A Portfolio Performance Index

Michael Stutzer

Fund managers may sensibly be averse to earning a time-averaged portfolio return that is less than the average return of some designated benchmark. When a portfolio is expected to earn a higher average return than the benchmark return, the probability that it will not approaches zero asymptotically at a computable exponential decay rate. The probability decay rate is thus proposed here as a new portfolio "performance index." In the widely analyzed special case in which returns are normally distributed, the new performance-index-maximizing portfolio is the same as the popular Sharpe-ratio-maximizing portfolio. The results of the two approaches generally differ, however, because of nonnormal levels of skewness and/or kurtosis in the portfolio attributable to large asymmetrical economic shocks or investments in options and other derivative securities. An illustrative example will show that the new index is easy to implement and, consistent with empirical evidence on portfolio choice, favors investments with positively skewed returns.

Pension and endowment portfolio analysts need a meaningful yet practical way to rank-order feasible portfolios. Perhaps the most widely used measure for this purpose is the Sharpe ratio. The Sharpe ratio of a portfolio is its expected excess return relative to a chosen benchmark (usually the "riskless" rate of interest) divided by the standard deviation of its excess return. When returns are individually and identically normally distributed, risk-averse investors will choose a portfolio that is mean–standard deviation efficient, and Sharpe (1994) summarized the foundational case for using the Sharpe ratio to evaluate (*ex ante*) the growth (i.e., mean) versus security (i.e., standard deviation) trade-off in these portfolios. In practice (*ex post*), a historical time series of portfolio returns minus benchmark returns is used to calculate historical average excess return and historical standard deviation as estimates of the unknown *expected* excess return and standard deviation. And in practice, portfolios are ranked by the size of this *ex post* Sharpe measure.

But what is to be done when the returns are not normally distributed? The theoretical foundation for the Sharpe measure does not apply when excess returns deviate from the normal because of large absolute values of skewness and/or kurtosis. Such nonnormalities in a portfolio may arise from large asymmetrical economic shocks, investments in options and other derivative securities with inherently asymmetrical returns, limited liability (bankruptcy) effects on asset returns, or other causes.[1] Moreover, a suggestion that has been made for a long time now is that investors value positive skewness of returns (e.g., Kraus and Litzenberger 1976). The Sharpe measure does not consider skewness at all.

Alternatives to the Sharpe Ratio

An alternative performance index should satisfy the following desiderata:

- The index should rank-order portfolios in accord with the Sharpe ratio when returns are normally distributed.
- When returns are not normally distributed, the index should reflect skewness preference while retaining the Sharpe ratio's useful statistical interpretation and ease of implementation.
- The index should be derived from a sound behavioral foundation that is free of unspecified and unknowable parameters and is relevant to many fund managers.

Three general approaches have been used to construct alternative performance indexes: *ad hoc* modifications to the Sharpe ratio, expected utility functions, and expected return relationships implied by perfect financial market equilibrium resulting from investors who maximize expected

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utility. Each approach has advantages and disadvantages. Accordingly, a single performance index is unlikely to be appropriate for all fund managers.

One way to construct an alternative performance index that retains statistical interpretation and ease of implementation is to modify the Sharpe ratio. Fishburn (1977) noted that "decision makers in investment contexts very frequently associate risk with failure to attain a target return" (p. 117). The Sharpe ratio, however, penalizes return variance and hence penalizes squared deviations from the mean return regardless of whether they are the "bad" deviations below a target return or the "good" deviations above it. To favorably weight positive skewness, one can replace the variance by a statistic that penalizes only the deviations below some cutoff value. Using only those squared deviations produces a semivariance, whose square root has been used as a measure of downside risk (Sortino and Van der Meer 1991) to replace standard deviation in the denominator of the Sharpe ratio. Ang and Chua (1979) noted, "Even if return distributions are symmetrical, the semivariance still yields information different from the variance" (p. 363). But they also pointed out that use of semivariance is "in a sense, *ad hoc* because of the absence of a theoretical basis" (p. 363), in violation of the third desideratum.

