

# Confronting the economic model with the data

Søren Johansen\*

Department of Applied Mathematics and Statistics  
University of Copenhagen

October 2005

## Abstract

Econometrics is about confronting economic models with the data. In doing so it is crucial to choose a statistical model that not only contains the economic model as a submodel, but also contains the data generating process. When this is the case, the statistical model can be analyzed by likelihood methods. When this is not the case, but likelihood methods are applied nonetheless, the result is incorrect inference. In his paper we illustrate the problem of possible incorrect inference with a recent application of a DSGE model to US data (Ireland, 2004). Specifically, this paper discusses two broad methodological questions.

- How should a statistical model be chosen to achieve valid inference for the economic model?
- Given a correctly chosen statistical model, can we rely on the asymptotic results found in the statistical literature for the analysis of the data at hand?

Using some simple examples, the paper first discusses some unfortunate consequences of applying Gaussian maximum likelihood when the chosen statistical model does not properly describe the data. It also demonstrates that even when the correct statistical model is chosen, asymptotic results derived for stationary processes cannot be used to conduct inference on the steady state value for a highly persistent stationary process.

**Keywords:** DSGE models, state space models, the role of likelihood methods, unit roots.

**JEL Classification:** C32, E13.

---

\*I would like to thank the Danish Social Sciences Research Council for continuing support, and David Colander, Massimo Franchi, Christian Groth, Henrik Hansen, Kevin Hoover, Katarina Juselius, Mikael Juselius and the members of the EFS-EMM network, as well as the audiences in Rome and Helsinki for useful comments.

# 1 Introduction

When taking an economic model to the data in order to conduct inference ‘we need a stochastic formulation to make simplified relations elastic enough for applications’, to quote Haavelmo (1943). Thus, the relations derived from theory need to be supplemented by some extra error terms to describe the stochastic variation of the measurements. We thereby embed the economic model in a statistical model. But the statistical model must also be chosen so that it describes the variation of the data, that is, it has to contain the density of the data generating process. In this case the empirical model provides a platform for conducting valid inference on the relevant parameters of the economic model, see Kapetanios, Pagan, and Scott, (2004) or Altuğ (1989) among many others.

The papers by Söderlind (1999) and Ireland (2001, 2004) suggest a particular method for taking a DSGE model to the data. The method is discussed in section 2, but can briefly be described as follows: The DSGE model delivers relations and first order conditions involving forward looking expectations, and the relations are linearized around their steady state values. This gives identities and stochastic difference equations driven by the technology shocks of the theoretical model. The identities are then made ‘more elastic’ by adding an autocorrelated error term, thereby extending the economic model to a statistical model. The statistical model is formulated as a state space model and analyzed by Gaussian maximum likelihood methods using the Kalman filter to calculate the likelihood function. The method is then applied to US data.

Most economists would find this approach appealing as it suggests a straightforward way of conducting inference on precisely defined economic relations and their coefficients. But, for this inference to be valid, and therefore reliable, one has to demonstrate that the statistical model adequately describes the stochastic variation of the data. The paper to be discussed did not demonstrate this, and therefore possibly provides misleading and irrelevant results.

## 2 Extending the DSGE model to a state space model

Let me begin by discussing some details of the method proposed and the findings by Ireland (2001,2004). He begins with a DSGE model that has a representative agent producing output  $Y_t$  with capital  $K_t$  and labor  $H_t$ , measured as hours worked, according to

$$Y_t = A_t K_t^\theta (\eta^t H_t)^{1-\theta}.$$

The coefficient  $\eta > 1$  measures the ‘gross rate of labor-augmenting technological progress’. The agent has preferences over consumption  $C_t$  and hours worked  $H_t$  and wants to maximize

$$E \left[ \sum_{t=0}^{\infty} \beta^t (\ln C_t - \gamma H_t) \right],$$

with respect to  $\{C_t, H_t\}_{t=0}^{\infty}$  subject to a number of constraints. The technology shocks  $A_t$  follow a stationary  $AR(1)$  process

$$\ln(A_t) = (1 - \rho) \ln A + \rho \ln(A_{t-1}) + \varepsilon_t, \quad |\rho| < 1.$$

The variables  $Y_t, K_t, C_t$  and investment  $I_t$  obey the identities

$$Y_t = C_t + I_t$$

and

$$K_{t+1} = (1 - \delta)K_t + I_t.$$

The first order conditions become

$$\gamma C_t H_t = (1 - \theta)Y_t,$$

and

$$1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} \left( \frac{\theta Y_{t+1}}{K_{t+1}} + 1 - \delta \right) \right].$$

The assumption of trend stationarity of the variables is added in order to linearize the solution of these equations around their steady state values. We define the means or steady states  $y, c, i, k, a, h$  of the variables  $Y_t/\eta^t, C_t/\eta^t, I_t/\eta^t, K_t/\eta^t, A_t, H_t$  respectively. For the observations  $Y_t, C_t,$  and  $H_t$ , expressed as deviations of their logarithm from their trend, the three variables

$$f_t = \begin{pmatrix} \log Y_t - t \log \eta - \log y \\ \log C_t - t \log \eta - \log c \\ \log H_t - \log h \end{pmatrix}$$

are introduced.

