Why the NPV Criterion does not Maximize NPV

Elazar Berkovitch
Interdisciplinary Center, Herzliya (IDC)

Ronen Israel
Interdisciplinary Center, Herzliya (IDC)

This article presents a theory of capital allocation that shows how the use of net present value (NPV) as an investment criterion leads to inefficient capital budgeting outcomes and how this criterion may be dominated by other capital budgeting criteria, like the internal rate of return and the profitability index. The essence of our theory is rooted in the mainstream paradigm of corporate finance: while firms use NPV to measure the addition to firm value from prospective projects, “classical” informational and agency considerations prevent it from implementing the optimal capital budgeting outcome. Our theory also identifies conditions when alternative criteria should be used. Finally, we characterize when direct monitoring through capital budgeting dominates compensation contracts in alleviating the agency problem.

The use of net present value (NPV) as a criterion for project selection is a standard tenet of scholars, students, and authors of corporate finance textbooks. However, this consensus regarding NPV within the academic community is at odds with findings on the actual practices of firms in making investment decisions. Indeed, firms use criteria like internal rate of return (IRR), payback period, and profitability index (PI) more often than they use the NPV criterion in selecting projects.1 This discrepancy

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1 For example, Stanley and Block (1984) report, for a survey of 121 multinational firms, that “The internal rate of return clearly dominates as the primary method with a 65.3 percent representation. The net present value approach, often preferred over other methods in published materials on the subject, was specified as the primary method in only 16.5 percent of the responses.” This report is consistent with the findings in Mao (1970), Schall, Sundem, and Geijsbeek (1978), Scott and Petty (1984), and Ross (1986). For example, Schall, Sundem, and Geijsbeek (1978) study a sample of 189 firms and report that 74% used payback (with only 2% using it as the sole criterion), 58% used accounting rate of return (4% using it alone), 65% used IRR (6% using it alone), and only 56% used NPV (with only 2% using it alone).
between theory and practice raises the question: Are firms unaware of academic recommendations regarding NPV as a criterion or are scholars and financial economists unaware of the complexity and specific capital budgeting problems and needs faced by firms?

Regarding the firm’s perspective, the influx of MBAs into financial management positions suggests that firms are aware of the academic consensus about the use of NPV as a capital budgeting criterion. Nevertheless, despite this awareness, firms have not adopted NPV as their dominant capital budgeting criterion. Indeed, in a recent exhaustive survey of multinational firms, Segelod (1995) finds that the IRR is still the dominant criterion. Although use of the NPV criterion has increased over the years (from 11% of firms requesting it in their capital budgeting forms in 1964, to 33% in 1977, to 52% in 1990), it is still clearly a secondary tool, as explained by the author: “The NPV criterion has also become increasingly popular, although not as popular as the table may imply. Some of the groups which use IRR use the NPV or the PI to interpolate the correct IRR. Both criteria are requested on their forms, but the IRR appears to be the principal criterion for these groups” (p. 85).

In this article we present a theory that explains why and how firms use different capital budgeting criteria in their capital allocation systems. We explain the selection of capital budgeting criteria as a way to implement an optimal capital allocation system. This approach enables us to identify scenarios where the NPV criterion leads to inefficient capital budgeting outcomes; this explains why NPV is often secondary in firms’ budgeting decisions to other criteria such as the IRR and the PI. Our theory is rooted in the mainstream paradigm of corporate finance: while firms use NPV to measure the addition to firm value from prospective projects, “classical” informational and agency considerations prevent it from implementing the optimal capital allocation outcome. In fact, in many cases, it implements strikingly inferior outcomes.

To explicate our theory, we develop a model of a firm that consists of two components: (1) top management, or “headquarters” and (2) divisional mangers, or “the manager.”  

2 This structure has been used frequently by researchers of corporate finance. For similar modeling in the context of capital budgeting, see, for example, Harris and Raviv (1996, 1998) and Stein (1997). Since we assume headquarters perfectly represents the interests of shareholders, this model is similar to classical principal-agent models [see Harris and Raviv (1979) and Holmstrom (1979)].
The difference between our approach and the “textbook” approach to capital budgeting is best illustrated with a simple example. If headquarters can observe, at no cost, all available projects, then the textbook NPV rule will lead headquarters to select the project that maximizes shareholder value. However, if headquarters cannot observe all available projects, then the manager may manipulate the selection process by presenting projects such that managerial utility is maximized.

To see how the manager may manipulate the selection process, suppose that the set of potential investment projects for the firm includes two mutually exclusive projects. The projects arrive randomly and independently; consequently one, both, or neither may become available. Suppose the manager prefers a project that requires a larger investment, although this project has a lower NPV. In this case, if both projects become available, and if headquarters uses the textbook NPV rule as its capital budgeting criterion, the manager may choose to request only the low NPV project. As a result, the set of projects headquarters considers will differ from the actual available set. Thus the NPV rule does a poor job in restricting manipulations by divisional managers who privately observe the availability of mutually exclusive projects. Depending on the underlying cash flow distribution and the severity of the agency problem between managers and headquarters, a different capital budgeting rule may be needed to ensure that the high-NPV project is selected. In other words, while NPV is the best way to measure value added, in many situations, it is not a good way to implement the selection of the highest NPV projects.