Another orthodox approach to rank ordering portfolios relies on the expected utility hypothesis. Investors who adhere to the Von Neumann–Morgenstern (1944) behavioral axioms act as if they rank-order the probability distributions of wealth generated by portfolios. Hence, each such investor's chosen portfolio should maximize the expected value of her or his utility function. Skewness preference is embodied in utility functions with positive third derivatives, but even if one ignores the impressive behavioral evidence calling the appropriateness of expected utility theory's behavioral axioms into question (e.g., Machina 1987), fund managers still must make two decisions before they can implement this approach: what utility function to use and what values to assign to the utility function parameters.

In lieu of dealing with the difficulties in making those decisions, a fund manager could start by eliminating portfolios that would never be chosen by any investor with increasing utility (first order stochastically dominated) or with increasing and concave (indicating risk aversion) utility (second order stochastically dominated). The result may not, however, provide much rank ordering of portfolios, in violation of the second desideratum. For example, in a study comparing the performance of 34 mutual funds with the DJIA, Joy and Porter (1974) concluded that, although no fund dominated the DJIA, "28 funds neither dominated the DJIA nor were dominated by it" (p. 31), using second order stochastic dominance.

The fund manager, then, cannot avoid choosing a specific utility function and its parameters. One widely prescribed utility function for managers of long-term funds is the logarithm, advocated centuries ago by Bernoulli as a solution to the St. Petersburg Paradox (for a survey of the log utility literature, see Hakansson and Ziemba 1995). It has several features of interest to pension and endowment managers. First, it has no additional parameters whose values must be specified to produce a rank ordering of portfolios. Second, like maximization of the Sharpe ratio, maximization of expected log utility has a nice statistical interpretation: It is equivalent to maximizing the expected (geometric) growth rate of wealth. Third, it induces a preference for positive skewness. So, it does satisfy the second and third desiderata. The complete emphasis of expected log utility on expected capital growth also appears to be particularly suitable for pension and endowment managers, rather than for investors with shorter horizons. MacLean and Ziemba (1999) emphasized, however, that its use "leads to riskier and less diversified portfolios than most other utility functions" (p. 222). The reason is that it ignores volatility and other higher-order moments of the capital growth rate. A lognormal example of Browne (1999) illustrates the phenomenon: An expected log-maximizing portfolio places an 89 percent weight on stocks when their returns have an 8 percent expected excess return over the risk-free rate and a standard deviation of 30 percent. Yet, there is still a 10 percent probability that the portfolio's wealth will not wind up more than a paltry 10 percent higher than a risk-free investment *after 98 years*.

A third approach to portfolio performance evaluation requires expected utility maximization for *all* investors, rather than only the subset of investors represented by a particular fund manager. Additional assumptions guaranteeing existence of perfect financial market equilibrium result in a predicted relationship between portfolio expected returns and a model-derived measure of risk. Grinblatt and Titman (1995) advocated this third approach. They argued that it

> seems more important to focus on the marginal contributions of a managed portfolio to the risk and expected return of an investor. This necessarily involves adjusting for risk with a marginal risk measure, like beta. (p. 582)

As noted by Leland (1999), "[M]ost practice is firmly rooted in the approach of the capital asset pricing model" (p. 27). He went on to note that superior portfolio performance according to the CAPM is measured by Jensen's alpha (Jensen 1969), which is the portfolio's expected return in excess of the return predicted from the security market line, evaluated at the portfolio's CAPM beta coefficient. Because the CAPM is based on the assumption that all investors care only about the mean and variance of their wealth, it cannot properly adjust for any investor preferences for positive skewness, so Ang and Chua proposed an analogous measure based on the Kraus and Litzenberger three-moment market equilibrium model.

Perhaps the most recent proposal in this vein was made by Leland and is heavily based on the findings of He and Leland (1993). They posited an expected-utility-maximizing, representative-agent exchange model with dynamically complete, frictionless, continuous-time trading possibilities. The key relevant finding was He and Leland's Equation 22, which showed that if the market portfolio's instantaneous rate of return has constant drift and volatility (i.e., if the market portfolio value is lognormally distributed), then the representative agent's utility function must have constant relative risk aversion (CRRA). Leland's proposal uses an equilibrium moment restriction that follows from He and Leland's Equation 22 to derive a performance index that replaces the CAPM beta coefficient in Jensen's alpha with a modified beta that also depends on the market portfolio's drift. In addition to the aforementioned argument of Grinblatt and Titman in favor of indexes of performance based on expected utility maximization with complete markets in equilibrium, the Leland proposal also embodies skewness preference because it is predicated on expected CRRA utility maximization (which has a positive third derivative). Hence, it satisfies the second and third desiderata.

