We analyze criteria for and effects of a country pegging (tying) its currency to a basket of foreign currencies. We demonstrate that there can be considerable benefits associated with this. As empirical cases we look at Turkey, Denmark, Greece, and Bangladesh. We find that (a) Turkey has a large idiosyncratic exchange rate risk and thus has a lot to gain from pegging its currency, the lira, to a basket representing its trade composition, (b) an insurance cost minimization-case can be made for the Danish krone keeping its peg to the euro, (c) if Greece leaves the euro but pegs its new currency to a trade weighted basket, the exchange rate risk costs are small, and (d) the cost-optimal basket peg for Bangladesh is a well-diversified portfolio (that isn’t exactly the trade weights) and switching to that gives an economically (as well as statistically) significant gain.

1 A peg to a basket

Introduction. Throughout history many countries have tried to peg (in some way) their currency to a larger, more dominant currency. Given what we know about the benefits of diversification — from Hans Christian Andersen’s fairy tale telling us not to
put all our eggs in one basket to Harry Markowitz’ work on mean/variance-optimization — it seems natural to ask if improvement (in some form) can be achieved by pegging to a diversified basket of currencies. For instance one that represents the trade composition of the country in question. This is what we analyze in this paper. The idea of pegging to a basket was floated in a number of academic papers in the 70’ies and 80’ies (for instance Flanders & Helpman (1979); their view is towards macroeconomic equilibrium, not directly (as is ours) volatility)) and in more applied papers following the South-East Asian currency crisis in the late 90’ies, but that was about how far the idea spread. However, with the increased focus on currency volatility — headline-making stories such as the ruble’s collapse, Switzerland removing its euro floor overnight, and Greece’s highly strained relationship with the euro — we hope that this omission will soon be rectified. This paper is a step in that direction.

**The mathematics of basket pegging.** A peg to a basket $b$ of $n$ currencies means that one unit of domestic currency pays $b_1$ units of foreign currency 1 and pays $b_2$ units of foreign currency 2 and … and $b_n$ units of foreign currency n. This means that the time $t$ exchange rate for foreign currency $i$ is

$$\text{peggedDOM}_{xxx_i}(t) = \left( \sum_{j=1}^{n} b_j \frac{xxx_i}{xxx_j}(t) \right)^{-1},$$

where we use the generic notation $xxx_i$ for the $i$th foreign currency, use DOM to denote domestic, and use the exchange rate notation where $xxx/yyyy$ means how many units of currency $xxx$ is needed to buy one unit of currency $yyy$, i.e. European as opposed to US or UK notation. To understand equation (1) start looking at the $j$th term in the sum, say $S$, on the right-hand side; $b_j$ is a number of units of currency $j$ that is then transferred into a number of units of currency $i$. The full sum $S$ then gives the value of the basket in currency $i$. So one unit of pegged domestic currency is worth $S$ units of foreign currency $i$, hence we quote the exchange rate as the reciprocal value of the sum, $1/S$.

**Practical and strategic considerations; or: What if this little peg went to market?** A central bank would keep a peg to a basket by being willing to both buy and sell one unit of foreign currency for the number of units of domestic currency given

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1Ngouana (2012) and Ma & Cheng (2014) are exceptions.
Turkey as a worked example. Looking at Turkey, Table 1 gives the exchange rate (currency lira, market abbreviation TRY) and trade trade weights for its six largest trading partners (where think of the eurozone as being one). The trade weights have been calculated as the sum of import and export with a specific counterpart relative to total import and export to the six counterparts. These six represented just over half of Turkey’s trade in 2013.

On Jan. 2, 2012 we could construct a basket that represents the trade weights and matches market exchange rates in this way:

\[ b_i = \frac{\text{trade weight}_i \text{TRYxxx}_i(\text{Jan. 2, 2012})}{0.1843, 2.9035, 0.5006, 0.0530, 0.0274, 0.0249}. \]

2Nobody – not least central banks — like to be reminded of their own fallibility. Nonetheless it remains as empirically solid a fact as there ever were.

3Speculative pressure can of course also occur with a single-currency peg. And markets are fickle, as this Commerzbank view and Hanke, Poulsen & Weissensteiner (2015) show in various ways in the case of the Swiss euro floor.
On June 1, 2012 the pegged-to-basket TRYEUR exchange rate was

\[
\text{peggedTRY}_{\text{EUR}}(\text{June 1, 2012}) = \left( \sum_{j=1}^{n} b_j \frac{\text{EUR}}{\text{TRY}}(\ldots) \frac{\text{TRY}}{\text{xxx}_j}(\ldots) \right)^{-1} = 2.41.
\]

Notice that when the basket is kept fixed over time (the numbers of units are the same), then the values of the different currencies in the basket relative to the value of the whole basket change because exchange rates change. (In the same way that relative wealth shares in a buy-and-hold stock portfolio change over time.) To demonstrate, the vector of value weights on Dec. 1, 2014 is (0.465, 0.116, 0.169, 0.106, 0.089, 0.055). This means that when we describe the basket in the intuitively by trade-weights, there must always an understanding of a reference date.