Our theory shows why the NPV rule cannot implement the optimal project selection outcome. If the NPV rule does not implement the selection of projects with the highest NPV, what mechanism does? In general, the rules governing the project selection process may be very complex, depending upon the economic environment. Therefore they may have many restrictions and detailed guidance. However, we identify economic environments where simple capital budgeting criteria like the IRR and PI implement the optimal outcome. In different economic environments, the firm has to choose between simple criteria and more complex criteria that restrict the manager more efficiently but are more expensive to implement. We argue that in many situations simple rules dominate more complex rules. Therefore firms often use these simple criteria.

Our theory has a number of implications for both researchers and firms. First, from an academic standpoint, our theory helps reconcile the textbook NPV criterion recommendation with actual firm capital budgeting practices. Second, it lends insight into the capital budgeting process by illustrating how modified forms of capital budgeting criteria can provide optimal outcomes. For example, firms will use a modified IRR rule where the hurdle rate is different than the risk-adjusted cost of capital. Thus our
theory can explain the puzzling finding in Poterba and Summers (1995) that U.S. firms use real hurdle rates that are considerably higher than a standard cost-of-capital analysis would suggest.\textsuperscript{3} Similarly, to the extent that multinational firms investing abroad confront more severe implementation problems, our theory provides a possible rationale for the often-criticized practice of adding a fudge factor to their cost of capital.

Our theory belongs to the growing capital budgeting literature. Antle and Eppen (1985) show that in a setting of asymmetric information it is optimal for firms to ration their investments in some states of nature. The firm can do so by using a hurdle rate that is higher than the cost of capital. In a related article, Harris and Raviv (1996) provide a theory of endogenous capital rationing as a way to curb empire-building managers while benefiting from their private information. In a later study, Harris and Raviv (1998) characterize when it is optimal for headquarters to delegate to the manager the allocation of a total budget for two independent investment projects.\textsuperscript{4} These studies show that NPV is an ineffective criterion because headquarters cannot know project NPV without verification costs. However, our theory shows that, even if NPV is known, it still will not implement the optimal outcome in these cases.

Narayanan (1985) rationalizes the use of the payback method by managers that prefer projects with early cash flows to enhance their reputation. Thakor (1990) provides a rationale for capital rationing in an environment where the firm is cash constrained and therefore prefers short-term projects in which the cash flow is realized sooner. This preference toward short-term projects may be implemented, for example, by the payback method. In contrast to these models, in our model there are no cash constraints and the capital allocation system is used to restrict managerial discretion. Hirshleifer and Suh (1992) show, in a setting similar to ours, how compensation contracts are used to induce self-serving managers to take high-value, high-risk projects. We endogenously solve for the optimal method, wage compensation or capital budgeting, to alleviate the agency conflicts.

In Stein (1997), a firm faces an exogenous capital constraint and its informed headquarters allocates the limited resources to the better project ("winner picking") at the expense of the other project ("loser sticking"). Trading off headquarters’ monitoring ability against the efficient allocation of resources, Stein explains optimal scale and scope for firms. In our model, headquarters does not face capital rationing, but can be manipulated by better-informed, lower-ranked managers.

\textsuperscript{3} Specifically, Poterba and Summers (1995) find that, during the past half-century, the average real discount rate was 12.2\%, distinctly higher than equity holders’ average rates of return of approximately 7\% and much higher than the return on debt of approximately 2\%.

\textsuperscript{4} In previous work, Harris, Kriebel, and Raviv (1982) show how transfer pricing is used in allocating funds within the firm in the presence of information and incentive problems.
Finally, our theory relates to the real option literature. This literature has also identified a problem with the NPV criterion. Specifically, simple NPV calculations can miss the real option value of a project, resulting in rejection of value-creating projects whose option value exceeds the absolute value of their naive negative NPV. In this case, the wrong decision results from miscalculation of the true NPV. McDonald and Siegel (1986) and McDonald (1999) show that when the firm cannot calculate the true NPV, it is optimal to use a hurdle rate or the PI criterion. In this respect, our results are similar to theirs. However, in our article, headquarters can calculate the NPV of all projects they observe.

The remainder of the article is organized as follows. Section 1 provides the basic structure of the capital allocation process and characterizes the optimal capital allocation system. Section 2 evaluates the effectiveness of common capital budgeting criteria in implementing the optimal outcome. Section 3 concludes. All proofs are relegated to the appendix.

1. A Model of the Capital Allocation Process

In this section we outline the model underlying our theory of optimal capital allocation. In this model we consider a firm consisting of (1) top management, or “headquarters” and (2) divisional managers, or “the manager.” In this firm, the objective of headquarters is to maximize shareholder value, while the objective of the manager is to maximize her own utility. To maximize shareholder value, headquarters sets up a capital allocation system specifying both rules for the capital budgeting process and compensation for divisional managers. To abstract away from exogenous capital rationing issues, we assume that headquarters faces no liquidity constraints. We assume that all agents in the economy are risk neutral.

The main idea behind our theory can be demonstrated by considering an investment opportunity set consisting of one business venture that is implemented by two mutually exclusive projects. The only difference between the two projects is the amount of capital required to operate the respective underlying technologies. One project, project H, is more capital intensive. The other project, project L, is less capital intensive. While the characteristics of each project are known, there is uncertainty about which combination of the two projects is actually available to the firm. The arrival of projects L and H follows a binomial process with independent probabilities of arrival, \( g_L \) and \( g_H \), respectively.

