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PRICES OF STATE CONTINGENT CLAIMS WITH INSIDER TRADERS, AND THE FAVOURITE-LONGSHOT BIAS*

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The literature on the pricing of state-contingent claims has been based almost exclusively on the framework of competitive equilibrium as pioneered by Arrow (1964) and Debreu (1959). This approach rests on the classical view of markets as a place which matches buyers and sellers at a common price – the price at which supply equals demand.

In practice, however, most financial transactions take place through an intermediary, such as the market maker, who holds inventories of assets, and who sets prices for buyers and sellers. Invariably, the price facing buyers differs from that facing sellers. One source of this divergence is the cost to the market maker of maintaining the inventory. However, a more potent source of this divergence is the incidence of insider trading. The market maker faces an adverse selection problem in which a customer may be trading on the basis of superior information. In this case the bid-ask spread is determined in a trade-off between setting a large spread so as to minimise the profit of insider traders, and setting the optimal spread against the noise or liquidity traders. Bagehot (1971) hints at this source of the bid-ask spread, and subsequent papers by Copeland and Galai (1983), Glosten and Milgrom (1985) and Kyle (1985) have provided formal treatments of the problem.

This paper is an attempt to draw together these two themes in the literature. It is concerned with the pricing of contingent claims when market makers set prices in the presence of insider traders. The specific setting for our investigation is the market for bets in a horse race, in which the role of the market makers are taken by the bookmakers, and the traders are played by the potential bettors (the 'punters'). Thus, our description of betting follows the system in the United Kingdom in which bookmakers set odds rather than the system in North America in which odds are determined by the parimutuel method in which prices are proportional to amounts wagered. There are several reasons for our choice of the betting market as a vehicle for the study of asset pricing with insider trading.

Firstly, the betting market is a particularly good example of a contingent claims market. In its simplest formulation, the market for bets in an n-horse race corresponds to a market for contingent claims with n states of the world, in which the ith state corresponds to the outcome in which the ith horse wins

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the race. Moreover, the basic securities (Arrow-Debreu securities) which pay a dollar if a particular state obtains and nothing otherwise, have their prices determined by the betting odds. Since odds are offered on each horse, all basic securities are traded, thereby ruling out the difficulties associated with incomplete markets.

Moreover, the betting market is a particularly simple example of a financial market. The market convenes for about half an hour, at the end of which there is a definite and commonly acknowledged outcome. This is in contrast to the complex decisions faced by traders in more sophisticated financial markets in which considerations of the distant future play an integral part in current decisions.

There is also a sizeable body of evidence, both systematic and anecdotal, which points to the prevalence of insider trading in the market for bets (see, for example, Crafts (1985)). No less a body than the Committee of Inquiry of the Jockey Club acknowledges that 'trainers and their staff are insufficiently paid for their services and the majority have to resort to betting to make ends meet' (1968, p. 91). It would seem, therefore, that the activity of insider traders influences the outcome in the betting market in a significant way.

Our chief concern in this paper is to address the well-known stylised fact that the percentage mark-ups in the prices over the true probabilities is not uniform. In general, prices exhibit *favourite-longshot bias* in which, the normalised prices on the favourites of the race understate the winning chances of these horses, while the normalised prices on the longshots exaggerate their winning chances. For the betting market in the United Kingdom, Dowie (1976) confirms this bias.

Betting odds at North American race tracks are determined by the parimutuel method in which prices are proportional to amounts wagered. However, even this system cannot be regarded as being completely immune from manipulation. The setting of morning line odds, for example, will influence the early stages of betting. Nevertheless, this distinction should be borne in mind when examining North American data (see Ali (1977), Asch *et al.* (1982) and Thaler and Ziemba (1988)).

The overview of our paper is as follows. In the next section, we present our model of the betting market in an n-horse race. The solution of the model follows in Section II, where we derive a necessary and sufficient condition for the favourite-longshot bias. Section III investigates the robustness of this result in a more general framework. By means of a comparative statics argument, we show that the bias survives in a generalised form.