The validity of an index derived via this third approach depends on the accuracy of its equilibrium model's derived relationship between passive portfolio expected returns and the model's measure of risk, because the index evaluates managed portfolio performance relative to this relationship (e.g., Jensen's or Leland's alpha). Unfortunately, these relationships have been shown to be very inaccurate. For example, Campbell, Lo, and MacKinlay (1997) reported that analogous representative-agent, complete-market, equilibrium exchange models using Leland's CRRA utility failed to pass the award-winning asset-pricing statistical tests of Hansen and Singleton (1982).[2] Specifically, Campbell, Lo, and MacKinlay reported that "the overidentifying restrictions of the model are strongly rejected whenever stocks and commercial paper are included together in the system" (p. 314).

Furthermore, the same models suffer from the "equity premium puzzle" discussed by Mehra and Prescott (1985). In a recent survey of the large body of literature surrounding this issue, Kocherlakota (1996) summarized the puzzle as follows: When the CRRA model's measure of risk is used, "stocks are not sufficiently riskier than Treasury bills to explain the spread in their returns" (p. 43). He also noted a "risk-free rate puzzle" implied by this model—that is, even if investors have a coefficient of relative risk aversion high enough to explain the spread between the expected stock return and the risk-free return, the model equilibrium risk-free return must be higher than observed.

Why should the measure of risk in Leland's CRRA representative-agent, complete-market, equilibrium exchange model be used to evaluate managed portfolio performance when analogous models that make those same assumptions cannot correctly predict the expected return of both a stock index portfolio and the risk-free rate?[3] With regard to the Leland model's lognormally distributed market portfolio, he noted that his model implies that the Black–Scholes model should correctly price European options on the market portfolio. In testing this proposition, the commonly adopted proxies for the market portfolio are broad-based stock indexes. Yet, as concluded by Rubinstein (1994) in his presidential address to the American Finance Association, "[T]here has been a very marked and rapid deterioration" (pp. 773–774) since 1986 in the applicability of Black–Scholes to S&P 500 Index options, a finding since confirmed by many others.[4]

In contrast, my proposal combines some advantages that semivariance-type and expected-utility indexes have for a long-horizon portfolio manager without requiring additional assumptions about the behavior of all other investors and the nature of any resulting financial market equilibrium. Like the semivariance-type indexes, my performance index explicitly incorporates the belief mentioned by Fishburn (i.e., that investors frequently associate risk with failure to attain a target) in an easily interpretable statistical criterion. Specifically, I model a manager who is concerned that the portfolio's discrete-time-averaged, individually and identically distributed (i.i.d) returns will be no greater than the corresponding time-averaged return earned by some benchmark (i.e., the target) that is either specified by the trustee or chosen by the manager. Such benchmarks are pervasive in the investment management industry, and fear of underperformance relative to benchmarks may account for some of the interest in protective put strategies.[5] When the investor holds a portfolio that is *expected* to return more than the benchmark, the probability of missing this time-averaged target return decays to zero as time passes at a computable exponential decay rate. To minimize this probability, the fund manager asks the analyst to find a portfolio that maximizes this decay rate, which is the proposed new portfolio performance index.

Following a summary of the results of Sharpe ratio maximization for the normally distributed case, I develop the alternative behavioral hypothesis and provide a simple, distribution-free approach to estimating the optimal portfolio when excess returns are *not* normally distributed.

Sharpe Ratio Maximization

Following Jobson and Korkie's (1982) notation, denote the expected excess returns of the joint normally distributed assets by the vector μ and the covariance matrix of the excess returns by Σ.[6] The Sharpe ratio for any portfolio p with asset proportions w_p summing to 1 (i.e., $w'_p \mathbf{e} = 1$, where $\mathbf{e}$ is a vector of 1's) is then

$$\lambda_p = \frac{w'_p \mu}{\sqrt{w'_p \sum w_p}}. \tag{1}$$

The tangency portfolio (i.e., the portfolio at the tangency of the capital market line and the risky assets' mean–variance frontier) with weights w_m maximizing the Sharpe ratio is

$$w_m = \frac{\sum^{-1} \mu}{\mathbf{e}' \sum^{-1} \mu}, \tag{2}$$

which (after a little algebra) can be shown to attain the following maximum attainable value, λ_m, of the Sharpe ratio:

$$\begin{aligned} \lambda_m &\equiv \frac{\mu_m}{\sigma_m} \\ &= \sqrt{\mu' \sum^{-1} \mu}, \end{aligned} \tag{3}$$

where σ_m is the standard deviation of the Sharpe-ratio-maximizing portfolio. In the presence of a riskless asset as the benchmark, the tangency portfolio that maximizes the Sharpe ratio is the optimal portfolio of normally distributed risky assets, in the sense that *any* mean–variance-efficient portfolio is a combination of this portfolio and the riskless asset. Thus, Sharpe summarized the case for using the *ex ante* Sharpe ratio to evaluate the relative performance of all risky-asset portfolios. In the CAPM, the portfolio that maximizes the Sharpe ratio is the equilibrium market portfolio of the risky assets.

Behavioral Hypothesis

Denote a portfolio p's *excess* (i.e., net of a benchmark) rate of return in any time period t by R_{pt}, and denote the time-averaged excess return it earns over T periods by

$$\overline{R_{pT}} \equiv \frac{\sum_{t=1}^{T} R_{pt}}{T}. \tag{4}$$

Now, assuming the portfolio has a positive expected excess return, the law of large numbers implies that $\text{prob}(\overline{R_{pT}} \le 0) \to 0$ as $T \to \infty$. In i.i.d. return processes and a wide variety of other return processes, this probability will eventually converge to zero asymptotically at a computable exponential rate I_p; that is,

$$\text{prob}(\overline{R_{pT}} \le 0) \approx \frac{c}{\sqrt{T}} e^{-I_p T}, \tag{5}$$

for large T, where c is a constant that depends on the return distribution. Therefore, the behavioral hypothesis is as follows:

> *A fund manager who is averse to receiving a nonpositive time-averaged excess return above some specified benchmark will direct analysts to select a portfolio,* m, *that makes the probability of such a return occurring decay to zero at the maximum possible rate,* I_m.

Discussion. Pension or endowment fund managers fit the characteristics of the "manager" in this hypothesis. Such funds are often thought to have a "long" investment horizon, but assuming that a pension or endowment fund manager knows the exact horizon length is unreasonable. By contract, the trustees may evaluate the manager's performance over a relatively short time (e.g., five years), but the manager hopes to get the contract renewed. Subsequent evaluations might examine the manager's performance since inception of the relationship rather than only the most recent five-year period. Therefore, assuming that the pension or endowment manager wants to avoid a nonpositive average excess return over the benchmark for much longer than five years is not unreasonable. That is, in computing excess returns, the manager will want to minimize $\text{prob}(\overline{R_{pT}} \le 0)$ over an indefinite time span T that is much longer than the contract interval.

Recent behavior by some actively managed mutual funds is consistent with this hypothesized emphasis on avoiding underperformance, albeit in a more extreme way. A recent article in the *New York Times* documented the practice of "closet indexing," in which supposedly active stock pickers invest heavily in stocks that replicate a benchmark index.[7]

The article reported Morningstar research findings that about 40 percent of funds investing in large-capitalization stocks closely track a large-cap index (i.e., regressing their funds' returns on the index returns resulted in an R^2 greater than 90 percent). The article suggested a few reasons for the documented increase in this behavior, including the desire of 401(k) plan sponsors to

> avoid the embarrassment of trying to explain why the large-cap growth fund they selected for the 401(k) is badly trailing the overall market.

This behavior was also attributed to the following managerial motive:

> But because so much of the actively managed fund is tied to an index, it will probably not trail the benchmark by much, thus avoiding the huge underperformance—say 10 percent or more—that prompts investors to withdraw their money. That is good news for the fund company.

The behavioral hypothesis of this article is not a model of closet indexers. The behavioral hypothesis reflects the desire to avoid underperformance of a designated benchmark, but it emphasizes growth (long-run, time-averaged returns) in excess of the benchmark. Nevertheless, closet indexing indicates that underperformance of benchmarks is an important concern of many fund managers. The behavioral hypothesis presented here is a model of fund managers who are willing to trade off some of their extreme concern about underperformance for the possibility of extra growth.