The actual levels of investment and cash flow of each project depend on a scale parameter, \( \alpha \), representing the level of demand for the final product of the firm. The cumulative distribution function of the scale parameter \( \alpha \) is \( F(\alpha) \). Specifically, the required investment for project \( m = L, H \) is \( I_m(\alpha) = \alpha I_m \), where \( I_L \) and \( I_H \) are scalars that represent the underlying
technology. We further specify that $I_H > I_L$, reflecting the assumption that project $H$ is more capital intensive than project $L$. This assumption implies that $I_H(\alpha) > I_L(\alpha)$. Regardless of the underlying technology, the expected cash flow over period $t$ is identical for both technologies. This expected cash flow is represented by $c_t(\alpha) = \alpha c_t$, where $c_t$ are known scalars. Note that these assumptions represent a constant return-to-scale production technology. Because of risk neutrality, the appropriate discount rate is the riskless rate, $\rho$, independent of the underlying technology. Thus the present value of future cash flow, denoted by $PV = \sum_{t=1}^{\infty} c_t/(1 + \rho)^t$, is also independent of the underlying technology. Therefore the NPV of a type $m$ project is given by

$$NPV^*_m(\alpha) = -aI_m + \alpha PV m = L, H.$$  \hspace{1cm} (1)

In addition to project value, the model also defines manager utility that creates an agency conflict. Specifically the manager’s utility is defined over her wage income, $W$, and firm size, as measured by the required investment, $I$, and is given by the following utility function:

$$U(W, I) = W + \psi I.$$  \hspace{1cm} (2)

We assume that $\psi$ is a positive scalar, implying that the manager benefits from larger-scale investment projects. For tractability, we assume that the manager has no personal wealth and her reservation utility is zero. The assumption that the manager has no personal wealth implies that $W \geq 0$.

The sequence of events and information structure of the capital allocation process consists of three stages. In stage 1, headquarters establishes a capital allocation system that specifies rules for the capital budgeting process and managerial compensation. In stage 2, the manager privately observes the characteristics of available projects. In this stage, the manager submits a capital budgeting request that specifies the required investment and future expected cash flow for each requested project. In stage 3, headquarters evaluates the capital budgeting request, makes acceptance decisions based on the capital budgeting rule, and pays the manager according to the compensation contract. In this process we assume that headquarters can perfectly verify the characteristics of requested projects and cannot discern the availability, and thus the characteristics of other

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5 The assumption that the future cash flow of project $L$ equals that of project $H$ is for convenience, and can be relaxed as long as $NPV_H(\alpha) < NPV_L(\alpha)$.

6 We use a linear utility function to simplify the analysis. The main results hold under a more general specification.

7 If the manager’s reservation utility were positive, headquarters could motivate the manager by shifting wages between various states without incurring additional costs. Nevertheless, because there are no negative wages, a positive reservation utility would not solve the incentive problem completely. Thus, to simplify the analysis, we assume away this possibility.
projects. This assumption is equivalent to assuming a zero verification cost to headquarters for requested projects, and infinite verification cost for other projects.\footnote{With finite verification cost, the optimal mechanism would be either the one in our model or a mechanism with costly verification, as in Harris and Raviv (1996), depending on the magnitude of the verification cost.}

The optimal capital allocation system implements the best outcome to shareholders, given the above information constraints and incentive problems. To describe this system, we solve the capital allocation problem as a direct revelation game, where the manager reports project availability and headquarters establishes a capital allocation system that maximizes shareholder value subject to the incentive compatibility constraint that the manager truthfully reports project availability.\footnote{By the revelation principle, we can solve for the best outcome to shareholders by considering only a direct revelation game.} We solve for the optimal capital allocation system conditional on $\alpha$, although neither the manager nor headquarters observe the realization of $\alpha$ directly. This is possible because both the manager and headquarters can infer the realization of $\alpha$ from the characteristics of a requested project by solving $\alpha = c_f(\alpha)/c_t$.\footnote{This assumption simplifies the analysis. The scale parameter $\alpha$ has an important role in the discussion of complexity of different capital budgeting criteria in the beginning of Section 2.}

After solving the revelation game, we relate the solution to actual capital allocation practices to evaluate the effectiveness of current allocation practices in maximizing shareholder value.

In the revelation game, headquarters chooses project acceptance probabilities and manager wages to maximize firm value.\footnote{Note that the manager cannot lie when no project is available, because headquarters can check any project she reports. For the same reason, the manager does not want to lie when only project L is available, because the only possibility to misrepresent the information is to report that no project is available. For the manager, this possibility is worse than reporting only project L. Therefore, in what follows, we will use the fact that the manager truthfully reports that only project L is available to set the acceptance probability of project L to one when the manager reports that only project L is available.} We denote the acceptance probabilities by $\lambda_L(LH, \alpha)$ and $\lambda_H(H, \alpha)$. Specifically, $\lambda_L(LH, \alpha)$ is the acceptance probability of project L when the manager reports both project L and project H. In this case, if project L is rejected, headquarters accepts project H.\footnote{In general, it is possible to put a lower probability on accepting project H, implying that both projects are rejected with a positive probability. However, since a higher probability of rejecting both projects decreases the expected utility to the manager from reporting both projects, a higher rejection probability decreases her incentive to report the truth. Furthermore, this also decreases the expected value to shareholders. Thus accepting project H when project L is rejected dominates.} Thus when the manager reports both projects, the acceptance probability of project H is $(1-\lambda_L(LH, \alpha))$. Likewise, $\lambda_H(H, \alpha)$ is the acceptance probability of project H when the manager reports only project H. We denote the manager’s wage when both projects are reported by $W(LH, \alpha)$.\footnote{Since the manager prefers to report project H, there is no reason to pay her any wage when she reports only project H.} Consequently headquarters’
problem is represented as follows:

$$\max_{\lambda_L(LH, \alpha), \lambda_H(H, \alpha), W(LH, \alpha)} V = \int [g_L(1 - g_H)NPV_L(\alpha) + \lambda_H(H, \alpha)g_H$$

$$\times (1 - g_L)NPV_H(\alpha) + g_Lg_H[\lambda_L(LH, \alpha)$$

$$\times NPV_L(\alpha) + (1 - \lambda_L(LH, \alpha))$$

$$\times NPV_H(\alpha) - W(LH, \alpha)]dF(\alpha)$$  \hspace{1cm} (3)

subject to

$$\lambda_L(LH, \alpha)\psi I_L(\alpha) + (1 - \lambda_L(LH, \alpha))\psi I_H(\alpha) + W(LH, \alpha) \geq \lambda_H(H, \alpha)\psi I_H(\alpha) \quad \forall \alpha$$  \hspace{1cm} (4)

$$0 \leq \lambda_L(LH, \alpha) \leq 1 \quad \forall \alpha$$  \hspace{1cm} (5)

$$0 \leq \lambda_H(H, \alpha) \leq 1 \quad \forall \alpha$$  \hspace{1cm} (6)

$$W(LH, \alpha) \geq 0 \quad \forall \alpha.$$  \hspace{1cm} (7)

Equation (3) represents the ex ante value to shareholders. Specifically this value consists of the expected NPV, net of managerial wage. Equation (4) represents the incentive compatibility constraints for the manager for each possible realization of $\alpha$.

We now outline the conditions for the optimal solution to the capital allocation problem [Equations (3)–(7)].

**Proposition 1.**  \hspace{1cm} (a) Suppose $\psi < 1$ and let $g_L^* = NPV_H/(\psi I_H + NPV_H)$.

(i) If $g_L \geq g_L^*$, then $W(LH, \alpha) = 0$, $\lambda_H(H, \alpha) = I_L/I_H$, and $\lambda_L(LH, \alpha) = 1$.

(ii) If $g_L < g_L^*$, then $W(LH, \alpha) = \alpha\psi(I_H - I_L)$, $\lambda_H(H, \alpha) = \lambda_L(LH, \alpha) = 1$.

(b) Suppose $\psi > 1$ and let $g_L^{**} = NPV_H/PV$.

(i) If $g_L \geq g_L^{**}$, then $W(LH, \alpha) = 0$, $\lambda_H(H, \alpha) = I_L/I_H$, and $\lambda_L(LH, \alpha) = 1$.

(ii) If $g_L < g_L^{**}$, then $W(LH, \alpha) = 0$, $\lambda_H(H, \alpha) = 1$, and $\lambda_L(LH, \alpha) = 0$.

We now examine the capital allocation process in greater detail. In Proposition 1(a)(i) and 1(b)(i), headquarters announces that it will directly intervene in project selection without paying a wage. Headquarters induces the manager to reveal project L by rejecting project H with a positive probability. This rejection probability is chosen to satisfy the incentive compatibility constraint of the manager. In particular, if the manager reports project L, this project is accepted, resulting in utility $\psi I_L(\alpha)$ to the manager. However, if the manager reports project H, project H is accepted with probability $I_L/I_H$, also giving the manager an expected utility of $\psi I_L(\alpha)$. These acceptance probabilities induce the manager to reveal project L when it is available. Note that the positive rejection
probability of project H requires headquarters to commit to an action that is ex post inefficient. Headquarters can be held to this commitment both indirectly and directly. First, if headquarters keeps deviating from this rule, it will lose its reputation and thus its ability to induce managers to reveal the best project. Second, shareholders may provide members of the board of directors with compensation packages that internalize the optimal rule. Since negotiation of the board of directors’ compensation before the decision on every project is practically impossible, this may induce the implementation of the optimal capital budgeting rule.\footnote{If it is impossible to implement a stochastic capital budgeting rule, the optimal rule will be similar, with acceptance probabilities restricted to either one or zero.}

Proposition 1(a)(ii) describes the situation where it is more efficient for headquarters to induce the manager to reveal project L by paying her the difference in her utility from accepting project L over project H, \( W(LH, \alpha) = \alpha \psi(I_H - I_L) \). Finally, Proposition 1(b)(ii) describes the situation where delegating project selection to the manager dominates direct intervention in the capital budgeting or giving incentives to the manager by paying her to reveal the availability of project L. The reason for this is that when \( \psi > 1 \), paying the manager her difference in utility from undertaking project L over project H is higher than the difference in the projects’ NPV.