I. THE MODEL

Our model describes a market for bets in a horse race with n horses. The market is organised as an extensive form game in which two bookmakers compete in setting odds on the horses in anticipation of the betting behaviour to follow. There are three players in the game – the incumbent bookmaker, a potential bookmaker, and the bettor with insider information (the 'Insider'). There is also a set of n individuals called 'outsiders', labelled by the set $\{1, 2, \ldots, n\}$. The outsiders are not modelled formally as players in the game. Rather, they are mechanical traders who do not act strategically. In particular, the ith outsider attaches probability 1 to the ith horse winning the race.

However, although each outsider (taken individually) may be quite irrational, the game is designed so that the market prices which follow from outsiders' demands for bets are fully revealing, in the sense that the equilibrium prices coincide with the true probabilities. This provides a benchmark for the general case in which distortions in market prices are introduced as a result of insider trading.

There are n types of tickets which are sold in the market. The ith type of ticket pays one dollar if the ith horse wins and zero otherwise. The price of this ticket is denoted by π_i. These prices correspond to betting odds in the usual way. Odds of k to l correspond to the price of $l/(k+l)$. Negative odds are ruled out in our model, so that $0 \leqslant \pi_i \leqslant 1$ for all i. Fractional quantities of tickets may be sold.

As in any financial market, the prices quoted are not valid for all quantities. In sophisticated financial markets, the quoted price varies with the quantity traded. For our purpose, we shall assume that the bookmakers accept unit bets of one dollar. This is consistent with the convention that bookmakers' odds are the prices at which a 'substantial sum' may be wagered. The implication is that there is a limit to the size of the wager accepted at the quoted price. The assumption of unit trades is widely used in models of financial markets to bound the positions taken by insiders and risk neutral traders. Examples include Copeland and Galai (1983) and Glosten and Milgrom (1985), cited earlier. Our motivation is similar.

The game is in extensive form and describes the encounter between one bookie and one punter. It has four stages.

Stage 1 (*Bidding Stage*). The two bookies – the incumbent and the potential entrant – bid for monopoly rights to the betting market. Each submits a sealed bid of a positive real number. The bookie who submits the lower number wins the bid. If the bids are identical, the incumbent wins. The bookie who loses the bid gets a payoff of zero.

Stage 2 (*Price-setting Stage*). The bookie who wins the bid at stage 1 sets prices $\pi_1, \pi_2, \ldots, \pi_n$ for the n types of tickets subject to the constraint that the sum $\Sigma_i \pi_i$ does not exceed the bid submitted in stage 1 and that $0 \leqslant \pi_i \leqslant 1$, for all i.

Stage 3 (*Nature's Choice of Winner and Punter*). Nature then performs two experiments. In the first, a winner of the race is chosen. In the second, precisely one punter is chosen to bet against the price-setting bookie. Thus, either the Insider is chosen to play, or one of the outsiders is chosen. The experiments are governed by the probability measure μ, given as follows. Denote by O_i the event in which the ith outsider is chosen, by I the event in which the Insider is chosen, and by H_i the event in which the ith horse is chosen as the winner. The event O is the union $\cup_i O_i$. We shall assume that $\mu(H_i) = p_i$, $\mu(I) = z$, and $\mu(O_i) = (1-z)p_i$. That is, the ith horse wins with probability p_i, the Insider is chosen with probability z, and the ith outsider is chosen with probability

$(1-z)p_i$. We shall assume that $p_i > 0$ for all i. In keeping with the idea that the outsiders are noise traders, if an outsider is chosen, his identity is independent of the winning horse. That is, for any k and i,

$$\mu(O_k \mid H_i) = \mu(O_k). \tag{1}$$

In contrast, the probability that the Insider plays may not be independent of the identity of the winning horse. We denote by z_i the probability that the Insider is chosen conditional on the ith horse being the winner. That is,

$$\mu(I \mid H_i) = z_i. \tag{2}$$

Stage 4 (*Betting Stage*). The punter chosen at stage 3 meets the bookie and a bet is placed. Crucially, if the Insider is chosen to play, she is permitted to observe the identity of the winning horse, and is free to buy tickets from the bookie at the posted prices up to the value of one dollar. On the other hand, the ith outsider is an expected payoff maximiser with the belief that the ith horse wins with probability 1. Thus, the ith outsider always bets a dollar on the ith horse.