We can perform the same calculations for all trade counterparts over the period from early 2012 to late 2014. The results are shown in Figure 1, where the left-hand panel shows the behavior of the six currency crosses with and without pegging. To the naked eye the fluctuations of the pegged exchange rates are lower than for the actual exchange rates. This is quantified in the right-hand panel, where estimated volatilities (understood as the annualized standard deviation of daily logarithmic exchange rate changes) are depicted. We see that these are uniformly lower; a bit for the ruble, down to a third for the euro (which has a large weight in the basket), and about halved for four other currencies. This shows that the Turkish lira has a large idiosyncratic (i.e. country specific) volatility component.

2 Two approaches to quantifying diversification gains

We let \( Z_i^b(k) \) denote daily (\( \Delta t = 1/252 \) for daily observations) log-increments of the \( b \)-pegged exchange rate against currency \( i \), i.e.

\[
Z_i^b(k) = \ln \left( \frac{\text{peggedDOM}/xxx_i(\Delta tk)}{\text{peggedDOM}/xxx_i(\Delta t(k - 1))} \right).
\]

Assuming these \( Z \)'s are independent over time, we define \( \Sigma \) as the annualized covariance matrix,

\[
\Sigma_{i,j}^b = 252\text{cov}(Z_i^b, Z_j^b),
\]

which we estimate by simple sample moments.
Figure 1: The left-hand panel shows the time-series behaviour of the pegged (red) and unpegged (black) version of the Turkish lira against major trading partners. The right-hand panel shows the individual volatilities for pegged (red) and unpegged (black) versions of lira.
Given a vector \( w \) of trade weights, we might be interested in choosing the basket \( b \) such that it minimizes the variance (i.e. the risk) of payments

\[
\min_b w^\top \Sigma^b w,
\]

(2)

where the minimization is performed over baskets with positive entries scaled such that the initial values of pegged and actual exchanges are equal. The variance minimization approach takes a bird’s eye view. Favorable movements in one currency can offset unfavorable movements in another; correlations matter. But that might not be a relevant or feasible view. The currency risk exposure comes from individual companies whose trade compositions do not match that of the whole economy. Hence another sensible measure of currency risk (towards a specific currency) would be how much it would cost to buy insurance against unfortunate movements. This can be quantified through option prices. More model specifically we do it by the Garman–Kohlhagen formula. The insurance cost/option price will depend on the \( i \)th diagonal element of the \( \Sigma \)-matrix only. We ignore interest rates, look at at-the-money options (meaning that put and call option prices are equal or that insurance is bought against changes relative to current exchange rate) with four month maturities (a typical time period for companies’ currency risk exposures). The overall basket-picking criterion then becomes minimizing the total insurance cost, i.e.

\[
\min_b \sum_{i=1}^n |w_i|\text{Garman} - \text{Kohlhagen}(\ldots, \sqrt{\Sigma^b_{i,i}}),
\]

(3)

Because (see Figure 2) short-term, at-the-money option prices are almost linear in volatility (\( \sigma \), not \( \sigma^2 \)), for all practical purposes solving (3) corresponds to minimizing trade weighted average volatility. This also allows a sensible partner-by-partner breakdown of the gains from pegging.

2.1 Which trade weights to use?

Define

\[
w^{\text{import}}_i = \frac{\text{import from counterpart } i}{\text{total import}}, \quad w^{\text{export}}_i = \frac{\text{export to counterpart } i}{\text{total export}}.
\]

We could then consider the more flexible or elaborate version of the optimization problem in (2):

\[
\min_b (\alpha w^{\text{import}} + \beta w^{\text{export}})^\top \Sigma^b (\alpha w^{\text{import}} + \beta w^{\text{export}}),
\]

(4)
where \((\alpha, \beta)\) represents our weighting of import and export. Our common choice is \(\alpha = \frac{\text{total import}}{\text{total im- and export}}\) and \(\beta = \frac{\text{total export}}{\text{total im- and export}}\), which represents a balanced view of import and export. But that isn’t the only possible choice; \(\alpha = 0\) would put the focus solely on risk-minimization export-wise, while \((\alpha, \beta) = (1, -1)\) would look only at net-exposure. The latter is sensible if individual companies’ trades mirror the whole country’s trading pattern. It is doubtful if that is the case.