The behavioral hypothesis also differs from conventional minimization of the "probability of ruin," as advocated in the Safety First principle of Roy (1952). In an apt critique of the expected utility hypothesis, Roy noted:

> In an economic world, disasters may occur if an individual makes a net loss as the result of some activity. For large numbers of people some such idea of a *disaster* exists, and the principle of Safety First asserts that it is reasonable, and probable in practice, that an individual will seek to reduce as far as is possible the chance of such a catastrophe occurring. (p. 432)

Unlike the Safety First principle, in the approach presented here, the "disaster level" is not a fixed minimum value of return or wealth but, rather, a long-run *time-averaged* excess return, over some benchmark, of zero. One might think that my approach still presumes an excessively conservative fear of downside risk and that such timid fund managers will never survive in the harshly competitive world of investment management. But as argued previously, a manager may be fired for choosing an excessively volatile portfolio that has a greater chance of underperforming the time-averaged return of its benchmark.

Recently, researchers in the rapidly developing field of behavioral finance have proposed the concept dubbed "loss aversion," which is part of the prospect theory of investor behavior. As defined in Benartzi and Thaler (1995), loss aversion is the tendency of individuals to be more sensitive to reductions than to increases in their levels of well-being. Consider the level of well-being attainable by individuals passively investing in the benchmark, who can thus ensure an expected excess rate of return of zero. If they are more sensitive to reductions in this level than to increases, as the hypothesis of loss aversion predicts, they may want fund managers to behave in accord with the behavioral hypothesis proposed here: Choose a portfolio that minimizes the long-run probability of earning a nonpositive time-averaged excess return over the benchmark. In this interpretation, the behavioral hypothesis presented here provides a preference-parameter-free alternative to prospect theory.

The Performance Index. As shown in Bucklew (1990), Cramer's Theorem can be used to provide the following computation of the performance index, I_p, for a portfolio denoted p with return R_p in excess of its benchmark:

$$I_p = \max_{\theta} -\log E\left(e^{\theta R_p}\right), \tag{6}$$

where θ is a number less than zero and E denotes the expected-value operation. In the important special case of normally distributed portfolio excess returns R_p, Bucklew showed how to compute the performance index (Equation 6). The result in this case is that

$$I_p = \frac{1}{2}\lambda_p^2, \tag{7}$$

that is, half the squared Sharpe ratio. So, the performance-index-maximizing portfolio (hereafter called "the performance portfolio") is the same as the Sharpe-ratio-maximizing portfolio (hereafter called "the Sharpe portfolio") given in Equation 2. It attains the maximum feasible performance index value given by Equations 3 and 7.

The intuition behind this result is straightforward: A portfolio with a large positive expected excess return has a small chance of producing a finite time series with negative time-averaged excess return. But so does a portfolio with a smaller standard deviation of excess return, for then a bad run of negative excess returns is less likely to drive the time-averaged excess return below zero. Because the Sharpe ratio, λ_p, is the ratio of these two statistics, it will be larger in either case than otherwise.

A hypothesis should be judged mainly by its results rather than by the realism of its assumptions.[8] I have just shown that the rank ordering of feasible normally distributed portfolios implied by the behavioral hypothesis results in the same rank ordering delivered by the Sharpe ratio—the most widely accepted method for evaluating normally distributed portfolios. This result is true even though the behavioral hypothesis *explicitly* stresses minimization of the probability that the riskless rate will not be exceeded, in a way that (the reader can now see) is *implicit* in the Sharpe ratio. More generally, remember that the behavioral hypothesis's minimization of the probability that the time-averaged benchmark will not be exceeded is equivalent to an emphasis on maximizing the probability that the time-averaged benchmark *will* be exceeded. Hence, it does not excessively emphasize security over growth considerations—or at least no more than the Sharpe ratio does.[9]

When excess returns are *not* normally distributed, the performance index I_p is not Equation 7. Instead, it will depend on the higher-order cumulants[10] of the nonnormal excess-return distribution (e.g., the function will reflect nonnormal skewness and kurtosis of the excess returns). As illustration, the computation of Equation 6 can be quickly rearranged to establish an equivalence to expected constant absolute risk aversion (CARA) utility:

$$-\frac{1}{e^{I_p}} = \max_{-c} E\left(-e^{-cR_p}\right). \tag{8}$$

The right side of Equation 8 is the CARA expected utility of portfolio R_p with a specific, computable value of its coefficient of risk aversion, c, greater than zero. The left side of Equation 8 increases with I_p, so this specific CARA function's rank ordering of portfolios will agree with the performance index's ordering.