Once the betting has been completed, the race is run according to script and the horse chosen by Nature is seen to win the race. The bookie then settles with the punter in accordance with the odds offered at stage 2.

Two comments are in order concerning the model. Firstly, we could interpret the model as one in which a bookie meets a large number of punters, where the probabilities denote the proportion of each type of punter in the population. The problem for the bookie is identical. Secondly, our main interest is in the determination of prices given a zero profit condition. An alternative modelling strategy would have been to allow fully-fledged price competition between the bookies. Provided that bookies offer the full menu of bets, the outcome of price competition coincides with that of our model.

II. DERIVATION OF EQUILIBRIUM PRICES

We now proceed to the solution of the game. Our solution concept is subgame perfect equilibrium. Although there are many equilibria of our game, they are all essentially identical in the sense that the prices are identical across all equilibria. We begin by solving for the bookie's maximisation at stage two given the winning bid β at stage 1. This yields expressions for the prices $\pi_1, \ldots, \pi_n$. By substituting these expressions into the bookie's profit function, we obtain an expression for the price-setting bookie's profit in terms of the bid β. By solving for the equilibrium bid β and substituting into the expressions for $\pi_1, \ldots, \pi_n$, we obtain explicit solutions for the equilibrium prices in terms of the parameters of the model.

We denote by $\mathbf{p}$ the vector of winning probabilities $(p_1, \ldots, p_n)$ and by $\boldsymbol{\pi}$ the vector of prices $(\pi_1, \ldots, \pi_n)$. We shall denote by $V(\boldsymbol{\pi})$ the expected profit of the price-setting bookie given prices $\boldsymbol{\pi}$, and denote by $V(\boldsymbol{\pi} \mid I)$ the expected profit conditional on the punter being the Insider. Similarly, we denote by $V(\boldsymbol{\pi} \mid O)$ the expected profit conditional on the punter being an outsider. Thus,

$$V(\boldsymbol{\pi}) = zV(\boldsymbol{\pi} \mid I) + (1-z)V(\boldsymbol{\pi} \mid O). \tag{3}$$

Any punter bets precisely one dollar on a particular horse, so that if the punter bets on the ith horse, $1/\pi_i$ units of the ith ticket are sold by the bookie. Thus, if the bookie pays out, he pays out $1/\pi_i$ dollars, where i is the index of the winning horse. Since revenue is constant at 1 and the bookie always pays out to the Insider, $V(\boldsymbol{\pi} \mid I)$ is given by $1 - \sum_i \mu(H_i \mid I)/\pi_i$. Some manipulation using (2) yields:

$$V(\pi \mid I) = 1 - \frac{1}{z} \sum_i \frac{z_i p_i}{\pi_i}. \tag{4}$$

Against the outsiders, the bookie pays out only in the events $\{H_i \cap O_i\}$. Thus, $V(\boldsymbol{\pi} \mid O)$ is given by $1 - \sum_i \mu(H_i \cap O_i \mid O)/\pi_i$. From (1), we have:

$$V(\boldsymbol{\pi} \mid O) = 1 - \sum_i \frac{p_i^2}{\pi_i}. \tag{5}$$

From (3), (4) and (5), the problem for the price-setting bookie at stage 2 of the game is to:

$$\underset{\boldsymbol{\pi}}{\text{maximise}} \quad 1 - \sum_i \frac{z_i p_i + (1-z) p_i^2}{\pi_i} \tag{6}$$

