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OPTIMAL BETTING ODDS AGAINST INSIDER TRADERS*

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Consider the following situation. A bookmaker at the race track is facing a group of gamblers, some of whom have insider information on the outcome of a race. The bookmaker cannot distinguish those gamblers who have insider information from those who do not, but has some idea of the proportion of one group to the other. In this situation, what odds should the bookmaker offer? Also, given the optimal decision of the bookie, what value can be imputed to the insider information? These are the questions addressed in this paper.

One of the central results of this paper is that when the above situation is modelled as a game between the bookmaker and the gamblers, the normalised betting odds diverge systematically from the true winning probabilities of the horses. In particular, it is optimal for the bookmaker to employ a 'square root rule' in which the ratio of posted prices is set equal to the square root of the ratio of winning probabilities. One consequence of this rule is that the betting odds tend to understate the winning chances of the favourites, and to overstate the winning chances of the longshots.

This tendency to understate the winning chances of the favourites and overstate the winning chances of the longshots has been confirmed in several empirical investigations of the betting market. The table below summarises Dowie's (1976) investigation into the British flat racing season of 1973, as presented by Crafts (1985, p. 300). It shows the ratio of winnings to wager for subsets of the runners ordered by their betting odds. There were approximately 3,000 races in the 1973 flat season with almost 30,000 runners.

Subset	Winnings/wager (%)
Up to evens	99·2
Up to 5/1	90·9
Up to 10/1	89·4
Up to 16/1	80·3
All	60·5

If the prices were proportional to winning probabilities, the winnings/wager ratio should be constant over all subsets. However, the odds were biased in such a way that the prices on the favourites were cheaper (relative to true probabilities) than the prices on the longshots. This is precisely the favourite–longshot bias.

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The description of betting given in this paper is modelled on the betting system in the United Kingdom in which bookmakers set odds. In North America, betting odds are determined by the parimutuel method in which prices are proportional to amounts wagered. However, even this system cannot be regarded as being completely immune to manipulation. The setting of morning-line odds, for example, will influence the early stages of betting. Nevertheless, this distinction should be borne in mind when examining North American data (see Ali, 1977; Asch *et al.* 1982; Thaler and Ziemba, 1988).

Although the discussion in this paper is couched in terms of gambling, the analysis is applicable to any situation in which an *uninformed* price-setter is facing a group of traders, some of whom have superior information. Then, the problem of setting optimal betting odds is analogous to posting optimal bid and ask prices against the traders.

The approach to insider information taken in this paper differs from the conventional one in which an *informed* price-setter faces *uninformed* traders. Examples of this approach include Gould and Verrecchia (1985), Grinblatt and Ross (1985) and Laffont and Maskin (1990). These papers focus on the issue of the revelation of information by the large, informed trader in its price-setting strategy. The large trader has the dual advantage of being the informed party as well as being the price-setter.

In contrast, the approach taken in our paper is motivated by the observation that, although the price-setter may have better information than the majority of small traders, there may be a few traders who have better information than the price-setter. This seems a reasonable hypothesis both on the race track and in the stock market.

More precisely, we postulate an 'informational hierarchy' which consists of three tiers. Given a space of possible states of the world, the *insiders* occupy the top tier of this hierarchy. They know the actual *realisation* of the state of the world. Below the insiders comes the price-setter, who knows the *true* probability distribution over the states of the world, but who does not know the realisation of the true state. Lastly, at the bottom of this informational hierarchy come the uninformed small traders (the outsiders). These traders hold diverse *subjective* probability distributions over the state space.

We begin by presenting the model and stating the results alluded to above in a more precise way. Section II contains the solutions of the model, and Section III contains the concluding comments.

I. THE MODEL

Our model describes the market for bets surrounding a two-horse race in which two types of tickets are traded. The first pays a dollar if the first horse wins while the second pays a dollar if the second horse wins. They pay nothing otherwise. We denote by π_1 the price of the first type of ticket, and by π_2 the price of the second. These prices correspond to betting odds in the usual way. Odds of k to l correspond to the price $l/(l+k)$, while the price π_i corresponds to odds of $(1/\pi_i)-1$ to 1.