2. Capital Budgeting Criteria and the Implementation of the Optimal Outcome

We now analyze the effectiveness of various well-known capital budgeting criteria in implementing the outcome of the optimal capital allocation system defined in Proposition 1. In particular, we compare the effectiveness of ratio criteria like IRR and PI to level criteria like NPV in implementing the optimal capital budgeting rule. However, before comparing the criteria, it is necessary to define the relationship between a capital allocation system and capital budgeting criteria. To define this relationship, we view capital budgeting criteria as a mechanism to implement the optimal capital budgeting rule specified in Proposition 1.

An important aspect of the implementation of a capital budgeting rule is how the headquarters’ project selection process is communicated to divisional managers. If this process is not communicated in a clear and understandable manner, managers will not be certain of the consequences of project manipulation. Thus the capital allocation system will not reduce manager manipulations of project information as it could. Our premise is that capital budgeting criteria are instrumental to communicate headquarters’ decision processes to divisional managers in a clear and understandable manner.

In most corporations, the capital budgeting rule is communicated formally via a capital budgeting manual that gives instructions on how to file
a request and what to include [see Segelod (1995)]. The capital budgeting criteria are one method to specify the capital budgeting rules. However, these criteria are often overly simplistic and thus should be dominated by more detailed, accurate capital budgeting rules. Thus we might ask why firms prefer the simple criteria to their more effective underlying counterparts. In the context of the current model, this question may be rephrased as, why should firms prefer to use simple capital budgeting criteria that are independent of the scale parameter $\alpha$ over more detailed capital budgeting rules that depend on the scale parameter $\alpha$?

However, the very simplicity of these capital budgeting criteria may provide a communication advantage. Since a firm’s capital budgeting manual may be complex and cumbersome, any simplification may enhance its communication value. Therefore we view the optimal capital budgeting criterion as the one that best communicates headquarters’ capital budgeting rules. In this respect, the trade-off in selecting a capital budgeting criterion is between simplicity (the cost of complexity) and precision. Given this trade-off, we now explore the efficiency of typical capital budgeting criteria in implementing the optimal system specified in Proposition 1. This analysis shows that some simple capital budgeting criteria, like the IRR and PI, can implement the outcome of the optimal system, while others, like the NPV rule, cannot.

To evaluate these criteria, we must first derive a definition of criterion complexity. A natural measure of complexity in our model is $\alpha$. In general, the optimal system depends on $\alpha$. This dependence creates complexity because both headquarters and the manager must perform the following two steps to implement a capital allocation system that depends on $\alpha$:

1. Indicate to the manager how headquarters defines and calculates $\alpha$.
2. Specify the set of contingencies, as prescribed by Proposition 1, on the calculated $\alpha$.

Note that if a criterion is independent of $\alpha$, headquarters does not need to specify these steps. Following this observation, we define two degrees of complexity: simple criteria, or criteria that are not conditioned on $\alpha$, and complex criteria, or criteria that are conditioned on $\alpha$.

Given the above measure of criterion complexity, we now explore specific capital budgeting criteria. We first examine the capital budgeting criteria as defined by typical finance texts. We next assess the effectiveness of the textbook versions and offer modified versions of these criteria. We then assess the effectiveness of the modified versions in implementing the optimal capital budgeting rule.

We focus our analysis on the most common capital budgeting criteria—NPV, IRR, and PI. The textbook NPV rule is to undertake the highest positive-NPV project. Similarly the textbook IRR rule is to accept the
project with the highest IRR, as long as it exceeds the cost of capital $\rho$, where the IRR of project $m = L, H$ is defined by

$$0 = -\alpha I_m + \sum_{t=1}^{\infty} \frac{\alpha c_t}{(1 + IRR_m(\alpha))^t} = -I_m + \sum_{t=1}^{\infty} \frac{c_t}{(1 + IRR_m)^t} \quad m = L, H.$$ (8)

Finally, the textbook profitability index (PI) rule is to accept the project with the highest PI, as long as its PI is larger than one. The PI for project $m = L, H$ is given by

$$PI_m(\alpha) = \frac{\alpha \sum_{t=1}^{\infty} c_t/(1 + \rho)^t}{\alpha I_m} = \frac{\sum_{t=1}^{\infty} c_t/(1 + \rho)^t}{I_m} \quad m = L, H.$$ (9)

We now show that the above criteria, in their textbook forms, cannot implement the best outcome. We first examine the effectiveness of the NPV rule where headquarters accepts the project with the highest positive NPV. The manager will take advantage of this rule by requesting project $H$ whenever it is available. Since the NPV of project $H$ is positive, and it is the only one that the manager requests, headquarters will accept it, leading to an inferior outcome. For the same reason, the textbook IRR and PI ratio also fail to implement the best outcome. This discussion is summarized in the following proposition.

**Proposition 2.** The textbook NPV, IRR, and PI cannot implement the best outcome.

We now examine whether simple modified versions of the above criteria can implement a better outcome by imposing stricter project acceptance standards. For example, headquarters can use a simple modified NPV criterion that specifies a critical value, NPV$^c$, such that a project is accepted with probability 1 if its NPV is above NPV$^c$. If the requested project’s NPV is lower than NPV$^c$, the project is accepted with the optimal probability, $\lambda$. However, this modification creates the following two deviations from the outcome of Proposition 1. First, for projects with high $\alpha$ values, the manager can manipulate headquarters to approve project $H$. This occurs when NPV$_H(\alpha) >$ NPV$^c$. Second, when $\alpha$ is low, good projects will be rejected. This occurs when NPV$_L(\alpha) <$ NPV$^c$. Thus the simple modified NPV rule is effective only when NPV$_H(\alpha) \leq$ NPV$^c \leq$ NPV$_L(\alpha)$, in which case, the manager requests only

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15 In the following discussion, we assume that the IRRs of projects are well defined and unique. When this assumption is violated, the IRR rule cannot implement the optimal mechanism.

