FinKont2 Hand-in Exercise #2: Coupon-Bond Options and Swaptions

First, remind yourself how options on coupon-bonds can be priced in the Vasicek model. i.e. how exercise 5 of Hand-In #1 was solved. The original, but hard-to-read, reference for the “find $r^*$”-trick is Jamshidian (1989).

Second, recall how swaptions work and that pricing them is equivalent to pricing options on coupon-bonds; relevant places to look are Björk (2004, Section 20.3, 25.7-10) and the lecture slides from Thursday February 26.

1-dimensional models and exact pricing

Questions/assignments:

- Implement the coupon-bond pricing technique for the Vasicek model.
- Would Jamshidian’s -trick work/be helpful in a CIR-model?
- Would Jamshidian’s trick (appear to) work in, say, a 2-dimensional Gaussian interest rate model?
- Write a program that simulates the Vasicek model and use it to verify the “Jamshidian’s trick”-solution.

1-dimensional models and approximate pricing

Read Schrager & Pelsser (2006, Section 4).
Questions/assignments:

- Prove equations (4.2) and (4.3).
- Explain the approximation in equation (4.4), and why that leads to their $\sigma_{n,N}$ as “the relevant volatility”.
- Prove (4.6). (A “standard result” about means of truncated normal variables is useful.)
• Implement the (approximative) formula and compare your results to Schrager and Pelsser’s Table 4.2.

Read Munk (1999, Section 1.1, 2.1 and 3.1).

Questions/assignments:

• How is stochastic duration defined?

• What is stochastic duration in a Vasicek model?

• How is the price of a coupon bond option approximated?

• What is the idea behind the approximation? (Focus on the 1-dimensional case for now.)

• Implement the stochastic duration approximation for the Vasicek-model. Compare numerical results to those from above (ie. Schrager and Pelsser’s Table 4.2 again).

• How many simulations are needed for the approximate prices to fall outside the, say, 95%-confidence interval of the simulation estimates?

2-dimensional models and approximate pricing

Read the appendix in Schrager & Pelsser (2006). (I believe that what they really mean on page 691 is $\hat{\Sigma} = \sqrt{\text{diag}(\Sigma \Sigma^\top)}$. As it stands, there are standard deviations on the left hand side, and variances on the right.)

Questions/assignments:

• How does “our favorite” 2-dimensional Gaussian model (lectures on Thursday March 5; or G2++ from Brigo & Mercurio (2001, Chapter 4) without time-dependence in the coefficients) fit into this framework?

• What is meant when they say (middle of page 691) “This is not important as long as the term structure generated by the model is known at time of valuation”?

• How does one get the rather horrible looking equation (A.3) on the middle of page 692?

• Implement the approximation for the 2-dimensional Gaussian model.

• Investigate the accuracy of the approximation. You could chose $r_0 = \delta = 0.05$ and our (rather crudely obtained) time-series estimated mean-reversion and volatility parameters from the lectures on Thursday March 5.
• What would happen if you just read off the 2-dimensional CIR-parameter values in Schrager and Pelsser’s Table 4.1 and said “I’ll use those (to the extend I can).”?

Reread Munk (1999, Sections 1.1, 1.2 and 3.1), now with a 2-factor state of mind.
Questions/assignments:

• How is stochastic duration calculated in the 2-dimensional Gaussian model?

• Which zero-coupon bond option formula must we plug into, ie. what is $C$ on the right hand side of equation (19)?

• Implement and compare the results to Schrager and Pelsser’s approximation and the simulated prices.

• Explain why Munk’s “promising candidate” $T^*$ on page 172 is not the stochastic duration in the 2-factor model.

• Implement a method for finding $T^*$, and investigate Munk’s claims that (i) there typically is little difference to using stochastic duration and (ii) where the is a difference, stochastic duration works better.

References