The actors in the model are as follows. There is a monopolist bookmaker (the *Bookie*) who sets odds on the race by posting selling prices of the two types of tickets. There is an informed punter (whom we shall call the *Insider*) who has privileged information concerning the outcome of the race, in a sense to be made precise below. The Insider has wealth of z dollars. Lastly, there is a continuum of uninformed punters (whom we shall call the *Outsiders*) indexed by the closed unit interval $[0, 1]$. The Outsider indexed by $q \in [0, 1]$ believes that the first horse wins with probability q and that the second horse wins with probability $1-q$. As a group, the Outsiders have a collective wealth of y dollars, and the distribution of wealth is uniform over $[0, 1]$. Without loss of generality, we shall assume that the total wealth of all punters is 1, so that $y = 1-z$.

Each Outsider is assumed to be an expected payoff maximiser (and hence risk-neutral), with beliefs given by the index $q \in [0, 1]$. Thus, given posted prices (π_1, π_2), the Outsider with index q acts according to the following rules.

(i) If $q > \pi_1$ but $1-q \leqslant \pi_2$, bet on the first horse.

(ii) If $1-q > \pi_2$ but $q \leqslant \pi_1$, bet on the second horse.

(iii) If $q > \pi_1$ and $1-q > \pi_2$, then bet on the first horse if $q/\pi_1 \geqslant (1-q)/\pi_2$ and bet on the second horse if $q/\pi_1 < (1-q)/\pi_2$.

(iv) If $q \leqslant \pi_1$ and $1-q \leqslant \pi_2$, do not bet at all.

The market is organised in the form of an extensive form game between the Bookie and the Insider. The Outsiders are not formally players in the game. They should be seen as 'noise traders' who cling stubbornly to their beliefs. The game consists of three stages.

Stage 1. Nature chooses a number p_1 from the open unit interval $(0, 1)$, according to the uniform density, where p_1 is the probability that the first horse wins the race, and $p_2 = 1-p_1$ is the probability that the second horse wins.

Stage 2. The Bookie observes Nature's choice of p_1 and posts prices $\pi = (\pi_1, \pi_2)$ for the two types of tickets, subject to the constraint that negative odds are not offered. That is $0 \leqslant \pi_i \leqslant 1$ for both horses i.

Stage 3. Nature then performs a Bernoulli experiment in which the winner of the race is chosen according to the probabilities (p_1, p_2). The Bookie cannot observe the outcome of this experiment, but the Insider is permitted to observe the identity of the winning horse, and is free to bet on the race within his budget of z dollars. The Outsiders bet on the race according to their beliefs in the way described above.

Once the betting has taken place, the race is run according to script, and the horse chosen by Nature is seen to win. The Bookie then settles with all the punters according to the prices offered at stage 2 of the game.

We shall be interested in two issues. The first is the form of the optimal prices in the game, and how they relate to the true winning probabilities p_1, p_2. The second is the value of the Insider's information. If we denote by V the Insider's equilibrium payoff in the game, then $(V-z)/z$ is the rate of return to information. We shall call this the *value of insider information*. Our conclusions are summarised in the following pair of propositions. We shall say that *bets are accepted on horse i* if the constraint $0 \leqslant \pi_i \leqslant 1$ does not bind. For the rest of this paper, the term 'equilibrium' is used to mean subgame perfect equilibrium.

PROPOSITION 1. *In the unique equilibrium,*

$$\pi_i = \min\{1, \sqrt{[p_i/(1-z)]}\}. \tag{1}$$

Hence, if bets are accepted on both horses, prices follow the square root rule;

$$\pi_1/\pi_2 = \sqrt{(p_1/p_2)}. \tag{2}$$

PROPOSITION 2. *The value of insider information is*

$$\tfrac{1}{3}(1-z)^2, \tag{3}$$

so that, as z becomes small, the value of insider information tends to 1/3.

Two things are worthy of note in these results. Firstly, the betting odds are biased indicators of the underlying winning probabilities, and will tend to understate the difference in winning probabilities between the two horses. This leads to the favourite-longshot bias mentioned in the introduction of this paper.

Secondly, there is an upper bound of one third on the value of insider information. From (3), it can be seen that the value of insider information is decreasing in z, reflecting the higher prices set by the monopolist bookie as the Insider plays a more prominent role in the market. The rate of return of 1/3 seems rather modest in the light of the privileged position occupied by the Insider in the market, and reflects the monopoly power of the Bookie.

