An ICM Satellite conference:

Operator Algebras, Dynamics and Groups

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Titles and abstracts

Oren Becker: *Character varieties of random groups*

The space $\mathrm{Hom}(\Gamma, G)$ of homomorphisms from a finitely-generated group Γ to a complex semisimple algebraic group G is known as the G-representation variety of Γ. We study this space when G is fixed and Γ is a random group in the few-relators model. That is, Γ is generated by k elements subject to r random relations of length L, where k and r are fixed and L tends to infinity.

More precisely, we study the subvariety Z of $\mathrm{Hom}(\Gamma, G)$, consisting of all homomorphisms whose images are Zariski dense in G. We give an explicit formula for the dimension of Z, valid with probability tending to 1, and study the Galois action on its geometric components. In particular, we show that in the case of deficiency 1 (i.e., $k - r = 1$), the Zariski-dense G-representations of a typical Γ enjoy Galois rigidity.

Our methods assume the Generalized Riemann Hypothesis and exploit mixing of random walks and spectral gap estimates on finite groups.

Based on a joint work with E. Breuillard and P. Varju.

Adrien le Boudec: *Commensurated subgroups and micro-supported actions*

A subgroup Λ of a group Γ is commensurated if all the conjugates of Λ are commensurable. We prove a theorem that relates the commensurated subgroups of a finitely generated group Γ with the topological dynamics of the minimal and micro-supported actions of Γ on compact spaces. As an application we obtain a criterion to exclude the existence of non-trivial commensurated subgroups in certain classes of groups. Examples include topological full groups of $\mathbb{Z}$-actions, and more generally topological full groups of amenable groups.

Although the theorem is about finitely generated discrete groups, an important role is played in the proof by non-discrete locally compact groups. We might also discuss how the notion of uniformly recurrent subgroups (URS) comes into play.

Joint work with Pierre-Emmanuel Caprace.

Bruno de Mendonca Braga: *Equi-approximability and vector measures in uniform Roe algebras*

The rigidity problem for uniform Roe algebras asked whether the existence of an isomorphism between the uniform Roe algebras of uniformly locally finite metric spaces implied the existence of a coarse equivalence between those spaces. A positive answer to this problem has recently been found. Two of the main ingredients in this proof are (1) the study of equi-approximable subsets of uniform Roe algebras and (2) the study of certain vector measures related to them. In this talk, I will discuss some further applications of those methods to the study of WOT-closed subalgebras of uniform Roe algebras.

This is based on an ongoing project with F. Baudier, I. Farah, A. Khukhro, A. Vignati, and R. Willett.

Léonard Cadilhac: *A noncommutative pointwise ergodic theorem for amenable groups*

The original ergodic theorem of Birkhoff asserts that ergodic averages associated with a measure-preserving transformation converge almost everywhere. I will present a generalization of this result where the single transformation is replaced with an action of an amenable group and the classical measure space is replaced with a tracial von Neumann algebra. The talk will start with a brief overview of the literature leading up to this result and then will discuss recent advances in noncommutative harmonic analysis on which the proof relies.

This is based on joint work with Simeng Wang.

Ionut Chifan: *Applications of wreath-like product groups to the study of von Neumann algebras*

In this talk we will present several new applications of wreath-like product groups to the structural theory of II_1 factors with property (T).

Using quotienting techniques from geometric group theory, including Dehn filling, we will introduce a new variant of wreath-like product groups which have peripheral structure. Exploiting this concept in combination with a new von Neumann algebraic reconstruction method we will show there exist uncountably many property (T) groups G which are completely recognizable from their von Neumann algebras, $L(G)$ (i.e., they satisfy Connes Rigidity Conjecture). In addition, we will show that many of these groups are reconstructible from their reduced C^*-algebras, $C_r^*(G)$ as well.

In a different direction, making use of the wreath-like product groups, we will prove that every separable tracial von Neumann algebra embeds into a II_1 factor with property (T) which can be taken to have trivial outer automorphism and fundamental groups.

This is based on an ongoing joint work with Adrian Ioana, Denis Osin and Bin Sun and a very recent joint work with Daniel Drimbe and Adrian Ioana.

Benoit Collins: *Operator norm convergence for tensors*

We will review recent results on the convergence of non-commutative polynomials in non-commutative iid tensors of unitary random matrice.

This is partly based on joint work with Charles Bordenave.