Because CARA utility has a positive third derivative, the performance index's ordering will reflect skewness preference, thus satisfying the second desideratum presented in the introduction. The behavioral hypothesis and the CARA-equivalence result provide a behavioral foundation relevant for long-horizon fund managers, thus satisfying the third desideratum.

Although, in general, no explicit computable formula exists for the performance index, its absence is not much of a disadvantage relative to the Sharpe ratio. Uses of the Sharpe ratio still require a time series of past portfolio returns to estimate the portfolio's mean and standard deviation. The next section shows that a time series of past returns also enables straightforward estimation of the performance index, I_p.

Finding the Optimal Portfolio: A Distribution-Free Approach

Denote a time series of the assets' returns in excess of the chosen benchmark by R_{it}, where $i = 0, \ldots, n$ and $t = 1, \ldots, T$. Then, a portfolio's excess return at t is the weighted average $\sum_{i=0}^{n} w_i R_{it}$. Because the portfolio weight w_0 is equal to $1 - \sum_{i=1}^{n} w_i$, this portfolio excess return at time t may be rewritten as

$$R_{pt} = \sum_{i=1}^{n} w_i (R_{it} - R_{0t}) + R_{0t}. \tag{9}$$

So, the estimate for the right side of Equation 6 based on historical data is

$$I_p = \max_{\theta} - \log \frac{1}{T} \sum_{t=1}^{T} e^{\theta R_{pt}}. \tag{10}$$

Substituting Equation 9 into Equation 10 shows that the performance portfolio weights are those w that solve

$$I_m = \max_{w_1, \ldots, w_n} \max_{\theta} - \log \frac{1}{T} \sum_{t=1}^{T} e^{\theta \sum_{i=1}^{n} w_i (R_{it} - R_{0t}) + R_{0t}}. \tag{11}$$

The easiest way to solve the numerical maximization problem (Equation 11) is by use of the optimizer features of a personal computer spreadsheet. This method generally requires a good initial guess at the solution. To the extent that a normal distribution even roughly approximates the historical return distribution, an estimate of the Sharpe portfolio (Equation 2) will be a good initial guess for the portfolio weights; a good initial guess for θ is –1 times that portfolio's mean excess return divided by its variance. Finally, constraints on the portfolio weights (e.g, on short sales) can be imposed through use of the spreadsheet optimizer.

Empirical Example

Following Kroll, Levy, and Markowitz (1984), I chose approximately 20 (23, to be exact) randomly chosen stocks' monthly returns between 1977 and 1997.[11] To investigate the effects of the choice of benchmark on the results, I first examined the zero benchmark (i.e., an under-the-mattress "investment" at an interest rate of zero). The results will later be contrasted to those obtained with a U.S. T-bill benchmark. I used a numerical optimizer to solve Equation 11 and to locate the Sharpe portfolio chosen from these 23 stocks. The two portfolios, which (in population) would be the same if returns were normally distributed, are compared in **Table 1**. As expected, the biggest difference in portfolio weights between the Sharpe portfolio and the performance portfolio is in the stock with the highest excess (i.e., nonnormal) skewness, which is stock KMB. Because positive skewness helps avoid the dreaded nonpositive time-averaged excess return, the performance portfolio invests 22 percent of funds in it, whereas the Sharpe portfolio invests only 16 percent. This difference is enough to make KMB the second largest holding in the performance portfolio but only the fourth largest holding in the Sharpe portfolio. The performance portfolio also gives a lower weight to the stock with the lowest (i.e., most negative) skewness, AT.

The performance portfolio attained a maximal I_m value of 12.1 percent—higher than the 11.7 percent achieved by the Sharpe portfolio. The implication is that, although the probability of realizing nonpositive time-averaged excess returns decays rapidly for both portfolios, the performance portfolio's probability decays faster (thus, it has a higher level of security). The high degree of risk aversion in the performance portfolio, $c = -\theta = 12.53$, reflects the desire to maximize the probability of outperforming a very conservative benchmark.