3 Three Cases: Denmark, Greece, and Bangladesh

**Denmark.** For more than 30 years, the Danish krone has been closely pegged to (first) the D-Mark and (then) the euro. In the following we let the trade weight vector \(w\) be the scaled version of Denmark’s five largest trading markets (import plus export; these are quite symmetric for Denmark); the eurozone, Sweden, Britain, Norway and China. In actual terms this comprises about 80\% of the total Danish trade. Table 2 below gives results of various analyses and optimizations. The different baskets are given (in rows 3-7) in terms of each currency’s (columns 2-6) initial relative share of the basket’s initial
Table 2: Effects of different basket choices for Danish krone against major export economies.

<table>
<thead>
<tr>
<th>Basket construction</th>
<th>EUR</th>
<th>SEK</th>
<th>GBP</th>
<th>NOK</th>
<th>CNY</th>
<th>Trade volatility</th>
<th>Insurance price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0174</td>
<td>0.0059</td>
</tr>
<tr>
<td>EUR peg</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0177</td>
<td>0.0059</td>
</tr>
<tr>
<td>Equal weights</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.0144</td>
<td>0.0083</td>
</tr>
<tr>
<td>Trade weights</td>
<td>0.562</td>
<td>0.176</td>
<td>0.100</td>
<td>0.091</td>
<td>0.070</td>
<td>0.0006</td>
<td>0.0070</td>
</tr>
<tr>
<td>Minimum variance</td>
<td>0.558</td>
<td>0.180</td>
<td>0.098</td>
<td>0.096</td>
<td>0.068</td>
<td>0.0004</td>
<td>0.0070</td>
</tr>
<tr>
<td>Minimum insurance price</td>
<td>0.973</td>
<td>0.011</td>
<td>0.007</td>
<td>0.003</td>
<td>0.004</td>
<td>0.0166</td>
<td>0.0057</td>
</tr>
</tbody>
</table>

The reported “Trade volatility” is the square root of the numerically optimized version of equation (2); the “Insurance price” is the optimized version of equation (3) which is expressed a fraction of total export.

There are several noteworthy things about the table. A peg to the euro does not remove all currency risk (row 3); almost half of the Danish export goes outside the eurozone. Linking to an optimally chosen basket (row 6) can remove practically all volatility at an aggregate level; volatility for the variance optimal basket is less than one half of one tenth of a percent. Using the trade weights directly (row 5) to construct the basket gets us very close to variance optimality. However, this low aggregate volatility comes about primarily because of the correlation structure, as shown in Figure 2 individual volatilities decrease at most by a quarter, and volatility against the euro even goes up from almost 0 to about 2%. This means that taking the “minimize insurance premium” point of view gives a quite different result for the optimal basket to peg to. To minimize the insurance cost the Danish krone should be almost perfectly (97+%) pegged to the euro, as seen in the last line of the table. This may seem a surprisingly close peg to the euro, but it happens because the insurance price criterion — while politically viable — introduces a first-past-the-post or winner-takes-it-all effect. Notice however, that the insurance costs differ little between the two optimal strategies (0.57% vs. 0.70%), which could lead to a suggestion of the minimum variance basket – or even the simple trade weighted basket – as a robust choice.

**Greece.** There are two characteristic things about the Greek trade balance (1) a
Volatilities when the Danish krone is pegged to the variance optimal basket (red) and when it isn’t (black).

Figure 3: The volatility effects of various pegs of the Danish krone.

massive trade deficit (in 2013 about USD 25 billion; import twice the size of export; Greek GDP about USD 240 billion in 2013), (2) a high degree of asymmetry between import and export countries. Point (1) makes it hard to run an economy; point (2) makes a reasonable choice of trade weights more tricky or debatable. However, we will stick with the import + export weights, but with the added assumption that the large energy imports from Russia and Iraq are settled in US dollars. The results are given in Table 3. Again, insurance price differences across baskets are economically small (costs range between 0.69% and 0.95%), while a considerable lowering of aggregate volatility (2.6% down to almost 0) is achieved through something resembling a trade weighted portfolio, which here means an about even peg to the euro and US dollar. Another way to interpret the results in the table is that if Greece leaves the euro, but pegs to (say) a trade weighted basket, the currency risk effects (understood as the insurance costs) are only marginally affected (the cost increase is 0.7 times one tenth of one percent of the trade balance).