Marius Dadarlat: *Cohomological obstructions to group stability with respect to the operator norm*

We exhibit topological obstructions to perturbing approximate unitary representations of groups to genuine representations with respect to the operator norm. We shall explain why quasidiagonality is relevant in this context and how it is used to find many examples of nonstable groups by exploiting interactions with the KK-theory approach to the Novikov conjecture and index theory.

Cornelia Drutu: *Connections between actions on Banach spaces, hyperbolic geometry and median geometry*

In this talk I shall explain how various forms of hyperbolicity, from the one of lattices in rank one simple groups to the one of acylindrically hyperbolic groups, present various degrees of compatibility with the median geometry and, as a result, various types of fixed point properties or various forms of amenability/proper actions on Banach spaces.

This is joint work with Indira Chatterji, and with John Mackay.

Jamie Gabe: *The Dynamical Kirchberg-Phillips Theorem*

The Kirchberg-Phillips theorem from the mid 1990's was one of the early major breakthroughs in the classification programme for nuclear C^*-algebras, giving a complete classification of Kirchberg algebras (in the UCT class) using K-theory. I will explain how this

theorem can be generalised to classify amenable and suitably outer locally compact group actions on Kirchberg algebras, which as a special case confirms a conjecture of Izumi.

This is joint work with Gábor Szabó.

Shirly Geffen: *Purely infinite C^*-algebras from tracially amenable actions on pure C^*-algebras*

We introduce the notion of tracial amenability for group actions, extending the notion of amenability. We show that for tracially amenable outer actions of groups containing F_2 on pure (e.g., classifiable) C^*-algebras, the attached crossed products are purely infinite. When the underlying C^*-algebra is purely infinite, this was obtained by Kishimoto. We cover the stably finite case.

This is joint work with Eusebio Gardella, Julian Kranz, Petr Naryshkin, and Andrea Vaccaro

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Cyril Houdayer: *ICM talk: Noncommutative ergodic theory of higher rank lattices*

I will survey recent results regarding the dynamics of positive definite functions and character rigidity of irreducible lattices in higher rank semisimple algebraic groups. These results have several applications to ergodic theory, topological dynamics, unitary representation theory and operator algebras. In the case of lattices in higher rank simple algebraic groups, I will explain the key operator algebraic novelty, which is a noncommutative Nevo–Zimmer theorem for actions on von Neumann algebras. I will also present a noncommutative analogue of Margulis' factor theorem and discuss its relevance regarding Connes' rigidity conjecture for group von Neumann algebras of higher rank lattices.

Adrian Ioana: *Wreath-like product groups and rigidity of their von Neumann algebras*

In this talk, I will introduce a new class of groups, called wreath-like products. These groups are close relatives of the classical (restricted) wreath products and arise naturally in the context of group theoretic Dehn filling. Unlike ordinary wreath products, many wreath-like products have Kazhdan's property (T). I will present several new rigidity results for von Neumann algebras of wreath-like products with property (T). In particular, we obtain the first examples of property (T) groups G which are W^*-superrigid, in the sense that the group von Neumann algebra $L(G)$ remembers the isomorphism class of G. We also compute the outer automorphism and fundamental groups of von Neumann algebras of a wide class

of wreath-like products. As an application, we show every finitely presented group can be realised as the outer automorphism group of $L(G)$ for a property (T) group G.

This is based on joint work with Ionut Chifan, Denis Osin and Bin Sun.

David Jekel: *The non-commutative Wasserstein distance, quantum information theory, and stability*

The Wasserstein distance between probability distributions μ and ν on $\mathbb{R}^d$ is the smallest possible value of $\|X-Y\|_{L^2}$, where X and Y are random variables with distributions μ and ν respectively (such a pair (X,Y) is called a coupling of μ and ν). Biane and Voiculescu in 2001 introduced an analog of the Wasserstein distance for non-commutative probability distributions (i.e., the moments of non-commuting random variables). In recent joint work with Wilfrid Gangbo, Kyeongisk Nam, and Dima Shlyakhtenko, we observed several connections between the non-commutative Wasserstein distance and aspects of operator algebras and quantum information theory. For instance, the refutation of the Connes embedding problem implies that there are non-commutative probability distributions μ and ν that individually model finite-dimensional tracial von Neumann algebras, but for which the Wasserstein distance can only be achieved by couplings in non-Connes-embeddable tracial von Neumann algebras. Moreover, it turns out that the Wasserstein distance gives a different topology on the space of non-commutative probability distributions than the weak-* topology, and in fact the points where these two topologies is closely related to amenability of the corresponding von Neumann algebras.