II. SOLUTION OF THE GAME

We employ the standard backward solution procedure for extensive form games. We begin by determining the betting behaviour at stage 3 given prices $\boldsymbol{\pi}$ and probabilities $\mathbf{p} = (p_1, p_2)$, and then solve for optimal prices at stage 2 in anticipation of the betting at stage 3. This yields solutions for prices and payoffs in terms of $\mathbf{p}$ and z. Finally, by integrating over the unit interval with respect to p_1, we obtain expressions for equilibrium payoffs in terms of the parameter z.

At stage 3, the Insider wagers her entire wealth on the winner. Thus, if the first horse is chosen to win, she buys z/π_1 units of the first type of ticket, and buys z/π_2 units of the second type of ticket if the second horse is chosen to win.

The demand for tickets from the Outsiders can be considered under two cases. We first consider the case in which $\pi_1 + \pi_2 \geqslant 1$. Then, the proportion $1-\pi_1$ of the Outsiders bet on the first horse, and proportion $1-\pi_2$ bet on the second. Since the distribution of wealth is uniform and the collective wealth of the Outsiders is $1-z$, the Bookie sells $(1-\pi_1)(1-z)/\pi_1$ units of the first type of ticket to the Outsiders and sells $(1-\pi_2)(1-z)/\pi_2$ units of the second type of ticket to the Outsiders.

For the case in which $\pi_1 + \pi_2 < 1$, every Outsider bets on one of the two horses. The Outsider with index $\bar{q} = \pi_1/(\pi_1+\pi_2)$ is indifferent between betting on the first horse and betting on the second, since $\bar{q}$ satisfies the equation $\bar{q}/\pi_1 = (1-\bar{q})/\pi_2$. Outsiders with index $q > \bar{q}$ bet on the first horse, and Outsiders with index $q < \bar{q}$ bet on the second. Thus, proportion $1-\bar{q}$ of the

Outsiders bet on the first horse, and proportion $\bar{q}$ bet on the second. Since the wealth of $(1-z)$ is uniformly distributed, $(1-\bar{q})(1-z)/\pi_1$ units of the first type of ticket are bought by the Outsiders, and $\bar{q}(1-z)/\pi_2$ units of the second type of ticket are bought by the Outsiders.

We now turn to the pricing decision of the Bookie at stage 2. The expression for the Bookie's expected profit at stage 2 of the game falls under two cases, depending on the sum of prices – namely, whether $\pi_1+\pi_2 < 1$ or $\pi_1+\pi_2 \geqslant 1$. Intuition should suggest that the former case is unlikely to obtain in equilibrium, since it implies the existence of 'Dutch books' – that is, portfolios which guarantee a payoff of one, but whose price is less than one. This intuition can be verified as follows. Suppose $\pi_1+\pi_2 < 1$. The expected revenue of the Bookie from the sale of tickets to the Outsiders is $(1-z)$, since all Outsiders bet. The bookie's cost depends on which horse wins the race, and is given by the number of tickets sold on the winning horse. By weighting the number of tickets sold by the appropriate probabilities, the Bookie's expected payoff at stage 2 against the Outsiders is:

$$(1-z)\left(1-p_1\frac{1-\bar{q}}{\pi_1}-p_2\frac{\bar{q}}{\pi_2}\right). \tag{4}$$

Since $\bar{q}=\pi_1/(\pi_1+\pi_2)$, this can be written as:

$$(1-z)\left[1-\frac{1}{\pi_1+\pi_2}\left(\frac{p_1\pi_2}{\pi_1}+\frac{p_2\pi_1}{\pi_2}\right)\right]. \tag{5}$$

Keeping the ratio π_1/π_2 constant and increasing the sum of prices will raise (5). Since the Bookie's expected profit against the Insider is increasing in both prices, we conclude that the Bookie's optimal prices at stage 2 lies in the region where $\pi_1+\pi_2 \geqslant 1$.

