Titles and abstracts — junior seminar

Joan Bosa: *Classification of monoids. Applications to the Cuntz semigroup.*

The Cuntz semigroup $W(A)$, in its classical format, fails to be a continuous functor. This shortcoming was remedied by Coward, Elliott and Ivanescu who, in 2008, introduced a new category (called Cu) and a semigroup attached to any C^*-algebra A (called $Cu(A)$), which belongs to Cu. The assignment $A \mapsto Cu(A)$ is now continuous. It was also shown that $Cu(A)$ can be identified with $W(A \otimes \mathcal{K})$.

We define a new category, that we call PreCu, where $W(A)$ naturally belongs. We show that there is a completion functor from PreCu to Cu, which is a left adjoint of the inclusion. With this construction, we obtain information about how $W(A)$ and $W(A \otimes \mathcal{K})$ are related, and recover some known results.

Bishan Jacelon: *A stably projectionless analogue of $\mathcal{O}_2$.*

Given $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$, there is an action $\alpha = \alpha^\lambda$ of $\mathbb{R}$ on the Cuntz algebra $\mathcal{O}_n = C^*(s_1, \ldots, s_n)$ defined by $\alpha_t(s_j) = e^{2\pi i t \lambda_j} s_j$ for $1 \leq j \leq n$ and $t \in \mathbb{R}$. Such actions are called 'quasi-free' and have been well-studied in the literature. If we assume that the action is nontrivial then we can normalise so that $\lambda_1 = 1$, and in the case $n = 2$ we then get a one-parameter family of crossed products $\mathcal{O}_2 \rtimes_{\alpha^\lambda} \mathbb{R}$. This crossed product is simple if and only if λ is irrational, and there is moreover a dichotomy between the cases $\lambda < 0$ and $\lambda > 0$. In both cases we have $K_*(\mathcal{O}_2 \rtimes_{\alpha^\lambda} \mathbb{R}) = K_*(\mathcal{O}_2) = 0$, but when $\lambda < 0$ we have $\mathcal{O}_2 \rtimes_{\alpha^\lambda} \mathbb{R} \cong \mathcal{O}_2 \otimes \mathscr{K}$, whereas if $\lambda > 0$ then $\mathcal{O}_2 \rtimes_{\alpha^\lambda} \mathbb{R}$ is stable and projectionless. It is this latter C^*-algebra that we would like to think of as an analogue of $\mathcal{O}_2$, and in this talk we will discuss possible strategies for investigating similarities between $\mathcal{O}_2 \rtimes_{\alpha^\lambda} \mathbb{R}$ and $\mathcal{O}_2$.

Miroslava Johanesova: *Kadison's Similarity Problem for certain C^*-algebras.*

We show that the tensor product of two unital C^*-algebras, one of which is nuclear and admits a unital embedding of the Jiang-Su algebra $\mathcal{Z}$, has Kadison's similarity property. In particular, so does every $\mathcal{Z}$-stable C^*-algebra.

Pawel Kasprzak: *A C^*-algebraic quantum Minkowski space.*

The aim of this talk is to present an example of C^*-algebraic quantum Minkowski space. It is obtained by the Rieffel deformation simultaneously applied to the Lorentz group and its action on the Minkowski space. The resulting C^*-algebra of the quantum Minkowski and that of the quantum Lorentz group can be described in terms of their generators satisfying the (p, q)-type commutation relations. We shall also describe the action of the quantum Lorentz group on the quantum Minkowski space in terms of these generators.

Magnus Norling: *Minimal idempotents in the Stone-Cech compactification of the integers, and the Kadison-Singer problem.*

The Stone-Čech compactifiaction of the integers, $\beta\mathbb{Z}$, posesses a rich topological semigroup structure that extends the group structure on $\mathbb{Z}$. Among other things, it has been shown to posess nonzero idempotents. This structure has earlier been used to prove some highly nontrivial results within combinatorial number theory - such as the van der Waerden Theorem - using quite simple proofs. Following work by Vern Paulsen, we look at a way this structure can be used to study some operator theoretical problems related to pure state extensions from a discrete maximal abelian von Neumann algebra. Whether or not pure states have unique extensions from such subalgebras to all of $B(H)$ is known as the Kadison-Singer problem.