Nadia Larsen: *Hecke C^*-algebras of right-angled Coxeter groups*

Hecke C^*-algebras associated to Coxeter groups are deformations of the corresponding group algebras in the presence of a family of parameters. The case of right-angled Coxeter groups is well-suited to a study using graph products of C^*-algebras. In a general setup of graph products, a Khintchine-type inequality can be established, leading to a Haagerup-type inequality for right-angled Hecke C^*-algebras of Coxeter groups. As an application, we investigate the unique trace property for this class of C^*-algebras.

This is joint work with Martijn Caspers and Mario Klisse.

Damian Osajda: *Elliptic actions on nonpositively curved spaces* — Cancelled

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For a finitely generated group Kazhdan's property (T) is equivalent to the property that every isometric action on the Hilbert space is elliptic, that is, it fixes a point. The Hilbert

space is a nonpositively curved (more precisely, CAT(0)) space of infinite dimension. There are striking differences between this case and the case of finitely dimensional nonpositively curved spaces. For example, conjecturally, every action of a finitely generated torsion group (i.e., every element has finite order) on the latter spaces is elliptic, whereas such torsion groups, even infinite Burnside groups, can act properly on the Hilbert space. I will present recent developments around related questions.

This is based on joint works with Karol Duda, Thomas Haettel, Sergey Norin, and Piotr Przytycki.

Gilles Pisier: *From local to global lifting?*

The main problem we will consider is whether the local lifting property (LLP) of a C^*-algebra implies the (global) lifting property (LP). Kirchberg showed that this holds if the Connes embedding problem has a positive solution, but it might hold even if its solution is negative. We will present several new characterizations of the lifting property for a C^*-algebra A in terms of the maximal tensor product of A with the (full) C^*-algebra of the free group $\mathbb{F}_\infty$. We will recall our recent construction of a non-exact C^*-algebra with both LLP and WEP. This prompted us to try to prove that LLP implies LP for a WEP C^*-algebra. While our investigation is not conclusive we obtain a fairly simple condition in terms of tensor products that is equivalent to the validity of the latter implication.

Sorin Popa: *On ergodic inclusions of factors*

An inclusion of von Neumann factors $M \subset \mathcal{M}$ is *ergodic* if it satisfies the irreducibility condition $M' \cap \mathcal{M} = \mathbb{C}$. We investigate the relation between this and several other (apriori stronger) ergodicity conditions, notably *R-ergodicity* which requires M to admit a copy of the hyperfinite II_1 factor $R \subset M$ that's ergodic in $\mathcal{M}$, and *MASA-ergodicity*, requiring existence of an abelian $A \subset M$ that's a MASA in $\mathcal{M}$. One of the results obtained is that for inclusions of continuous factors MASA-ergodicity implies R-ergodicity. This shows in particular that any non type I factor contains an ergodic copy of R.

Sven Raum: *Simplicity and the ideal intersection property for essential groupoid C^*-algebras*

To every étale groupoid with locally compact Hausdorff space of units, one can associate an essential groupoid C^*-algebra, which is a suitable quotient of the reduced groupoid C^*-algebra by an ideal of singular elements. For Hausdorff groupoids, it equals the reduced

groupoid C^*-algebra. Until recently, it had been an open question to characterise simplicity of such essential groupoid C^*-algebras.

In this talk, I will report on joint work with Matthew Kenney, Se-Jin Kim, Xin Li and Dan Ursu, which characterises étale groupoids with locally compact Hausdorff space of units whose essential groupoid C^*-algebra is has the ideal intersection property. Our characterisation is phrased in terms of what is called essentially confined amenable sections of isotropy groups, a notion that can be checked in concrete cases. This provides a complete solution of the open problem, combining the ideal intersection property with the dynamical requirement of minimality. In particular, it comes as a surprise that non-Hausdorff groupoids fit well into this general picture. Our work completes, extends and unifies previous results concerning C^*-simplicty of topological dynamical systems (Kawabe), Hausdorff groupoids with compact space of units (Borys) and groupoids of germs (Kalantar-Scarparo). Even the notion of essential groupoid C^*-algebras for non-Hausdorff groupoids was only developed recently (Kwaśniewski-Meyer).

An interesting consequence of our work is the fact that a relative Powers averaging property can be proven. To establish this result, the full extend of our work is necessary. As an application we prove relative Powers averaging property for suitable unitary representations of Thompson's group T into the Cuntz algebra $\mathcal{O}_2$.