The two variables  $A_t$  and  $K_t$  are considered unobserved and define

$$s_t = \begin{pmatrix} \log K_t - t \log \eta - \log k \\ \log A_t - \log a \end{pmatrix}.$$

A linearization of the model equations around the steady state of the variables gives equations of the form

$$\begin{aligned} s_t &= \mathcal{A}s_{t-1} + \mathcal{B}\varepsilon_t, \\ f_t &= \mathcal{C}s_t. \end{aligned}$$

Here the matrices  $\mathcal{A}, \mathcal{B}$ , and  $\mathcal{C}$  and the steady state values are computable functions of the parameters of the model:  $\beta, \gamma, \theta, \eta, \delta, \gamma, A, \rho, \sigma$ .

Because  $\mathcal{C}$  is  $3 \times 2$  and  $f_t = \mathcal{C}s_t$ , there must be a vector  $\xi$  so that  $\xi' \mathcal{C} = 0$ , and hence  $\xi' f_t = 0$ . No such identity holds in the data, so the method suggested for

taking the model to the data is to enlarge the system by an autoregressive noise term  $u_t$  :

$$\begin{aligned} s_t &= \mathcal{A}s_{t-1} + \mathcal{B}\varepsilon_t, \\ f_t &= \mathcal{C}s_t + u_t, \\ u_t &= \mathcal{D}u_{t-1} + \xi_t, \end{aligned}$$

where  $u_t$  is an unobserved  $AR(1)$  process with innovations  $\xi_t$ , i.i.d.  $N_3(0, V)$ . Thus the statistical model for the five variables,  $Y_t, K_t, C_t, H_t$ , and  $A_t$  are driven by four errors,  $\varepsilon_t, \xi_{1t}, \xi_{2t}$ , and  $\xi_{3t}$ . It is also assumed that  $\varepsilon_t$  is i.i.d.  $N(0, \sigma^2)$ , and independent of  $\xi_t$ . A model of this form can be written as a state space model and Gaussian inference can be conducted using the Kalman filter.

He then fits the model to US data, see [www2.bc.edu/~irelandp](http://www2.bc.edu/~irelandp). The economic model is supplemented with the assumption that  $\beta = 0.99$  and  $\delta = 0.025$ , as these coefficients were difficult to determine from the data. The main finding is that the model seems to predict well out of sample as compared to some competitors, a VAR(1) and a VAR(2) model and a model with diagonal  $D$ .

His finding, however, is based upon the assumption that the model is correctly identified. As a check for model misspecification he finds that by comparing the estimated values of  $\varepsilon_t$  and  $\xi_t$ , the estimated correlations were rather small  $(-0.0634, 0.0133, 0.0010)$ , so the assumption of independence seems reasonable. He also finds, using Wald tests, that the parameters of the economic model before and after 1980 have hardly been the same for the whole period. The estimated values of  $\rho$  (0.9987) and the largest eigenvalue of  $D$  (0.9399) are very close to one. These findings mean that the stationarity assumption is hard to justify. The data has also been investigated by Meeks (2005, personal communication) who showed by recursive estimation, that the assumption of constant parameters is indeed not possible to maintain.

Nevertheless, the conclusions of the paper are stated as if the model describes the data.

### 3 Model building and the role of maximum likelihood

In order to discuss in more detail what are the consequences of not choosing a model that describes the data, we consider in the next section a very simple example, where the (simulated) data is analysed by two different Gaussian maximum likelihood methods, but first we formulate two assumptions that need to be satisfied by the statistical model before we can apply the likelihood methodology.