$$\text{subject to} \quad \sum_i \pi_i \leqslant \beta \quad \text{and} \quad 0 \leqslant \pi_i \leqslant 1, \quad \text{for all } i,$$

where β is the winning bid made at stage 1. The feasible set is the intersection of the halfspace $\{\boldsymbol{\pi} \mid \sum_i \pi_i \leqslant \beta\}$ and the unit cube $\{\boldsymbol{\pi} \mid 0 \leqslant \pi_i \leqslant 1, \forall i\}$. It is the intersection of two convex sets, and hence is itself convex. The objective function is strictly concave in $\boldsymbol{\pi}$ as can be verified from the Hessian which is a diagonal matrix with negative entries. Thus, any solution to (6) is unique. We must now consider the solution of the game for two cases – namely, the case in which the optimal $\boldsymbol{\pi}$ lies in the interior of the unit cube and the case in which it lies on the boundary of the unit cube.

The solution for the case in which the equilibrium $\boldsymbol{\pi}$ lies on the boundary of the unit cube is presented in the appendix. For the main body of the paper, we shall concentrate on the case in which bets are accepted on all horses. Since $V(\boldsymbol{\pi})$ is strictly increasing in the prices, the constraint $\sum_i \pi_i \leqslant \beta$ binds, and the optimal $\boldsymbol{\pi}$ satisfies the first-order conditions. Solving for π_i,

$$\pi_i = \frac{\beta \surd [z_i p_i + (1-z) p_i^2]}{\sum_s \surd [z_s p_s + (1-z) p_s^2]}. \tag{7}$$

Substituting into $V(\boldsymbol{\pi})$, we have the following expression for expected profit in stage 1 in terms of β.

$$1 - \frac{1}{\beta} \left\{ \sum_s \surd [z_s p_s + (1-z) p_s^2] \right\}^2. \tag{8}$$

The payoff of both bookmakers must be zero in any equilibrium. It is at least zero since a bookie gets a payoff of zero by bidding a larger number than its rival. It is at most zero since the rival can undercut. Thus, in any equilibrium, the winning bid β is the number which sets (8) equal to zero. That is,

$$\beta = \left\{ \sum_s \sqrt{[z_s p_s + (1-z) p_s^2]} \right\}^2. \tag{9}$$

Substituting (9) into (7), we can solve for π_i in terms of the parameters of our model,

$$\pi_i = \sqrt{[z_i p_i + (1-z)] p_i^2} \left\{ \sum_s \sqrt{[z_s p_s + (1-z) p_s^2]} \right\}. \tag{10}$$

In the benchmark case in which $z = 0$ (when the Insider plays no part in the game), we have $\boldsymbol{\pi} = \mathbf{p}$, so that prices coincide with the true probabilities. Prices in this case mimic the determination of competitive prices in an economy consisting of the n outsiders, where the ith outsider is endowed with share p_i of the portfolio $(1, 1, \ldots, 1)$. Then, the total demand for the ith type of ticket is $p_i(\sum_s \pi_s)/\pi_i$, so that market clearing ensures $\pi_i = p_i \sum_s \pi_s$, and prices are proportional to the true probabilities. Notice also that the normalised price on the ith horse is identical to the proportion of the total wealth wagered on the ith horse. In other words, competitive prices are the parimutuel betting odds (see Eisenberg and Gale (1959)).

For the general case in which $z > 0$, we shall be interested in identifying the conditions under which prices exhibit favourite-longshot bias. Formally, we shall say that the equilibrium prices exhibit *favourite-longshot bias* when, $\pi_i/\pi_j < p_i/p_j$ if and only if $p_i > p_j$. In other words, the betting odds understate the winning chances of a favourite (horse i) relatively less than the winning chances of a longshot (horse j). A necessary and sufficient condition is identified in the following proposition.