16 When there are more than two alternative projects, the manager may include other available projects in the capital budgeting request as long as their NPV is lower than that of her preferred project.
project L, although both projects are available. Proposition 3 summarizes this discussion.17

**Proposition 3.** *A simple modified NPV criterion cannot implement the best outcome.*

We now examine the effectiveness of modifications of the IRR and PI criteria. Of interest is that although a simple modified NPV rule cannot implement the outcome of the optimal capital budgeting rule, simple modified versions of the IRR and PI rules can implement the outcome specified in Proposition 1(a)(i) and Proposition 1(b)(i). Proposition 4 shows how a simple modified IRR criterion implements this outcome.18

**Proposition 4.** *Suppose the conditions of either Proposition 1(a)(i) or Proposition 1(b)(i) hold. Then headquarters can implement the optimal outcome of the capital budgeting process through the following rules. If the capital budgeting request includes one project, a project with IRR = IRR_L is accepted with probability 1, while a project with IRR = IRR_H is accepted with probability \( \lambda_H = I_L / I_H \). If it includes two projects, the project with IRR = IRR_L is accepted. The wage is set at \( W = 0 \).*

Proposition 4 shows scenarios in which simple modified IRR and PI criteria can be used to implement the best capital budgeting outcome when a simple modified NPV criterion cannot.19 The generality of the example we analyze here is important to the conclusions we can draw from the analysis. How general are these scenarios? Can this example explain why the IRR criterion is widely used in practice? We have used a constant return to scale example to show two properties simultaneously. First, that the IRR and PI criteria dominate the NPV criterion, and second, that both can implement the best outcome. We believe that the first property,
namely, the dominance of the IRR and PI over the NPV criterion, applies to many situations. This dominance, however, does not imply that the IRR and PI criteria are part of firms’ optimal capital allocation systems. Thus, to conclude that our theory explains why so many firms use the IRR and PI criteria requires that they implement the best outcome in many different scenarios. How plausible is this condition?

To answer this question it is important to look at the sensitivity of our results to changing the constant return to scale assumption, in which case both the investment level and future cash flow will be affected by market factors, but at a different rate. For example, both the levels of investment and future cash flow will move up with a positive demand shock, but the impact on future cash flow is likely to be stronger. We can capture it in our model by having different α’s multiply c and I, where the one that multiplies c is higher. As a result, the NPV, IRR, and PI all increase in α. However, the IRR and PI, being ratios, are less sensitive to changes in α than the NPV. Moreover, it is possible that if the difference in the underlying technologies is big enough, the highest IRR for all possible α’s under the inferior technology (H) is lower than that under the superior one (L). In this case, the capital budgeting system of Proposition 4 applies. Even when such a “clean” separation is not feasible, the IRR and PI, both being less sensitive to demand shocks and simple, may be optimal over more complex rules. However, it is plausible that in other situations, more complex rules are optimal. In this respect, our theory may reconcile research findings on the actual use of capital budgeting criteria within organizations that most corporations do not rely solely on the NPV rule and use multiple capital budgeting criteria in their practice of capital budgeting. See, for example, Segelod (1995).

3. Conclusion

Our theory of capital allocation shows that there are plausible scenarios where the well-known and often criticized capital budgeting criteria like IRR and PI will perform better than the NPV criterion in implementing a value-maximizing project selection process. The main contribution of this theory is the idea that although the NPV is the right statistic to measure the value added to a firm from undertaking a certain project, it often does a poor job in implementing a selection process that maximizes firm value. Consequently NPV, although a useful valuation tool, is ineffective as the core of a capital allocation process. Our theory is consistent with evidence that many companies do not use the NPV criterion at all in their capital allocation process, or only rely on it partially.

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20 However, it is possible that under some other scenarios, with different information structure and incentives, the NPV criterion does better than the IRR and PI criteria.
An important explanation derived from our theory is why firms may use modified versions of textbook budgeting criteria. For example, some firms may use a modified NPV criterion, where they accept projects only if the NPV is above a threshold level different from zero. Or they may use a modified PI criterion, where they set the cutoff for accepting projects at a PI level different than one. Finally, they may employ a modified IRR rule, where the hurdle rate is different than the cost of capital. Our theory shows how all of these modifications are attempts to implement the optimal project selection process. Given this expanded understanding of the rationale behind capital budgeting criteria choices, our theory can explain the puzzling finding in Poterba and Summers (1995) that U.S. firms use real hurdle rates that are considerably higher than standard cost-of-capital analysis would suggest. Similarly, to the extent that multinational firms investing abroad confront more severe implementation problems, our theory provides a possible rationale for the often-criticized practice of adding a fudge factor to their cost of capital.

In sum, our analysis shows that the capital allocation process is more complicated than is traditionally taught, and that seemingly irrational budgeting practices may have a logical rationale behind them. This analysis also makes the point that market imperfections play a role in firms’ capital allocation processes. Since firms operate in imperfect capital markets, it should not be surprising that firms don’t rely on the NPV rule, just as it is not surprising that firms care about their capital structure when markets are imperfect.