Eduard Ortega: *The Corona Factorization property, Stability, and the Cuntz semigroup of a C^*-algebra"*

The Corona Factorization Property, originally invented to study extensions of C^*-algebras, appears to convey essential information about the intrinsic structure of the C^*-algebras.

We show that the Corona Factorization Property of a σ-unital C^*-algebra A is completely captured by its Cuntz semigroup $W(A)$ of equivalence classes of positive elements in matrix algebras over A. The corresponding condition in $W(A)$ is a (weak) comparability property that is termed the Corona Factorization Property (for the semigroup). Using this result one can for example show that all unital C^*-algebras with finite decomposition rank have the Corona Factorization Property.

Applying similar techniques we study the related question of when C^*-algebras are stable.

This is a joint work with Mikael Rørdam and Francesc Perera.

Leonel Robert: *Hilbert C^*-modules over a commutative C^*-algebra*

A Hilbert C^*-module over a commutative C^*-algebra may be expressed as a supremum of locally trivial vector bundles defined on open subsets of the spectrum of the algebra. This point of view may be exploited to transplant results from the theory of vector bundles to the setting of Hilbert C^*-modules over commutative C^*-algebras. For example, one can always embed a Hilbert C^*-module into another one with sufficiently larger fibre dimension (depending on the covering dimension of the base space). One can use clutching functions to construct new Hilbert C^*-modules from old ones. If the base space has dimension at most 3, all the isomorphism classes of Hilbert C^*-modules may be described in terms of cohomological data of the base space. I will talk about these and other results obtained recently in collaboration with Aaron Tikuisis.

David Robertson: *C^*-algebras generated by C^*-correspondences and applications to noncommutative geometry.*

C^*-correspondences generalise the theory of Hilbert spaces by replacing the field of scalars with an arbitrary C^*-algebra. In the talk I will recall the definition of a C^*-correspondence and show how one can associate a C^*-algebra to it as a universal object. I will also state a result showing that this process is functorial and respects pull-backs, and briefly explain how the theory can be applied to the study of noncommutative spaces. This is joint work with Dr. Wojciech Szymanski.

Louis Santiago: *Classification of a certain class of non-simple C^*-algebras.*

We show that non-simple inductive limits of tree algebras are classified by their Cuntz semigroup. More generally, using the Cuntz functor we classified *-homomorphisms—up to approximate unitary equivalence—from inductive limits of tree algebras to a C^*-algebra with stable rank one.

Adam Sierakowski: *Purely infinite crossed product C^*-algebras.*

When imposing properties that ensure A separates the ideals in the C^*-algebra crossed product $A \rtimes_r G$ we are able to show that the crossed product $A \rtimes_r G$ is purely infinite if and only if the non-zero positive elements in A are properly infinite viewed as elements in $A \rtimes_r G$ — in the case A is separable and has the ideal properly. As an application of this result we show that for a particular class of crossed products, where a discrete group G acts in a particular way on the Cantor set X, the C*-algebra $C(X) \rtimes_r G$ is purely infinite if and only if it is traceless.

Piotr Soltan: *Quantum spaces with no quantum group structure.*

I will show how some tools form C^*-algebra theory can be used to prove that some well known quantum spaces (objects of the category dual to the category of C^*-algebras) do not admit any quantum group structure.

Karen Strung: *Minimal dynamics and $\mathcal{Z}$-stable classification*

Let $\mathcal{S}$ be a class of separable, nuclear C^*-algebras that contains all unital hereditary

C^*-subalgebras of C^*-algebras in $\mathcal{S}$. We say that a C^*-algebra A is TA$\mathcal{S}$ if it can be tracially approximated by C^*-algberas in $\mathcal{S}$. For a C^*-algebra $A = C(X) \rtimes_\alpha \mathbb{Z}$ arising from a minimal dynamical system of an infinite compact metric space, we show that $A \otimes (\otimes_{n=1}^\infty M_q)$ is TA$\mathcal{S}$ whenever a certain tracially large C^*-subalgebra is TA$\mathcal{S}$. As an application, when $\mathcal{S}$ is the set of finite-dimensional C^*-algebras, we obtain classification results for the C^*-algebras associated to minimal dynamical systems of infinite compact metric spaces, up to tensoring with the Jiang-Su algebra $\mathcal{Z}$.

