Riding the Yield Curve

First some notation. Slightly cumbersome, but we need it. For a calendar date $t$ and a time to maturity $\tau$, let $Y_t(\tau)$ be the continuously compounded zero-coupon (spot) rate for time to maturity $\tau$, i.e. the (annualized) rate of return we get by investing (at time $t$) in zero coupon bonds maturing at time $t + \tau$ and holding them until expiry. The mapping

$$\tau \mapsto Y_t(\tau)$$

we call the (time-$t$ zero-coupon) yield curve. In terms of zero-coupon bond prices (current time $t$, maturity-date $T$):

$$P_t(T) = e^{-(T-t)Y_t(T-t)}.$$ 

Suppose we want to invest at time 0, look one year ahead, and have at our disposal zero-coupon bonds for all possible maturity-dates. The rate of return we get from investing in a maturity-date $T$ zero-coupon bond is

$$i_1(T) = \frac{P_1(T) - P_T(T)}{P_0(T)} = \frac{P_1(T)}{P_0(T)} - 1$$

A natural question is: Can we maximize this by choosing an appropriate $T$? The short answer is "No, not without being able to look into the future". Remember, not until time 1 do we know what $P_1(T)$ is. So we need to make further modelling assumptions. A natural first step is the hypothesis

\[ H_0: \text{The time-1 yield curve will be the same as the time-0 yield curve} \]

At time 1, a maturity-date $T$ zero-coupon bond has time to maturity $(T - 1)$, so under the $H_0$ hypothesis we have

$$P_1(T) = e^{-(T-1)Y_0(T-1)} = P_0(T - 1).$$

Thus the rate of return becomes known. It is in fact our old friend the (1-year-ahead) forward rate,

$$i_1(T) = \frac{P_0(T - 1)}{P_0(T)} - 1 = f_0(T - 1).$$
Figure 1: Zero-coupon and (1-year-ahead) forward rate curves estimated from UK Government bonds mid-October 2010.

So to maximize we should invest according to the highest forward rate. (Or more precisely: Find the time to maturity for the maximal forward rate and then invest in zero-coupon bonds with 1 year more to maturity than that.) And that forward rate will then be our return. This strategy is called ”riding the yield curve”. Note that it can be carried out whether $H_0$ holds or not — but only in the former case are we sure what our return will be.

In words, this strategy says that to maximize investment returns, go not where the yield curve is at its highest, but where it is at its steepest. If the yield curve is ”truly curved” then this can have surprising effects. A good example is provided by the UK yield curve from mid-October 2010; the one we found in Course Work #2. The zero-coupon and (1-year-ahead) forward curves are shown in Figure 1. The circles are (more or less) observed points on the zero-coupon curve. The smooth curve was fitted through them with an interpolation technique called cubic spline. The
smooth curve was then used to calculate forward rates. (Notice how the forward rate curve is considerably less smooth than the zero-coupon rate curve. How to deal with this is the focus of numerous research articles.)

We see that the forward rate curve attains its maximum around 7.5 years, and that the maximal value (5.4% continuously compounded) is considerably above any zero-coupon rate; that curve maxes out at 4.2% for 30-year maturities. Could this trick be repeated over and over for, say, 30 years £1 would grow to $e^{30 \times 0.054} = 5.08$, while investing in maturity-date 30 zero coupon bonds and holding pays back only 3.57. That difference should make any pension fund manager sit up and take notice.

Magic? Alchemy? Or: Is yield curve riding really the free lunch that is seems to be? Of course not.

First, it’s risky. We only get the return we think if hypothesis $H_0$ holds, i.e. if the yield curve does not move. And that is a big if. And the longer the maturity of the bond we have invested in, the greater the sensitivity.

Second, we may reverse the question and ask: How should the yield curve move for all 1-year returns to be the same? It turns out that if future zero-coupon spot rates are realized at the current forward rates then no gains be achieved by short-term riding or rolling. (It should be added: "for appropriately matched times to maturity and years-ahead". To make the statement precise we’d need three time indices, so we’ll spare the reader.) This then leads to a the counter-argument called the unbiased expectations hypothesis.

$H_1$: Forward rates are expected future zero-coupon spot rates

So which hypothesis is it then? Well, the truth (if such a thing exists at all) is somewhere in between. First, these are technical arguments (to do with expectations of non-linear functions, Jensen’s Inequality, and absence of arbitrage) against the unbiased expectations hypothesis. But the main reason is risk-aversion: If all bonds give the same expected return, then why invest in risky ones at all? Thus prices of long-term bonds will be "low" and one is rewarded for taking the riskier positions — such as riding the the yield curve. (But, arguably, riding the yield curve gives "double exposure": It’s risky — and to the extend that it’s not, movements will go against you!)