Masterclass on groups, boundary actions and group C^*-algebras: Titles and abstracts

Emmanuel Breuillard: *Random Walks and Spectral Gaps*

The theory of Random Matrix Products deals with the behavior of random walks on $\mathrm{GL}(n)$ and its subgroups. It can be used for example to give a proof of the Tits alternative and hence of the non-amenability of non virtually solvable subgroups of $\mathrm{GL}(n)$. Non amenability is equivalent to the existence of a spectral gap for the regular representation. It turns out that random walks can also be used to prove spectral gaps for other types of unitary representations, coming from group actions or expander graphs. I will describe these ideas and results in some detail, and also make a connection with M. Kennedy's lectures on C^*-simplicity.

Vadim Kaimanovich: *Boundaries of groups: geometry, probability and analysis*

I will survey various approaches to boundaries of infinite groups based on geometrical, probabilistic and analytic considerations. The Poisson boundary of random walks on groups will be at the centre of the exposition. I will discuss its properties and criteria for its identification (in particular, triviality). I will also talk about the links between the Poisson boundary and several other constructions (Busemann, Martin, Furstenberg boundaries).

Matthew Kennedy: *Boundaries, injectivity and C^*-simplicity for discrete groups*

In this series of talks I will introduce an operator-algebraic approach to Furstenbergs theory of topological boundaries for discrete groups. I will discuss applications of these ideas to the study of crossed products and group C^*-algebras, and in particular present our recent result characterizing the discrete groups which are C^*-simple and have the unique trace property.

Joint work with Emmanuel Breuillard, Mehrdad Kalantar and Narutaka Ozawa.

Robin Tucker-Drob: *Group invariant means and inner amenability*

The starting point of this series of talks will be the notion of amenability of a group action, i.e., actions which admit an invariant mean. We will observe that even nonamenable groups can have amenable actions, and discuss how this tension can be exploited to deduce a wealth of information about the acting group. By applying these techniques to inner amenability, we will be able to describe all inner amenable subgroups of $\mathrm{GL}(n)$ and also answer several fundamental questions regarding the measured group theoretical structure of inner amenable groups, e.g., regarding cost and cocycle superrigidity.