The Constant Relative Risk-Aversion Utility Function

The benchmark utility function has marginal utility

\[ m(x) = x^{-b}, \]

and as by definition \( m = u' \), we have

\[ u(x) = \begin{cases} 
\frac{1}{1-b}x^{1-b} & \text{for } b \neq 1 \\
\ln(x) & \text{for } b = 1.
\end{cases} \]

Note the affine invariance.
Relative risk-aversion is commonly defined as

\[ RRA(x; u) = -\frac{xu''}{u'} \).

In this case \( RRA \) is simply \( b \).

Whenever Sharpe writes risk-aversion, he refers to relative risk-aversion.
Reservation Prices

A specific agent’s reservation price for a specific security is the price at which he wants to trade exactly 0 units, i.e. he wants to sell at higher prices, he wants to buy at lower prices.

A state-\( j \) claim (aka Arrow-Debreu security) pays 1 in state \( j \), 0 in all other states.

Agent \( k \)’s reservation price for a state-\( s \) claim is

\[
k r_j = \frac{\pi_j d_j m(kX_j)}{m(kX_1)},
\]

where \( kX_j \) is the agent’s wealth/consumption in state \( j \).
Agent $k$'s reservation price for the $i$'th security, which pays $Z_{i,j}$ in time-1 state $j$, is

$$k R_i = \sum Z_{i,j} \cdot k r_j$$
The Trading Algorithm

Sharpe’s Section 3.7 and the note. And Hand-In # 1.

Trading is completely sequential (I think), i.e. one round for one security at a time.

Excess buy or sell orders are served proportionally.
Today: The Rest of Sharpe’s Chapter 3

Case 2: Rich siblings, or what constant relative risk-aversion does. (Exact scaling and proportional serving is important.)

Case 3: Quadratic utility. Increasing relative risk-aversion. (I’m rich, I don’t have to take chances!) Not as bad as it looks though. (Use locally. So mean/variance efficient portfolios are relevant.)

Case 4: Decreasing relative risk-aversion.

Case 5: Kinked utility; agents with a reference level.
Today, as time: Sharpe’s Chapter 4

Incomplete markets: (Many) More states than assets.

Complete markets: All state claims are traded.

Case 6: Trade to equilibrium, but everybody is not perfectly happy.

Case 7: Market completion. (Financial ingenuity increases welfare!)