Bangladesh. The following figure shows the volatilities of BDTxxx exchange rate crosses (corresponding to the 8 biggest trade partners; export + import) with different choices for the basket to which BDT is pegged. The trade figures are from 2013, the
<table>
<thead>
<tr>
<th>Basket construction</th>
<th>USD</th>
<th>EUR</th>
<th>TRY</th>
<th>CNY</th>
<th>Trade volatility</th>
<th>Insurance price</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR peg</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.026</td>
<td>0.0070</td>
</tr>
<tr>
<td>USD peg</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.036</td>
<td>0.0095</td>
</tr>
<tr>
<td>Trade weights</td>
<td>0.2944</td>
<td>0.31</td>
<td>0.109</td>
<td>0.066</td>
<td>0.0010</td>
<td>0.0077</td>
</tr>
<tr>
<td>Minimum variance</td>
<td>0.2928</td>
<td>0.525</td>
<td>0.119</td>
<td>0.063</td>
<td>0.0008</td>
<td>0.0077</td>
</tr>
<tr>
<td>Minimum insurance price</td>
<td>0.016</td>
<td>0.982</td>
<td>0.003</td>
<td>0.006</td>
<td>0.025</td>
<td>0.0069</td>
</tr>
</tbody>
</table>

Table 3: Effects of different basket choices for a Greek currency against major trade partners.

Figure 4: The volatility effects of various pegs of a Greek currency.
exchange rates range from Jan. 2013 to Feb. 2015, and the top eight trade weights – export plus import, listed in the order indicated on the x-axis – are (0.22, 0.14, 0.13, 0.31, 0.07, 0.06, 0.04, 0.04) and represent about 75% of Bangladesh’s trade. Volatilities are generally lowered, but neither the volatility nor insurance price minimizing baskets achieves a uniform decrease in volatility. This is because the BDT has been following the USD (and thus CNY). This means that if the currency peg regime is changed it will be to the disadvantage of some companies, but from an overall perspective there will still be an advantage.

The economy-wide gains towards the eight largest trading partners are quantified in Table 4. Whether one focusses on column two or column three depends on how broad a view of the economy one is allowed to take. In column two (“trade volatility”), gains and losses in different currencies are allowed to offset each other; column three gives the (total) cost of insuring every individual trade. We see that pegging to a balanced basket (basket compositions are not reported, but even for the two last rows, they are close to the trade weights), overall trade volatility is lowered markedly; from about 0.05 down to 0.001 when the volatility minimizing basket or simply the trade weights
<table>
<thead>
<tr>
<th>Type of peg for BDT</th>
<th>100 * Trade volatility</th>
<th>Insurance price (% of trade)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>5.0</td>
<td>1.74</td>
</tr>
<tr>
<td>To USD</td>
<td>4.9</td>
<td>1.63</td>
</tr>
<tr>
<td>To EUR</td>
<td>6.2</td>
<td>1.79</td>
</tr>
<tr>
<td>Trade weights</td>
<td>0.1</td>
<td>1.55</td>
</tr>
<tr>
<td>Min. volatility-weights</td>
<td>0.1</td>
<td>1.55</td>
</tr>
<tr>
<td>Min. insurance price</td>
<td>2.0</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Table 4: Effects of different basket choices for Bangladeshi taka against major trade partners. Volatility and insurance are measured for the top eight trading partners.

are used. In short, there are strong gains from diversification.\(^4\) Looking at the more conservative measure of insurance costs, the gains are smaller but still clearly visible. Using trade weights to construct the basket would lower economy-wide insurance cost against the top eight trade partners from 1.74% of the trade balance to 1.55% of the trade balance, which given the size of the Bangladeshi economy corresponds to USD 119 million.

It is a well-documented fact that volatilities and correlations in exchange rate markets fluctuate over time, rather than being stable as the analysis above implicitly assumes. To investigate that stability of the results, we performed the exact same analysis with the input data used to estimate correlations and volatilities stemming from eight non-overlapping three-month periods, i.e. a quarter-by-quarter analysis. Figure\(^5\) below shows the results. The left-hand panel show the estimated gains (in mUSD/year) for each of the eight estimation quarters; the red curve is the gain when the insurance price optimal weights are used, the blue curve is for simple trade weights. The right-hand panel shows the time-development of weights in the eight different currencies; the flat blue lines are trade weights, the black curves are volatility optimal weights (black and blue are practically indistinguishable) and the red curves are the insurance price optimal weights. We see some variation in the estimated gains (between USD 70 million and 220 million per year), but the average gains (USD 138 million for insurance price

\(^4\)The trade weights are the outset for GCU’s construction of so-called G-rates, but to better reflect markets conditions, the latter are adjusted more frequently than the weights in the analysis
Figure 6: Stability analysis for Bangladesh. The right-hand panel shows estimated gain (red for insurance cost optimal baskets, black for the trade weight basket) based on non-overlapping three-month intervals during 2013-14. The left-hand panel are graphs showing the development over time of trade (flat black), minimum variance (black) and insurance price optimal (red) baskets.
optimal weights, USD 119 million for trade weights) should still give a good impression of the gains. (And again, this is quite a conservative measure of gains from a central bank’s point of view.) The estimated insurance price optimal weights do fluctuate, but they always represent well-diversified basket (only two weights are ever above 40%, none above 50%).

References