PROPOSITION 1. *Suppose $z > 0$. Then, equilibrium prices exhibit favourite-longshot bias if and only if*:

$$z_i/p_i < z_j/p_j \Leftrightarrow p_i > p_j. \tag{11}$$

Proof. Since $z > 0$ and all components of $\mathbf{p}$ are non-zero, the ratio of prices in (10) is given by:

$$\frac{\pi_i}{\pi_j} = \sqrt{\left[\frac{z_i p_i + (1-z) p_i^2}{z_j p_j + (1-z) p_j^2}\right]} = \frac{p_i}{p_j} \sqrt{\left[\frac{1 - z + (z_i/p_i)}{1 - z + (z_j/p_j)}\right]}. \tag{12}$$

Thus, if (11) holds, $p_i > p_j \Leftrightarrow z_i/p_i < z_j/p_j \Leftrightarrow \pi_i/\pi_j < p_i/p_j$, which is the bias. The converse is immediate from (12).

Since $z_i = \mu(I \mid H_i)$, the condition identified in (11) rules out those cases in which the incidence of insider trading is substantially larger when a favourite is tipped to win than when a longshot is tipped to win. Since, in general, we would expect insider trading to be more prevalent given that a longshot is tipped to win, this condition accords with our intuitions. Moreover, condition (11) is satisfied even if z_i rises with p_i, provided that the ratio z_i/p_i is falling with p_i. Thus, the favourite-longshot bias would seem to be a fairly general feature of models of this kind. Notice, in particular, that (11) is satisfied when z_i is identical across all horses i. If we interpret our model as one in which the bookie meets a large number of punters, where the probabilities denote the proportion of each type of punter in the population, a low p_i would correspond to a low demand for the ith basic security from the set of outsiders. Then, (11) is satisfied if insider trading is more prevalent in 'thin' markets. Again, this accords with our intuition.

III. A MORE GENERAL ARGUMENT

The explicit solution for equilibrium prices has been obtained at the cost of attributing extreme probability beliefs to the noise traders. We shall now examine how our results are affected by a more general formulation of outsiders' beliefs.

We shall continue to assume that the outsiders are expected payoff maximisers according to their subjective probability judgements but we allow arbitrary sets of outsiders. However, we shall impose the condition that, in the absence of insider trading, prices are proportional to true probabilities. By insisting on this benchmark, any deviation from proportionality will be attributable to insider trading. We shall continue with the notation $V(\boldsymbol{\pi} \mid O)$ to denote the expected profit of the price-setting bookie against the set of outsiders. Our assumptions are:

(A1) $V(\boldsymbol{\pi} \mid O)$ is differentiable on the interior of the unit cube;

(A2) If β is the equilibrium bid of the game, then $\beta\mathbf{p}$ maximises $V(\boldsymbol{\pi} \mid O)$ subject to $\sum_i \pi_i \leqslant \beta$.

(A2) is a condition on all the subgames with bid β. Since not all of these subgames are reached in equilibrium, (A2) should be seen as a counterfactual statement concerning the set of outsiders. We note that our original model satisfies both (A1) and (A2).

Although we cannot specify the equilibrium prices without specifying $V(\boldsymbol{\pi} \mid O)$ in more detail, we have the following comparative statics result which states that, if condition (11) holds, the bookie's expected profit increases as prices move from $\beta\mathbf{p}$ toward the centre of the β-simplex. To say that prices move toward the centre of the simplex is to say that the prices on the favourites are lowered and the prices on the longshots are raised. Thus, our result can be paraphrased as saying that the introduction of favourite-longshot bias raises the bookie's expected profit.

PROPOSITION 2. *Assume* (A1) *and* (A2), *and suppose* z_i/p_i *is decreasing in* p_i. *Then, for* $\mathbf{p} \neq (1/n, \ldots, 1/n)$, *there exists a point* $\tilde{\pi}$ *on the line segment joining* $\beta\mathbf{p}$ *and the centre of the* β*-simplex* $(\beta/n, \ldots, \beta/n)$ *for which* $V(\tilde{\pi}) > V(\beta\mathbf{p})$.