Mikael de la Salle: *ICM talk: Analysis with simple Lie groups and lattices*

In this survey talk, I will present a tool to perform analysis with simple Lie groups and their lattices, which has been applied in various contexts: non-unitary representations, operator algebras and their approximation properties, Fourier analysis, geometry of Banach spaces... The idea, which originates from the work of Vincent Lafforgue, is to first restrict to compact subgroups and then exploit how they sit inside the whole group.

Christopher Schafhauser: *Proper asymptotic unitary equivalence and $\mathcal{Z}$-stability*

Let A and B be separable C^*-algebras with B stable. If $\phi, \psi\colon A \to \mathcal{M}(B)$ are representations with $\phi(a)-\psi(a) \in B$ for all $a \in A$, then ϕ and ψ define an element $[\phi, \psi] \in KK(A, B)$. Dadarlat and Eilers have shown that $[\phi, \psi] = 0$ if and only if there if there is a $\theta\colon A \to M(B)$ such that $\phi \oplus \theta$ and $\psi \oplus \theta$ are asymptotically unitarily equivalent via a one parameter family of unitaries in the minimal unitization of $M_2(B)$, and in fact, θ can be taken to be any absorbing representation (i.e., one satisfying the conclusion of Voiculescu's Theorem).

When ϕ and ψ are absorbing, can the direct summand θ be removed? Dadarlat and Eilers have shown this is the case when $B = \mathcal{K}$, but the general case is still open. I'll discuss

how, in general, the direct summand θ can be replaced with a tensor factor of $\mathcal{Z}$ in this setting.

This is joint work with José Carrión, Jamie Gabe, Aaron Tikuisis, and Stuart White and forms a key step in our abstract approach to the classification of simple nuclear C^*-algebras.

Dimitri Shlyakhtenko: *On Voiculescu's free entropy dimension*

Voiculescu's free entropy dimension is a free probability analog of various classical notions of fractal dimension. We discuss some estimates on this number, as well as alternative definitions.

Karen Strung: *Crossed products of commutative C^*-algebras by Hilbert bimodules* — Cancelled

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Given a Hibert $C(X)$-bimodule, one can construct a crossed product which generalizes the usual crossed product by a homeomorphism. When the Hilbert bimodule comes from a minimal homeomorphism of mean dimension zero twisted by a line bundle, the resulting C^*-algebra absorbs the Jiang–Su algebra. We can also classify their orbit-breaking subalgebras. In this case we furthermore have a nice description of them as locally subhomogeneous C^*-algebras. With no assumptions on the mean dimension, the tensor product of two or more such C^*-algebras also absorbs the Jiang–Su algebra. This entails their classification by the Elliott Invariant.

This is based on joint work with Jeong and Forough as well as work with Adamo, Archey, Forough, Georgescu, Jeong, Viola.

Stefaan Vaes: *Superrigidity for group actions on the n-sphere and their skew products*

An essentially free group action of a discrete group Γ on a measure space (X, μ) is called orbit equivalence (OE) superrigid if the associated orbit equivalence relation entirely remembers the group Γ and its action on (X, μ). I will present a joint work with Daniel Drimbe, providing several OE superrigidity theorems for dense subgroups of Lie groups and their actions on homogeneous spaces, in particular on hyperbolic n-space and the n-sphere. These actions are typically of type II_∞ or of type III_1. I will then discuss a recent joint work with Bram Verjans, providing OE superrigidity for actions of type III_0 with a prescribed associated Krieger flow, using a skew product construction.

Dan Voiculescu: *A noncommutative nonlinear condenser capacity*

The quasicentral modulus is key in many questions about commutants mod normed ideals and in normed ideal perturbations of n-tuples of operators. I have extended the definition to that of a condenser quasicentral modulus and I will point out a noncommutative analogy with condenser capacity in nonlinear potential theory.

Stuart White: *Tracially Complete C^*-algebras*

Over the last decade a major theme has been the appplication of von Neumann methods in the structure and classification of stably finite C^*-algebras, often with the idea of explicitly lifting results back from the von Neumann level to the C^*-algebra setting. This works most crisply for C^*-algebras with a unique trace, where one works with the associated GNS representation. For more complicated trace simplices, one needs more complicated enveloping objects to handle all traces simultaneously, as exemplified by Ozawa's theory of W^*-bundles associated to C^*-algebras whose trace simplex is Bauer. In this talk, I'll introduce the framework of tracially complete C^*-algebras as a general bridge between the W^* and C^*-theory, and discuss structure and classification results in this setting.

This is joint work with Carrión, Castillejos, Evington, Gabe, Schafhauser and Tikuisis.

Wilhelm Winter: *Weakening the UCT problem*