Hannes Thiel: *Torsion in the Elliott invariant and dimension theories of C^*-algebras.*

The Elliott conjecture predicts that nuclear (simple) C^*-algebras are classified by their Elliott invariant. It is therefore natural to ask, how properties (like dimension) of a C^*-algebra are reflected in its Elliott invariant.

While for commutative spaces there is only one (natural) concept of dimension, there are various dimension theories for C^*-algebras (e.g. real and stable rank). We consider the AH-dimension (ASH-dimension), which is $\leq n$ if the C^*-algebra is a direct limit of n-dimensional homogeneous (subhomogeneous) algebras. The famous AF-agebras are exactly the algebras with ASH-dimension $= 0$.

We show that low AH- and ASH-dimension is connected to torsion-freeness of the K_0 and K_1-groups. In a classifiable setting the connection is so tight that we can easily read off the ASH-dimension from the Elliott invariant.

Moritz Weber: *Twists of Isometries.*

In 1981, J. Cuntz gave a construction of a twisted tensor product $A \times \mathcal{O}_n$ of a C^*-algebra A with the Cuntz algebra $\mathcal{O}_n$. Given n automorphisms $\alpha_1, \ldots, \alpha_n \in \mathrm{Aut}(A)$ of A, he defined $A \times \mathcal{O}_n$ by adjoining the n Cuntz isometries $S_1, \ldots, S_n$ to the algebra A, implementing the automorphisms $\alpha_1, \ldots, \alpha_n$ (the ith automorphism α_i modeled by S_i). This generalizes the concept of the crossed product $A \rtimes_\alpha \mathbb{Z}$ of an algebra A with the group $\mathbb{Z}$, given an automorphism α on A.

As a special case ($A = \mathcal{O}_m$), $\mathcal{O}_m \times \mathcal{O}_n$ for $2 \leq n, m < \infty$ may be seen as the universal C^*-algebra generated by n Cuntz isometries $S_1, \ldots, S_n$ (i.e., $\sum_{i=1}^n S_i S_i^* = 1$) together with m Cuntz isometries $S'_1, \ldots, S'_m$ ($\sum_{j=1}^m S'_j S'^*_j = 1$), such that $S_i S'_j = \lambda_{ij} S'_j S_i$ holds for fixed $\lambda_{ij} \in S^1 \subset \mathbb{C}$, for $1 \leq i \leq n, 1 \leq j \leq m$. This is the twist with respect to the automorphisms $\alpha_i(S'_j) = \lambda_{ij} S'_j$, $1 \leq i \leq n, 1 \leq j \leq m$ on $\mathcal{O}_m$.

Although this looks like a generalization of the tensor product $\mathcal{O}_m \otimes \mathcal{O}_n$ in the spirit of the rotation algebra A_θ seen as a twist of the tensor product $\mathcal{C}(S^1) \otimes \mathcal{C}(S^1)$, it doesn't lead to a new object. Due to Kirchberg's result of 1994/1995, the algebras $\mathcal{O}_m \otimes \mathcal{O}_n$ and $\mathcal{O}_m \times \mathcal{O}_n$ are completely classified by their K-groups since they are unital, separable, nuclear (fulfilling the UCT), simple and purely infinite. Using a sequence in K-theory analogous to the six term sequence of crossed products by $\mathbb{Z}$, it turns out that these two algebras have the same K-theory, hence they are isomorphic.

I will introduce the object $\mathcal{O}_m \times \mathcal{O}_n$, roughly explain why it is nuclear and purely infinite, compute its K-theory and conclude, that it is isomorphic to $\mathcal{O}_m \otimes \mathcal{O}_n$.