Book of Abstracts for the Hybrid Masterclass/Workshop

Mean Curvature Flow and Related Topics

GeoTop, University of Copenhagen

March 7th-11th, 2022

Mini Courses

Speaker: Francisco "Paco" Martín (University of Granada)

Title: Some Morse theory for minimal surfaces in 3-manifolds.

Abstract: In this mini-course, we will present some facts about Morse theory of minimal surfaces in 3-manifolds that are useful in order some local geometric properties of minimal surfaces. These results have interesting applications, not only in the study of minimal surfaces in Euclidean 3-space, but also in improving our knowledge about the local behavior of solitons of the mean curvature flow, like shrinkers and translators.

Speaker: Or Hershkovits (Hebrew University)

Title: Mean curvature flows asymptotic to cylinders.

Abstract: The first two talks in this mini course will constitute a crash course in mean curvature flow, which should be approachable to anyone with a basic knowledge in differential geometry, and which hopefully will nevertheless be of some interest to PhD students working in the field. The last two talks will concern the classification of ancient mean curvature asymptotic to the cylinder in $\mathbb{R}^3$. I hope to be able to convey how linear analysis, in addition to coarse geometric information, can be used in order to "read off" geometric properties of the solution. These last two talks will be based on Section 3, 4.1, 4.2 and 5 of the paper "Ancient low entropy flows, mean convex neighborhoods, and uniqueness" written jointly with Kyeongsu Choi and Robert Haslhofer.

Contributed Talks

Time: Monday, March 7th, 15:30–16:15.

Speaker: Michele Rimoldi (Politecnico di Torino)

Title: Quantitative index bounds for weighted minimal hypersurfaces via topology

Abstract: The recent impressive developments in the existence theory for minimal immersions have motivated a renewed interest in studying estimates on the Morse index of these objects. One possible way to control instability is through topological invariants (in particular the first Betti number) of the minimal hypersurface. This was first investigated by A. Ros for immersed minimal surfaces in $\mathbb{R}^3$, or a quotient of it by a group of translations, and then, in higher dimension, by A. Savo when the ambient manifold is a round sphere. In this talk we will first discuss how the method of Savo can be generalized to study the Morse index of self-shrinkers for the mean curvature flow and, more generally, of weighted minimal hypersurfaces in a weighted Euclidean space endowed with a convex weight. When the hypersurface is compact, we will show that the index is bounded from below by an affine function of its first Betti number. When the first Betti number is large this improves index estimates known in literature. In the complete non-compact case, the lower bound is in terms of the dimension of the space of weighted square integrable f-harmonic 1-forms. In particular, in dimension 2, the procedure gives an index estimate in terms of the genus of the surface.

Combining this technique with an adaptation to the weighted setting of Li-Tam theory we will also discuss some recent quantitative estimates on the Morse index of translators for the mean curvature flow with bounded norm of the second fundamental form via the number of ends of the hypersurface.

This talk is based on joint works with Debora Impera and Alessandro Savo.

Time: Monday, March 7th, 16:30–17:15.

Speaker: Otis Chodosh (Stanford University)

Title: Mean curvature flow of generic initial data and applications

Abstract: Mean curvature flow can develop complicated singularities. However, one can hope that a "generic" initial data produces a "better" mean curvature flow than the worst-case scenario. I will discuss the status of this problem (including some joint work with Kyeongsu Choi, Christos Mantoulidis, and Felix Schulze) and describe some potential (and actual) applications.

Time: Wednesday, March 9th, 15:30–16:15.

Speaker: Jingxuan Zhang (University of Copenhagen/University of Toronto)

Title: Asymptotic stability of cylindrical singularities

Abstract: We study the rescaled mean curvature flow (MCF) of hypersurfaces that are global graphs over a fixed cylinder of arbitrary dimensions. We construct an explicit stable manifold for the rescaled MCF of finite codimensions in a Gaussian weighted Sobolev space. For any initial hypersurface from this stable manifold, we construct a unique global dissipative solution to the rescaled MCF. This proves asymptotic stability of cylindrical singularities of arbitrary dimensions under generic (i.e. finite-codimensional) initial perturbations.