The proof consists in showing that the directional derivative of $V(\boldsymbol{\pi})$ at $\beta\mathbf{p}$ toward the centre of the β-simplex is positive. We start by noting the following consequence of (11).

LEMMA. *If* z_i/p_i *is decreasing in* p_i, *then*

$$\sum_i \frac{z_i}{p_i}\left(\frac{1}{n} - p_i\right) > 0.$$

Proof. By hypothesis, for any $i \neq k$, $(z_i/p_i - z_k/p_k)(p_i - p_k) < 0$. In particular, for

$$p_k = \frac{1}{n}, \quad \text{we have} \quad (z_i/p_i - nz_k)\left(\frac{1}{n} - p_i\right) > 0.$$

Multiplying out and summing over i,

$$\sum_i \frac{z_i}{p_i}\left(\frac{1}{n} - p_i\right) - nz_k \sum_i \left(\frac{1}{n} - p_i\right) > 0.$$

But the second term is zero, thereby proving the claim.

Proof of Proposition 2. The directional derivative of a function f in the direction x evaluated at y is given by the inner product of the gradient ∇f at y with the unit vector $x/\|x\|$. Let $\mathbf{c} \equiv (\beta/n, \ldots, \beta/n)$ be the centre of the β-simplex. We are interested in the directional derivative of $V(\boldsymbol{\pi})$ in the direction $\mathbf{c} - \beta\mathbf{p}$ evaluated at $\beta\mathbf{p}$. This directional derivative exists by (A 1), and is given by:

$$\nabla V(\beta\mathbf{p}) \frac{\mathbf{c}-\beta\mathbf{p}}{\|\mathbf{c}-\beta\mathbf{p}\|} = [(1-z)\,\nabla V(\beta\mathbf{p} \mid O) + z\nabla V(\beta\mathbf{p} \mid I)] \frac{\mathbf{c}-\beta\mathbf{p}}{\|\mathbf{c}-\beta\mathbf{p}\|}. \tag{13}$$

By (A 2), $\beta\mathbf{p}$ maximises $V(\boldsymbol{\pi} \mid O)$ over the β-simplex. Since $\mathbf{c}$ lies on the β-simplex and $V(\boldsymbol{\pi} \mid O)$ is differentiable,

$$\nabla V(\beta\mathbf{p} \mid O)\,(\mathbf{c} - \beta\mathbf{p}) = 0. \tag{14}$$

Thus, (13) has the same sign as $\nabla V(\beta\mathbf{p} \mid I)\,(\mathbf{c} - \beta\mathbf{p})$. From the expression for $V(\boldsymbol{\pi} \mid I)$ given by (4),

$$z\nabla V(\beta\mathbf{p} \mid I)\,(\mathbf{c} - \beta\mathbf{p}) = \sum_i \frac{z_i p_i}{\beta^2 p_i^2}\left(\frac{\beta}{n} - \beta p_i\right) = \frac{1}{\beta}\sum_i \frac{z_i}{p_i}\left(\frac{1}{n} - p_i\right). \tag{15}$$

Thus, by the lemma, $z\nabla V(\beta\mathbf{p} \mid I)\,(\mathbf{c} - \beta\mathbf{p}) > 0$, so that from (13), (14) and (15), the directional derivative of $V(\boldsymbol{\pi})$ in the direction $\mathbf{c} - \beta\mathbf{p}$ evaluated at $\beta\mathbf{p}$ is positive. Since $V(\boldsymbol{\pi})$ is differentiable, there exists a point $\tilde{\pi}$ lying on the line $\beta\mathbf{p} + t(\mathbf{c} - \beta\mathbf{p})$, $(t \geqslant 0)$ for which $V(\tilde{\pi}) > V(\beta\mathbf{p})$.

