Masterclass on Sofic groups and applications to operator algebras

Titles and abstracts

David Kerr: *Soficity, dynamics, and entropy.*

It has recently been realized, beginning with the breakthrough work of Bowen, that the classical theory of entropy for actions of amenable groups can be extended to the much broader realm of sofic acting groups. Sofic entropy is based on the Boltzmann idea of counting discrete models, which had only been present in the amenable theory in an implicit way. In the amenable case these models are partial orbits over Følner sets and hence are internal from the perspective of the group, while the sofic case externalizes this picture by replacing the Folner sets with arbitrary finite sets on which the group acts in an approximate way. This distinction between internal and external modeling is closely paralleled in the continuous setting of operator algebras via the notions of hyperfiniteness and Connes embeddability for finite von Neumann algebras and of nuclearity and the MF property for C^*-algebras.

The expansion of entropy theory beyond amenability has opened the door to a variety of novel applications and directions based around the analytic, probabilistic, and geometric study of groups and their actions. I will discuss some of these developments and how they connect to operator algebra theory, both at a conceptual level thorugh the parallels mentioned above and at a technical level by way of rigidity theory, algebraic actions, and the Fuglede-Kadison determinant in group von Neumann algebras.

Narutaka Ozawa: *Connes' Embedding Problem and its equivalent.*

Connes' embedding problem is considered as one of the most important open problems in the field of operator algebras. It asks whether every finite von Neumann algebra is approximable by matrix algebras in suitable sense. This problem is parallel to the soficity problem asking whether every group is sofic, i.e., approximable by finite groups. It turns out, most notably by Kirchberg, that Connes' embedding problem is equivalent to surprisingly many other important conjectures. Connes' embedding problem is related to almost all the subfields of operator algebras and also to other branches of mathematics such as quantum information theory and algebraic geometry. I will try to reduce operator algebra prerequisite as possible.

Andreas Thom: *Finite-dimensional approximation properties of groups and their applications.*

In this course I will explain how finite-dimensional approximation properties can help to understand infinite discrete groups and associated objects such as group rings. I will discuss Kaplansky's Direct Finiteness Conjecture, the Kervaire-Laudenbach Conjecture about equations in groups and the Algebraic Eigenvalue Conjecture. More concretely, the plan for the four lectures is:

1) Metric approximation properties — sofic, weakly sofic, hyperlinear invariant length functions, sofic groups, hyperlinear groups, finite simple groups, Theorem of Liebeck-Shalev, structure of the ultraproducts, metric ultraproducts, concentration of measure, Levy families

2) The Kaplansky's Direct Finiteness Conjecture. The group von Neumann algebra, the trace, spectral measure, direct finiteness of group rings in characterstic zero, Elek's proof in characteristic p for sofic groups, open problems, reverse engineering in characteristic two

3) The Kervaire-Laudenbach Conjecture. Equations in groups, free groups and free products, one relator groups, Theorem of Gerstenhaber-Rothaus, extension to the hyperlinear case, open problems about the unitary group of finite von Neumann algebras

4) The Algebraic Eigenvalue Conjecture. Spectral measures of elements in the integral group ring, special case of sofic groups, proof of the Algebraic Eigenvalue Conjecture, Theorem of Friedman, approximation of determinant of laplacians, free spanning forrest

Gábor Elek: *Invariants and spaces of finite graph sequences and sofic approximations.*

In the first talk I will give a very gentle introduction to finite graph sequences and their invariants. I will explain the notion of convergence, limits, and the most interesting invariants. Based on this rather elementary notions one can study the compact metric space of sofic representations and their invariants. That will be the subject of the second talk.

Eberhard Kirchberg: *TBA.*

Nicolas Monod, I: *Simple amenable groups.*

We provide the first examples of finitely generated simple groups that are amenable (and infinite). This follows from a general existence result on invariant states for piecewise-translations of the integers, and from work of Matsui. The states are obtained by constructing a suitable family of densities on the classical Bernoulli space. Joint work with Kate Juschenko.

Nicolas Monod, II: *Simple non-amenable groups.*

For many decades, it was not known whether all paradoxical decompositions are due to

free groups. This problem was finally solved through the heroic work of Ol'shanskii and Adyan. I will present a new solution which is simple (in the non-mathematical sense!). The method is inspired by a Geneva book: Frankenstein.