Time: Wednesday, March 9th, 16:30–17:15.

Speaker: Steen Markvorsen (Technical University of Denmark)

Title: Conductive Riemannian Manifolds

Abstract: In this talk we present a couple of results which show how conductive Riemannian manifolds generalize the notion of weighted metric measure spaces. Firstly, the natural conductive Laplacian gives rise (via its corresponding Bochner formula) to a Ricci type curvature, which, when bounded appropriately, implies Myers-Ambrose type compactness results for the Riemannian manifolds in question.

Secondly, we note how the conductive Laplacian induces inequalities and comparison results for the conductive capacity (for metric annuli) and for the conductive exit times (for metric balls), respectively.

Time: Thursday, March 10th, 15:30–16:15.

Speaker: Ali Muhammad (University of Copenhagen)

Title: Entropy bounds for embedded self-shrinkers with rotational symmetry.

Abstract: In this talk, we study the space of complete embedded rotationally symmetric self-shrinking hypersurfaces in $\mathbb{R}^{n+1}$. First, we derive explicit upper bounds for the entropy of all such self-shrinkers. Second, as an application we prove a smooth compactness theorem on the space of all such shrinkers. We also prove that there are only finitely many such self-shrinkers with an extra reflection symmetry.

This talk is based on joint work with John Ma and Niels Martin Møller.

Time: Thursday, March 10th, 16:30–17:15.

Speaker: Eric Ling (Rutgers, the State University of New Jersey)

Title: Remarks on the cosmological constant appearing as an initial condition for Milne-like spacetimes

Abstract: Abstract: Milne-like spacetimes are a class of $k = -1$ FLRW spacetimes which admit continuous extensions through the big bang. In previous work, it's been shown that the cosmological constant appears as an initial condition for Milne-like spacetimes. We generalize this statement to spacetimes which share similar geometrical properties with Milne-like spacetimes but without the strong isotropy assumption associated with them. We discuss an interesting relationship between the strength of the crushing singularity and the existence of spacetime extensions.

Time: Friday, March 11th, 10:00-10:45.

Speaker: Alexander Mramor (Johns Hopkins University)

Title: Some new applications of the MCF to self shrinkers.

Abstract: In this talk I'll discuss some applications of the mean curvature flow to the study of self shrinkers in $\mathbb{R}^3$ and $\mathbb{R}^4$.

Time: Friday, March 11th, 11:00-11:45.

Speaker: Jason Lotay (Oxford University)

Title: Translators in Lagrangian mean curvature flow

Abstract: Translating solutions to mean curvature flow are well-known to provide models for singularity formation along the flow when the singularity is Type II. One of the great challenges in the subject is to understand when a Type II singularity is necessarily modelled by a translator. In this talk, I will describe some recent joint work with Felix Schulze and Gabor Szekelyhidi which helps shed light on this problem in Lagrangian mean curvature flow, and I will discuss the role of translators more generally in this setting.

Time: Friday, March 11th, 13:15-14:00.

Speaker: Ilyas Khan (Oxford University)

Title: The structure of mean curvature flow translators with finite total curvature.

Abstract: In this talk, I will describe the asymptotic structure of 2D mean curvature flow translators embedded in $\mathbb{R}^3$ which have finite total curvature, which turns out to be highly rigid. I will outline the proof of this asymptotic description, in particular focusing on some novel and unexpected features of this proof. These include the construction of barriers for graphical translators and the use of Leon Simon's approximate graphical decomposition lemma to find blow-down limits.

Organizers:

- John Ma (GeoTop, U Copenhagen)
- Niels Martin Møller (GeoTop, U Copenhagen)