This argument is limited in that it is a comparative statics argument rather than one which relies on the solution of the bookie's maximisation problem at stage 2. Indeed, it would only be by accident that the bookie's optimal $\boldsymbol{\pi}$ lies on the line segment joining $\beta\mathbf{p}$ and $\mathbf{c}$. Nevertheless, the result shows in an economical way the incentives at work. Also, the fact that this argument does not rely on a particular distribution of outsiders' beliefs focuses attention on the condition that z_i/p_i is a decreasing function of p_i.

IV. CONCLUDING REMARKS

A number of features make the betting market particularly tractable for formal investigation. Chief among these is the fact that prices are linear, in contrast to price schedules of more sophisticated financial markets where unit prices depend on quantities traded. Thus, betting markets form a natural bridge between the literature on linear pricing based on competitive equilibrium and that on insider trading. The necessary and sufficient condition for the favourite-longshot bias identified in this paper seems to be a natural one in this context.

Whatever one feels about the model presented in this paper, one lesson of general importance is that any departure from the classical view of markets must be accompanied by a detailed analysis of the 'market microstructure'. It is hoped that the insights gained here may contribute to the understanding of more sophisticated financial markets.

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APPENDIX

Corner Solution of $\boldsymbol{\pi}$

If all the constraints $0 \leqslant \pi_i \leqslant 1$ bind, then the optimal $\boldsymbol{\pi}$ is given by $(1, 1, \ldots, 1)$ since $V(\boldsymbol{\pi})$ is increasing in prices. Next, if all but one of the constraints $0 \leqslant \pi_i \leqslant 1$ bind, then the optimal $\boldsymbol{\pi}$ is determined by the constraint $\Sigma_i \pi_i \leqslant \beta$ and will be of the form:

$$[1, 1, \ldots, (\beta - n + 1), 1, \ldots, 1].$$

Otherwise, we can partition the set $\{1, 2, \ldots, n\}$ into A and B where $A = \{i \mid \pi_i < 1\}$ and $B = \{i \mid \pi_i = 1\}$. Since $V(\boldsymbol{\pi})$ is increasing in prices, the constraint $\Sigma_i \pi_i \leqslant \beta$ binds, and any π_i, π_j where $i, j \in A$ satisfy the respective first-order conditions. Thus, for any $i, j \in A$,

$$\frac{\pi_i}{\pi_j} = \sqrt{\left[\frac{z_i p_i + (1-z) p_i^2}{z_j p_j + (1-z) p_j^2}\right]}. \tag{16}$$

Denote by $\bar{B}$ the cardinality of the set B. Since the sum of prices is β and $\pi_i = 1$ for $i \in B$, the sum $\Sigma_{i \in A} \pi_i$ is given by $\beta - \bar{B}$. Thus, for $i \in A$,

$$\pi_i = \frac{(\beta - \bar{B}) \sqrt{[z_i p_i + (1-z) p_i^2]}}{\sum_{s \in A} \sqrt{[z_s p_s + (1-z) p_s^2]}}. \tag{17}$$

By substituting (17) into $V(\boldsymbol{\pi})$, the bookie's expected profit at stage 1 is:

$$1 - \frac{1}{(\beta - \bar{B})} \left\{\sum_{s \in A} \sqrt{[z_s p_s + (1-z) p_s^2]}\right\}^2 - \sum_{s \in B} [z_s p_s + (1-z) p_s^2], \tag{18}$$

which is zero in any equilibrium. Solving for $\beta - \bar{B}$ and substituting into (17), we arrive at the following solution for the equilibrium price $\pi_i (i \in A)$, in terms of the parameters of the model.

$$\pi_i = \sqrt{[z_i p_i + (1-z) p_i^2]} \left\{\frac{\sum_{s \in A} \sqrt{[z_s p_s + (1-z) p_s^2]}}{1 - \sum_{s \in B} [z_s p_s + (1-z) p_s^2]}\right\}. \tag{19}$$

We note that when B is empty, (19) reduces to the interior solution (10), as we would expect.

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