Appendix

Proof of Proposition 1. For convenience we first rewrite headquarters’ constrained optimization problem given by Equations (3)–(7):

$$\max_{\lambda_L(LH, \alpha), \lambda_H(H, \alpha), W(LH, \alpha)} V = \int [g_L(1-g_H)NPV_L(\alpha) + \lambda_H(H, \alpha)g_H(1-g_L)NPV_H(\alpha)$$

$$+ g_Lg_H(\lambda_L(LH, \alpha)NPV_L(\alpha) + (1 - \lambda_L(LH, \alpha))NPV_H(\alpha)$$

$$- W(LH, \alpha)]dF(\alpha)$$

subject to

$$\lambda_L(LH, \alpha)\psi L(\alpha) + (1 - \lambda_L(LH, \alpha))\psi H(\alpha) + W(LH, \alpha) \geq \lambda_H(H, \alpha)\psi H(\alpha) \quad \forall \alpha$$

$$0 \leq \lambda_L(LH, \alpha) \leq 1 \quad \forall \alpha$$

$$0 \leq \lambda_H(H, \alpha) \leq 1 \quad \forall \alpha$$

$$W(LH, \alpha) \geq 0 \quad \forall \alpha.$$  

We next denote the LaGrange multiplier of Equation (11) by $\mu_1$; the LaGrange multipliers of the two constraints in Equation (12) by $\mu_2$ and $\mu_3$; the LaGrange multipliers of the two constraints in Equation (13) by $\mu_4$ and $\mu_5$; and the LaGrange multiplier of Equation (14) by $\mu_6$. To simplify notation, we use $\lambda_L$ for $\lambda_L(LH, \alpha)$, $\lambda_H$ for $\lambda_H(H, \alpha)$, and $W$ for $W(LH, \alpha)$.

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We next identify the Kuhn-Tucker conditions of the above problem:

\[
\frac{\partial L}{\partial W} W = W[-g_L g_H + \mu_1 + \mu_6] = 0
\]  

(15)

\[
\frac{\partial L}{\partial \lambda_L} \lambda_L = \lambda_L[g_L g_H(I_H(\alpha) - I_L(\alpha)) + \mu_1(\psi I_L(\alpha) - \psi I_H(\alpha)) - \mu_2 + \mu_3] = 0
\]

(16)

\[
\frac{\partial L}{\partial \lambda_H} \lambda_H = \lambda_H[(1 - g_L) g_H NPV_H(\alpha) - \mu_1 \psi I_H(\alpha) - \mu_4 + \mu_5] = 0
\]

(17)

\[
\frac{\partial L}{\partial \mu_1} \mu_1 = \mu_1[\lambda_L \psi I_L(\alpha) + (1 - \lambda_L) \psi I_H(\alpha) + W - \lambda_H \psi I_H(\alpha)] = 0
\]

(18)

\[
\mu_2[1 - \lambda_L] = 0
\]

(19)

\[
\mu_3[\lambda_L] = 0
\]

(20)

\[
\mu_4[1 - \lambda_H] = 0
\]

(21)

\[
\mu_5[\lambda_H] = 0
\]

(22)

\[
\mu_6[W] = 0.
\]

(23)

We first check for possible solutions that satisfy the Kuhn-Tucker conditions of Equations (15)–(23) when \( W > 0 \), and then check for possible solutions when \( W = 0 \).

We first suppose that \( W > 0 \). In this case it follows that the bracketed expression in Equation (15) equals zero. Similarly we know from Equation (23) that \( \mu_6 = 0 \). Substituting zero for the above two components in Equation (15) yields \( \mu_1 = g_L g_H > 0 \). Consequently the bracketed expression in Equation (18) equals zero, implying that

\[
W = [-\lambda_L \psi I_L(\alpha) - (1 - \lambda_L) \psi I_H(\alpha) + \lambda_H \psi I_H(\alpha)].
\]

(24)

We now show that \( \lambda_L = 1 \). Suppose instead that \( \lambda_L = 0 \). Under this supposition, Equation (24) implies that \( W = (\lambda_H - 1) \psi I_H(\alpha) \leq 0 \), a contradiction. Thus \( \lambda_L > 0 \). It follows from this that the bracketed expression in Equation (16) equals zero. Furthermore, we know that Equation (20) implies that \( \mu_3 = 0 \). We can now substitute \( \mu_1 = g_L g_H \) and \( \mu_3 = 0 \) into the bracketed expression in Equation (16). Since we know that the bracketed expression in Equation (16) equals zero, we obtain

\[
\mu_2 = g_L g_H(I_H(\alpha) - I_L(\alpha))(1 - \psi) \neq 0.
\]

(25)

From Equations (25) and (19), it follows that \( \lambda_L = 1 \).

