

Yamabe Problem on Various Domains

Jie (Frank) Xu
Boston University

In this talk, I will discuss the prescribed constant scalar curvature λ under pointwise conformal change $\tilde{g} = e^{2f}g = u^{p-2}g$. It is equivalent to find out a real, positive, smooth solution of the nonlinear elliptic PDE

$$-\frac{4(n-1)}{n-2}\Delta_g u + S_g u = \lambda u^{\frac{n+2}{n-2}} \quad (1)$$

possibly with appropriate boundary conditions. I will discuss the results (i) in a small Riemannian domain (Ω, g) ; (ii) on closed manifolds; (iii) on compact manifolds with smooth boundary. In contrast to the classical methods by global analysis in calculus of variation, I will discuss how to apply local analysis, iterative scheme and barrier methods to obtain the solvability of Yamabe equation directly in different cases associated with different boundary conditions, like Dirichlet or Robin conditions (if necessary). Some results above are due to a joint work with my advisor S. Rosenberg.