

Master Class on von Neumann algebras and groups  
Copenhagen, January 25-29, 2010  
Titles and abstracts — junior seminar

**Paramita Das** (K.U. Leuven) :

*Affine category over a planar algebra.*

We will define the affine category over a planar algebra and their representations. Given a finite index extremal subfactor, we will show that the space of affine morphism at the *zero level* is given by the fusion algebra of the bimodule category appearing in the standard invariant of the subfactor. This is a joint work with Shamindra Kumar Ghosh and Ved Prakash Gupta.

**Wojciech Dybalski** (Technical University, München) :

*Continuous spectrum of automorphism groups.*

This talk presents a general framework for refined spectral analysis of a group of isometries acting on a Banach space, which extends the spectral theory of Arveson. The concept of continuous Arveson spectrum is introduced and the corresponding spectral subspace is defined. The absolutely continuous and singular-continuous parts of this spectrum are specified. Conditions are given, in terms of the transposed action of the group of isometries, which guarantee that the pure-point and continuous subspaces span the entire Banach space. In the case of a unitarily implemented group of automorphisms, acting on a  $C^*$ -algebra, relations between the continuous spectrum of the automorphisms and the spectrum of the implementing group of unitaries are found. The group of spacetime translation automorphisms in quantum field theory is analyzed in detail. In particular, it is shown that the structure of its continuous spectrum is relevant to the problem of existence of (infra-)particles in a given theory.

**Caleb Eckhardt** (Université de Franche-Comté) :

*A Noncommutative extension of the Gauss map.*

The Gauss map for continued fractions naturally defines a Markov operator (with respect to Gauss measure) on  $C[0, 1]$ . We discuss an extension of this map to a Markov operator on a certain  $AF$ -algebra.

**Pierre Fima** (K.U. Leuven) :

*$W^*$ -superrigidity and groups acting on trees.*

I will present a joint work with Stefaan Vaes. Recently Sorin Popa and Stefaan Vaes have obtained  $W^*$ -superrigidity results for certain actions of certain free products. We extend this results to the case of groups acting on trees.

**Shamindra Ghosh** (K.U. Leuven) :

*Subfactors, bicategories and planar algebras.*

After recalling the definition of planar algebras (introduced by Vaughan Jones), we will outline a construction of the planar algebra arising from a 'singly generated pivotal bicategory'. We will discuss the important properties of this example associated to the bicategory of bimodules of a subfactor.

**Lukasz Grabowski** (Göttingen University) :

*Turing machines and dynamical systems*

In the talk I will explain what a Turing machine and a Turing machine with an oracle are. Then I will explain how these objects can be naturally seen as dynamical systems on compact abelian groups. Finally I will show how analysis of dynamical properties of these systems can give information about possible values of von Neumann dimensions of kernels of elements of integral group rings (i.e. about so called  $l^2$ -Betti numbers).

**Pawel Kasprzak** (Southern Denmark University) :

*Rieffel Deformation of Homogeneous Spaces.*

Let  $G_1$  be a closed subgroup of a locally compact group  $G$  and let  $X = G/G_1$  be the quotient space of the left cosets. Let  $(C_0(X), \Delta_X)$  be the corresponding left coaction of  $(C_0(G), \Delta)$  on  $C_0(X)$ . Suppose that  $\Gamma$  is an abelian closed subgroup of  $G_1$  and let  $\Psi$  be a 2-cocycle on the dual group of  $\Gamma$ . Using these data we may define the left coaction of the quantum group  $G^\Psi$ - the Rieffel deformation of  $G$  - on the Rieffel deformation of  $C_0(X)$ . On the other hand, we may perform the Rieffel deformation of the subgroup  $G_1$  obtaining the quantum subgroup  $G_1^\Psi$ , which in turn, by the results of Vaes leads to the C\*-algebraic quotient  $G^\Psi/G_1^\Psi$ . The aim of this talk is to show that two construction described above give the isomorphic quantum homogeneous spaces.

**Sören Möller** (Southern Denmark University) :

*A law of large numbers for the free multiplicative convolution.*

In classical probability the law of large numbers for multiplicative convolution follows directly from the law for additive convolution. In free probability, this direct relationship is missing, so the multiplicative law requires a proof of its own. We provide such a proof

by using the  $S$ -transform of probability measures on  $(0, \infty)$ .

**Henrik Densing Petersen** (University of Copenhagen) :

*The Dixmier unitarisability problem and  $\ell^2$ -Betti numbers.*

The Dixmier unitarisability problem asks whether every (say, countable) group all of whose uniformly bounded representations as invertible operators on Hilbert space are similar to unitary representations (called unitarisable groups), is amenable. The converse was essentially shown to hold by Sz.-Nagy in 1947.

I will discuss a result by Epstein and Monod, showing that for residually finite unitarisable groups the first  $\ell^2$ -Betti number vanishes. In particular I will focus on the connections between  $\ell^2$ -Betti numbers and random spanning forests, and if time allows, perhaps also some notes on higher  $\ell^2$ -Betti numbers.

**Nansen Petrosyan** (K.U. Leuven) :

*New action-induced nested classes of groups and jump (co)homology.*

Using fixed-point-free group actions, we set up a scheme to define nested classes of groups indexed over ordinals. Restricting to cellular actions on  $CW$ -complexes, we find new classes as well as new characterizations for some well-known classes, such as virtually polycyclic groups. We generalize properties of the virtual cohomological dimension of a group to groups with jump (co)homology and prove that a core subclass of a new class of groups has jump (co)homology.

**Sven Raum** (K.U. Leuven) :

*Fusion rules of free orthogonal quantum groups.*

Free orthogonal quantum groups are a class of subquantum groups of the free orthogonal Wang algebra which can be described by combinatorial means. Actually, Banica and Speicher were able to classify them, and it turns out that there are only six such algebras. As these quantum groups are more easily accessible than other ones it is natural to ask questions about their structure. We continued work of Banica, Vergnioux and others by calculating the fusion rules of all not yet considered free orthogonal quantum groups and of their free complexifications.

**Sergei Silvestrov** (Lund University) :

*Topological dynamics and maximal commutative subalgebras in crossed products.*

Description and mutual position of Maximal abelian subalgebras and ideals are important in construction and classification problems for Von Neumann Algebras and  $C^*$ -algebras and in applications in Representation Theory and Quantum Physics. Connection with properties and classification of dynamical systems and actions play central role in this context. In this talk, I will present a review and some new recent results on interplay

between properties of invertible and non-invertible dynamical systems and actions and intersection properties and structure of ideals and maximal abelian subalgebras in crossed products and their generalizations.

**Piotr Soltan** (University of Warsaw) :

*Examples of quantum homogeneous spaces.*

I will describe two examples of quantum homogeneous spaces of non-regular (non-compact) quantum groups. I will also discuss the action of the quantum groups on their homogeneous spaces and focus on the notion of continuity of actions. The technical tools I will use will include unbounded elements affiliated with  $C^*$ -algebras and algebras generated by such elements.