Table 2 shows the results for the two portfolios when the one-month T-bill return was used as the benchmark. Again, the greatest difference between the performance and Sharpe portfolios is the heavier weight the performance portfolio gives to the most favorably skewed stock, KMB. Because the T-bill benchmark has a positive mean return, the probability of realizing nonpositive time-averaged excess returns is higher in this test. As a result, the maximum attainable performance index value, I_m, drops from 12.1 percent to 6.4 percent when the benchmark is the one-month T-bill return. The performance portfolio's degree of risk aversion also drops, to 6.65, in reflection of the desire to maximize the probability of outperforming a tougher benchmark than before.

Table 1. Monthly Return Statistics with Benchmark Return of Zero, January 1977–December 1996

					Weight in Optimal Portfolio	
Ticker Symbol	μ	σ	Excess Skewness	Excess Kurtosis	Sharpe Ratio	Performance Index
KU	0.012	0.040	–0.138	0.481	0.34	0.34
BUD	0.017	0.063	0.326	0.649	0.19	0.21
BMY	0.015	0.058	0.056	–0.173	0.17	0.19
KMB	0.017	0.062	**0.868**	3.020	**0.16**	**0.22**
CRS	0.013	0.078	–0.128	1.540	0.12	0.11
CC	0.035	0.126	0.115	1.020	0.12	0.12
AT	0.016	0.060	**–0.217**	0.555	**0.11**	**0.08**
DG	0.028	0.121	0.377	–0.013	0.08	0.09
CPL	0.013	0.050	0.012	0.651	0.07	0.08
NCC	0.014	0.063	–0.162	1.168	0.06	0.04
MUR	0.015	0.090	0.055	0.062	0.04	0.05
ORU	0.011	0.042	0.490	2.683	0.03	0.02
GD	0.019	0.091	0.361	1.320	0.02	0.03
BK	0.018	0.082	0.155	1.100	0.02	0.04
AMD	0.024	0.163	0.260	0.519	0.01	0.00
APD	0.013	0.076	–0.021	0.899	–0.01	–0.01
ALK	0.015	0.105	0.557	1.850	–0.02	–0.05
KOW	0.018	0.113	0.159	0.600	–0.02	–0.02
LTD	0.026	0.129	0.073	0.498	–0.05	–0.05
BCL	0.014	0.076	0.273	2.193	–0.06	–0.10
DCN	0.012	0.080	0.168	0.951	–0.08	–0.11
ACK	0.014	0.088	0.088	0.936	–0.12	–0.12
BAX	0.011	0.073	–0.029	–0.129	–0.15	–0.15

Table 2. Monthly Return Statistics with a One-Month T-Bill Benchmark, January 1977–December 1996

					Weight in Optimal Portfolio	
Ticker Symbol	μ	σ	Excess Skewness	Excess Kurtosis	Sharpe Ratio	Performance Index
BUD	0.017	0.063	0.326	0.649	0.28	0.30
KMB	0.017	0.062	**0.868**	3.020	**0.27**	**0.32**
CC	0.035	0.126	0.115	1.020	0.23	0.24
BMY	0.015	0.058	0.056	–0.173	0.21	0.24
KU	0.012	0.040	–0.138	0.481	0.21	0.21
AT	0.016	0.060	–0.217	0.555	0.19	0.18
DG	0.028	0.121	0.377	–0.013	0.15	0.17
CPL	0.013	0.050	0.012	0.651	0.14	0.15
CRS	0.013	0.078	–0.128	1.540	0.10	0.10
BK	0.018	0.082	0.155	1.100	0.07	0.09
MUR	0.015	0.090	0.055	0.062	0.03	0.02
GD	0.019	0.091	0.361	1.320	0.02	0.04
AMD	0.024	0.163	0.260	0.519	0.02	0.01
NCC	0.014	0.063	–0.162	1.168	0.01	–0.02
LTD	0.026	0.129	0.073	0.498	–0.03	–0.03
ALK	0.015	0.105	0.557	1.850	–0.05	–0.07
KOW	0.018	0.113	0.159	0.600	–0.05	–0.06
APD	0.013	0.076	–0.021	0.899	–0.06	–0.05
ORU	0.011	0.042	0.490	2.683	–0.07	–0.10
ACK	0.014	0.088	0.088	0.936	–0.12	–0.12
BCL	0.014	0.076	0.273	2.193	–0.13	–0.17
DCN	0.012	0.080	0.168	0.951	–0.18	–0.21
BAX	0.011	0.073	–0.029	–0.129	–0.25	–0.26

Conclusions

Endowment or pension fund managers may have sensible reasons to be averse to earning a time-averaged portfolio return that is not greater than the average return of some trustee-designated benchmark and, therefore, will choose a portfolio with a positive expected excess return over the benchmark. In that case, the probability that the portfolio will not earn a higher average return than the benchmark approaches zero asymptotically at a computable exponential decay rate. Accordingly, portfolios with high probability decay rates are preferable to portfolios with low probability decay rates. I have thus proposed the probability decay rate as a new portfolio performance index.