For this reason, we confine our attention to the paths in the game tree in which $\pi_1+\pi_2 \geqslant 1$. The expression for the Bookie's expected profit in this case is derived as follows. Consider the Bookie's expected profit from the sale of the ith ticket to the Outsiders. The revenue from the sale of the ith ticket to the Outsiders is $(1-z)(1-\pi_i)$, while the Bookie pays out $(1-z)(1-\pi_i)/\pi_i$ if the ith horse wins. Weighting this cost by the probability of the ith horse winning and summing over $i = 1, 2$, we have the following expressions for the Bookie's expected profit from the sale of tickets to the Outsiders.

$$(1-z)\left(2-\pi_1-\pi_2-p_1\frac{1-\pi_1}{\pi_1}-p_2\frac{1-\pi_2}{\pi_2}\right). \tag{6}$$

Against the Insider, the Bookie receives constant revenue of z, since the Insider always bets. However, the Bookie pays out $1/\pi_i$ if the ith horse wins. Weighting this cost by the probability p_i and summing over i, the Bookie's expected profit from the sale of tickets to the Insider is

$$z\left(1-\frac{p_1}{\pi_1}-\frac{p_2}{\pi_2}\right). \tag{7}$$

The Bookie's overall expected profit at stage 2 is given by the sum of (6) and (7). It can be verified that this function is strictly concave in π in the feasible set. Thus, there is a unique optimum for the Bookie, and if the optimum lies in the interior of the unit square, it satisfies the first-order condition:

$$-(1-z)+\frac{p_i}{\pi_i^2}=0. \tag{8}$$

If the constraint $0 \leqslant \pi_i \leqslant 1$ binds, $\pi_i = 1$ is optimal. This implies that the optimal π_i is given by

$$\pi_i = \min\{1, \surd[p_i/(1-z)]\}. \tag{9}$$

Bets are accepted on the ith horse provided that $z < 1-p_i$. That is, provided that the Insider's role is small. If bets are accepted on both horses, prices follow the square root rule; $\pi_1/\pi_2 = \surd(p_1/p_2)$. Optimal prices are depicted in Fig. 1.

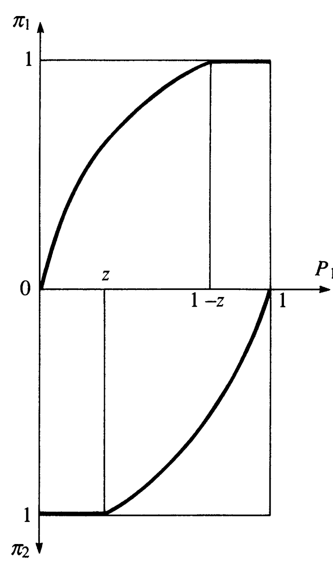

Fig. 1. Optimal prices (bold lines).

Optimal prices are indicated by the bold lines. Bets are accepted on both horses only if p_1 lies in the open interval $(z, 1-z)$, and this interval shrinks as z becomes large.

We now turn to the Insider's payoff in the game. The Insider's payoff given p is $z(p_1/\pi_1+p_2/\pi_2)$. The Insider's expected payoff for the game as a whole is obtained by integrating this expression over the unit interval with respect to p_1. From the fact that $\pi_i = \min\{1, \surd[p_i/(1-z)]\}$ and $p_2 = 1-p_1$, the Insider's expected payoff for the game as a whole is given by $z+z(1-z)^2/3$. Thus, the value of insider information is $(1-z)^2/3$, proving Proposition 2. As expected, the value of insider information is decreasing in z. This is due to the higher prices set by the Bookie in anticipation of the increased role of the Insider.

III. CONCLUDING REMARKS

The most prominent feature of the discussion in this paper has been the systematic divergence of the ratio of posted odds from the ratio of true probabilities. In particular, this divergence has been shown to follow the 'square-root rule' $\pi_1/\pi_2 = \sqrt{(p_1/p_2)}$, which tends to understate the difference in the winning chances of the two horses, and so leads to the favourite–longshot bias.

The intuition for the square root rule lies in the comparative rates of change of revenue and costs for the Bookie. Whereas the revenue which the Bookie collects from the sale of the ith ticket falls linearly (at the rate $1-z$) as the price of the ith ticket rises, the cost to the Bookie if the ith horse wins the race is given by the number of the ith ticket sold. In turn, the number of tickets sold is given by an expression in which π_i appears in the denominator. Thus, as π_i rises, the cost to the Bookie if the ith horse wins falls at the rate $1/\pi_i^2$. Since this expression is large when π_i is small, there is a tendency for the Bookie to avoid setting very long odds.

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