

# THE COMPARISON PROPERTY OF AMENABLE GROUPS

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This presentation is based on a joint work with Guohua Zhang.

Let a countable amenable group  $G$  act on a zero-dimensional compact metric space  $X$ . For two clopen subsets  $A$  and  $B$  of  $X$  we say that  $A$  is *subequivalent* to  $B$  (we write  $A \preceq B$ ), if there exists a finite partition  $A = \bigcup_{i=1}^k A_i$  of  $A$  into clopen sets and there are elements  $g_1, g_2, \dots, g_k$  in  $G$  such that  $g_1(A_1), g_2(A_2), \dots, g_k(A_k)$  are disjoint subsets of  $B$ . We say that the action *admits comparison* if for any clopen sets  $A, B$ , the condition that for every  $G$ -invariant probability measure  $\mu$  on  $X$  we have the sharp inequality  $\mu(A) < \mu(B)$  implies  $A \preceq B$ . Comparison has many desired consequences for the action, such as the existence of a systems of tilings with arbitrarily good Følner properties, which are factors of the action. We also study a purely group-theoretic notion of comparison: if every action of  $G$  on any zero-dimensional compact metric space admits comparison then we say that  $G$  has the *comparison property*. Classical groups  $\mathbb{Z}$  and  $\mathbb{Z}^d$  enjoy the comparison property, but in the general case the problem remains open. Our main result states that groups whose every finitely generated subgroup has subexponential growth have the comparison property.