The proposed performance index produces the same rank order of normally distributed feasible portfolios as the Sharpe ratio, thus satisfying the first desideratum in the introduction. But returns are not always normally distributed—because of skewness and/or kurtosis attributable to large asymmetrical economic shocks, limited-liability (bankruptcy) effects on asset returns, or strategies involving options and other derivative securities with inherently asymmetrical returns. When excess returns are not normally distributed, the performance index will reflect skewness preference, unlike the Sharpe ratio, but implementation of this index requires the same historical return data and personal computer spreadsheet software that is needed to maximize the Sharpe ratio. Moreover, maximizing the decay rate is equivalent to maximizing expected CARA utility, *with a specific, computable coefficient of risk aversion.* So, the index has a relevant behavioral foundation (with both statistical and expected utility interpretations) that is free of unspecified parameters. Apparently, the new performance index is the only known measure that meets the desiderata. Still, one should acknowledge the judgment of Sortino and Forsey (1996) that for performance measurement, "no one risk measure is the be-all and end-all" (p. 41).[12]

The example of an optimal portfolio formed from 23 randomly selected stocks showed that the performance portfolio did indeed place more weight on assets with positively skewed returns than the Sharpe portfolio did.

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Notes

1. Maximization of expected quadratic utility "rationalizes" the mean–variance criterion, even when portfolio returns are not normally distributed, but a quadratic utility function is problematic regardless of how its parameter values are chosen. It always has a "bliss" point (i.e., a level of wealth high enough that investors would never want to exceed it), and as noted by Pratt (1964) and others, increases in wealth below the bliss point result in counterintuitively higher aversion to inherent investment risks.
2. He and Leland did not try to explain the level of the risk-free rate, nor did they try to explain the intertemporal spending of their investors, whereas the analogous equilibrium models do.
3. Moreover, changing Leland's lognormal market and CRRA features and permitting continuous consumption will not eliminate the following counterfactual implication of *all* expected-utility-maximizing, representative-agent exchange models with complete markets: All of these models imply that spending by any investor is perfectly correlated with spending by any other investor. Graphs of all investors' intertemporal spending ought to move up and down in lockstep. They do not. In international asset-pricing models, each country's aggregate consumption should be perfectly correlated with any other's. They are not.
4. Leland did cite empirical evidence that "daily market returns are not lognormal but for longer periods (e.g., three months), returns are quite 'close' to lognormally distributed" (Note 7). This statement does not, however, mitigate the significance of his model's counterfactual implication about Black–Scholes pricing of options on this stock market. For example, the failure might be a result of his assumption of continuous-time, frictionless trading rather than nonlognormality (but I doubt it).
5. The conventional mean–variance tools cannot properly evaluate the nonnormal, positively skewed returns induced by these strategies, however, so the conventional tools are biased against their adoption.
6. This notation is also in accord with longstanding decrees of the Intra-Fraternity and Pan-Hellenic Councils.
7. October 10, 1999:BU17.
8. Otherwise, the expected utility hypothesis would also have been dismissed long ago.
9. I thank an anonymous referee for motivating this illustration of the behavioral hypothesis's sensibility despite its stress on the probability of underperformance.
10. Coefficients that arise in the series expansion of the logarithm of the moment-generating function.
11. Kroll et al. used 20 randomly chosen stocks. When I made a data request for 20 randomly chosen stocks' monthly returns off the CRSP tapes, 23 were delivered by someone thinking I would be pleased to get the extra data. To prevent even the slightest appearance of data snooping, I used all 23 rather than pare 3 from the set.
12. Finally, should one insist on an index implied by a perfect financial market equilibrium that results when *all* investors behave this way; it is easy to find one by applying Rubinstein's (1973) calculations to this specific CARA utility.

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