Similarly, to show that \( \lambda_H = 1 \), we suppose instead that \( \lambda_H = 0 \). Under this supposition, and since \( \lambda_L = 1 \), Equation (24) implies that \( W = -\psi I_L(\alpha) < 0 \), a contradiction. Thus \( \lambda_H > 0 \). It follows from this that the bracketed expression in Equation (17) equals zero. Furthermore, Equation (22) implies that \( \mu_5 = 0 \). We can now substitute \( \mu_1 = g_L g_H \) and \( \mu_5 = 0 \) into the bracketed expression in Equation (17). Since we know that the bracketed expression in Equation (17) equals zero, we obtain:

\[
(1 - g_L) g_H NPV_H(\alpha) - g_L g_H \psi I_H(\alpha) = \mu_4.
\]

(26)

It follows from Equation (26) that \( \mu_4 \neq 0 \), except, possibly, for one special case. Since \( \mu_4 \neq 0 \), Equation (21) implies that \( \lambda_H = 1 \).

We can now substitute \( \lambda_L = \lambda_H = 1 \) into Equation (24) to obtain \( W = [\psi I_H(\alpha) - \psi I_L(\alpha)] \).

Thus the resulting expected value to shareholders, when \( W > 0 \), is

\[
V_w(\alpha) = g_L NPV_L(\alpha) + (1 - g_L) g_H NPV_H(\alpha) - g_L g_H [\psi I_H(\alpha) - \psi I_L(\alpha)].
\]

(27)
Equations (10)–(14):
1. \( \lambda_H = \lambda_L = 0 \)
2. \( 1 > \lambda_L > 0 \) and \( 1 > \lambda_H > 0 \)
3. \( \lambda_H = \lambda_L = 1 \)
4. \( \lambda_H = 1 \) and \( \lambda_L < 1 \)
5. \( \lambda_H < 1 \) and \( \lambda_L = 1 \).

We first examine case 1. Evaluation of case 1 shows that it cannot be a solution to the problem because increasing \( \lambda_H \) from zero to one increases the objective function of Equation (10), while Equations (11)–(14) continue to hold.

Likewise, we find that case 2 cannot be a solution to the problem. Under the assumptions of case 2, Equations (19)–(22) imply that \( \mu_2 = \mu_3 = \mu_4 = \mu_5 = 0 \). However, substituting \( \mu_2 = \mu_3 = \mu_4 = \mu_5 = 0 \) into the bracketed expression in Equations (16) and (17) (note that these expressions equal zero) yields conflicting solutions for \( \mu_1 \). Thus the system of equations does not hold for case 2.

In addition, we find that case 3 cannot be a solution to the problem. In this case, the incentive compatibility constraint of Equation (11) is violated.

We now consider case 4. This case presents a possible solution to the problem. The suppositions that \( W = 0 \) and \( \lambda_H = 1 \), and the incentive compatibility constraint of Equation (11), imply that \( \lambda_L = 0 \). The resulting expected value to shareholders is

\[
V_{c4}(\alpha) = g_L(1 - g_H)NPV_L(\alpha) + g_H NPV_H(\alpha). \tag{28}
\]

Finally, we consider case 5. This case also presents a possible solution to the problem. To evaluate case 5, we first show that \( \lambda_H > 0 \). If \( \lambda_H = 0 \), then the incentive compatibility constraint of Equation (11) holds as a strict inequality, implying that the objective function evaluate case 5, we first show that

\[
V_{c5}(\alpha) = g_L NPV_L(\alpha) + \frac{I_L}{I_H} g_H (1 - g_L) NPV_H(\alpha). \tag{29}
\]

To prove Proposition 1(a), suppose that \( \psi < 1 \). We now use Equations (27)–(29) to fully characterize the optimal solution for the optimization problem.

A comparison of Equations (27) and (28) yields that \( V_{c4} < V_w \) iff \( g_L g_H(I_H(\alpha) - I_L(\alpha))(1 - \psi) > 0 \). Since \( \psi < 1 \) and \( I_H(\alpha) > I_L(\alpha) \), this inequality holds for all parameter values, implying that \( V_{c4} < V_w \). Thus it remains to compare \( V_w \) to \( V_{c5} \) of Equation (29). Using Equations (27) and (29), we find that \( V_{c5} > V_w \) iff:

\[
g_L \geq g_L^* = \frac{NPV_H}{\psi I_H + NPV_H}. \tag{30}
\]

Therefore, for \( g_L \geq g_L^* \), the optimal solution is \( W = 0 \), \( \lambda_L = 1 \), and \( \lambda_H = I_L/I_H \). This proves Proposition 1(a)(i). For \( g_L < g_L^* \), the optimal solution is \( W = \alpha \psi(I_H - I_L) \), \( \lambda_L = \lambda_H = 1 \). This proves Proposition 1(a)(ii).

To prove Proposition 1(b), suppose that \( \psi > 1 \). We know from comparing Equations (27)–(28) that \( V_{c4} < V_w \). Thus it remains to compare \( V_{c4} \) of Equation (28) to \( V_{c5} \) of Equation (29). Comparing these equations we find that \( V_{c5} > V_{c4} \) iff:

\[
g_L \geq g_L^{**} = \frac{NPV_H}{PV}. \tag{31}
\]

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Therefore, for $g_L \geq g_L^*$, the optimal solution is $W = 0$, $\lambda_L = 1$, and $\lambda_H = (I_L / I_H)$. This proves Proposition 1(b)(i). For $g_L < g_L^*$, the optimal solution is $W = 0$, $\lambda_L = 0$, and $\lambda_H = 1$. This proves Proposition 1(b)(ii).

For proofs of Propositions 2-4, see the detailed discussion prior to each of the propositions.

References


