson conjecture. It would be interesting, but maybe impossible, to find an interpretation of the higher equivariant Euler characteristics. Depending on how this goes, we intend also to consider other posets or categories of subgroups and look for connections between topology and group theory.

The time table is

First year: Learn background material, take courses
Second year: Start writing thesis, change of environment
Third year: Finish thesis.

rese-proj

2. Possible research projects

If you look at (some of) these problems you may find one that you are particularly interesyed in:

(1) Compute cohomology (singular, K, or Morava K) of $\mathcal{S}_G^{p+*}$ or its homotopy orbit space. Can K_G be replaced by H_G? What does H_G [2] mean? Is it connected to the Grothendieck construction $(\mathcal{S}_G^{p+*})_{hG}$?
(2) Describe the equivariant homotopy type of $\mathcal{S}_G^{p+*}$ with its G-action. Is $(\mathcal{S}_G^{p+*})^H$ for $H \leq G$ always spherical? If P is a p-group, is it always true that the Euler characteristic of $(\mathcal{S}_G^{p+*})^P$ is a power of p?
(3) What is the group theoretic interpretation of the rth equivariant Euler characteristic of the Brown poset of a group? There are interpretations for $r = 1, 2$. What about $r = 3$?
(4) Compute $\widetilde{\chi}_2(\mathcal{S}_G^{p+*})$ for the simple groups and verify the Knörr–Robinson conjecture. This is done in https://arxiv.org/abs/1502.01317 for some groups. In particular, G could be one of the sporadic simple groups or the alternating groups.
(5) Define the equivariant Euler characteristics of a finite G-category (not just a G-poset). Compute these invariants for some fusion categories.
(6) Investigate the poset of prime power index subgroups [14]. Equivariant Eucs of the poset of prime power index subgroups (eg for $\mathrm{GL}_n^+(\mathbf{F}_q)$). Is there a relation similar to Browns that the reduced Euler characteristic is always divisible by $|G|_p$? There is a variant of the Quillen conj: $\widetilde{\chi} = 0 \implies O^pG = 1$. What is the 2nd equivariant Euler characteristic? Does it have any group theoretical meaning?
(7) Equivariant Euler characteristic of a sequence of representations of a finite group in $\mathrm{GL}_n^+(\mathbf{F}_q)$ - or maybe all the equivariant Euler characteristics of a representation?
(8) The equivariant Lefschetz indices of a G-map? The indices of the identity map should be the equivariant Euler characteristics. Wasn't this studied somewhere?
(9) Is it true that any finite poset is a subgrop poset?
(10) Is there a Knörr–Robinson conjecture for blocks? The conjecture is probably indeed formulated for blocks. Is there a group action on something such that $\widetilde{\chi}_2$ of this action is equivalent to the Alperin Weight Conjecture?
(11) Equivariant Euler characteristic of finite groups of Lie type in cross characteristic case.

[8] [6]

First steps:

1. Explain the equivariant htpy equivalences from subposets of elementary abelian or radical subgroups.

2. Write down an account of K-theory Euler characteristic of the Brown poset [3, 4]. See [12, p 225] [1, 2] for information about Euler characteristics of quotient and fixed spaces. Compute som examples.

3. Symond's thm about the orbit space $|\mathcal{S}_G^{p+*}|/G$. What about the homotopy orbit space $|\mathcal{S}_G^{p+*}|_{hG}$?

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https://arxiv.org/pdf/1612.04302.pdf

https://arxiv.org/pdf/math/0210001.pdf

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