The starting point for building a statistical model is that we have an economic model given by economic relations, which may include stochastic error terms. We call this the substantive model. We also have data  $(x_1, \dots, x_T)$ , which we assume are measurements of the economic variables. Let the (unknown) data density be

denoted by  $f(x_1, \dots, x_T)$ . We choose a statistical model as a class of densities

$$p(x_1, \dots, x_T, \theta), \theta \in \Theta,$$

where  $\Theta$  is the parameter space. We then need to make two important assumptions:

**The substantive assumption:** *The statistical model embeds the substantive model, that is, the economic relations are given by some parameter restrictions.*

**The likelihood assumption:** *We assume that the density  $f(x_1, \dots, x_T)$  of the data is a member of the statistical family to which we apply the methodology. This means that there is some parameter value  $\theta_0$ , the true value, for which the density  $p(x_1, \dots, x_T, \theta_0)$  is the same as the density of the data:*

$$p(x_1, \dots, x_T, \theta_0) = f(x_1, \dots, x_T).$$

The first assumption is naturally satisfied if the statistical model is constructed as an extension of the economic model. It is the second assumption that will be discussed here. The likelihood function  $L(\theta) = p(x_1, \dots, x_T, \theta)$  is the density considered as a function of  $\theta$ , and a submodel is defined by as subset  $\Theta_0 \subset \Theta$ . Thus, under the substantive assumption, the economic model is given as a submodel of the statistical model.

The method of maximum likelihood provides, under certain stationarity and regularity conditions, the following useful statistical results:

- The maximum likelihood estimator  $\hat{\theta}$  is calculated by optimizing the likelihood function  $L(\theta)$  over the parameter set  $\Theta$ , and the estimator is consistent and asymptotically Gaussian.
- The observed information,  $I(\hat{\theta}) = -d^2 \log L(\theta) / d^2 \theta |_{\theta=\hat{\theta}}$ , provides an estimate of the asymptotic variance of  $\hat{\theta}$ , given by  $I(\hat{\theta})^{-1}$ .
- The maximum likelihood estimator of  $\theta \in \Theta_0$  is  $\check{\theta}$ , and the likelihood ratio test of  $\theta \in \Theta_0$ :

$$-2 \log LR(\Theta_0 | \Theta) = -2 \log(L(\check{\theta}) / L(\hat{\theta}))$$

is asymptotically distributed as  $\chi^2$ , when  $\theta \in \Theta_0$ .

Thus, once we can apply this methodology, we have general theorems from statistics and probability, which ensure that we have answers to a number of relevant questions in inference about the substantive model, without having to do more than finding a program to optimize the likelihood function and print out  $\hat{\theta}$ ,  $I(\hat{\theta})^{-1}$  and  $L(\hat{\theta})$ , for the various hypotheses we want to test.

This looks like a free lunch, but of course the method has a price. To derive useful results from our statistical model, we have to choose a reasonable model on

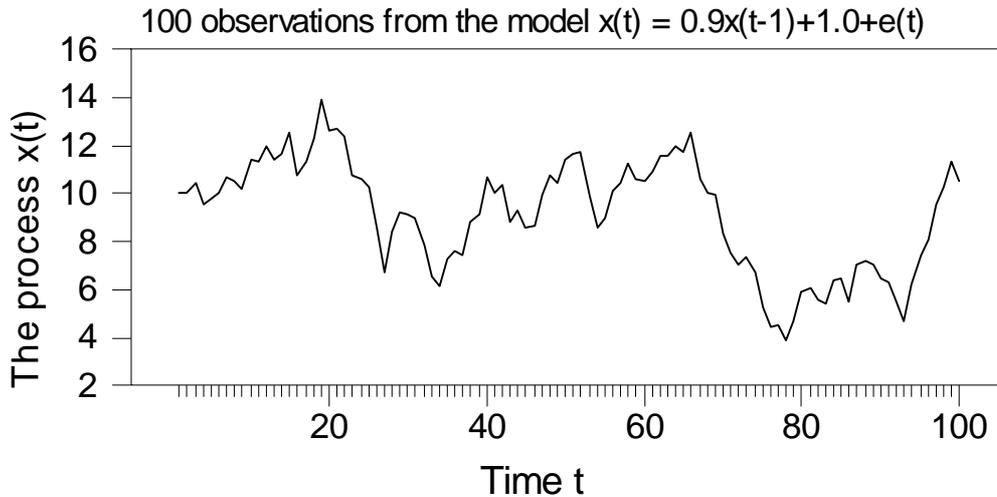


Figure 1: A simulation of 100 observations from the univariate  $AR(1)$  process(1)

which to base our likelihood analysis, and it is in the art of model building that the cost involved in likelihood based inference is found.

In order to illustrate the implications of the likelihood assumption we consider a very simple situation, where we have given some (simulated) data and have to choose between just two different statistical models from which to derive our inferences. One model satisfies the likelihood assumption and the other does not.

### 3.1 A simple example of the role of the likelihood assumption

Consider data generated as a univariate time series by the equations:

$$x_t = 0.9x_{t-1} + 1.0 + \varepsilon_t, \quad t = 1, \dots, 100, \quad x_0 = 10, \quad (1)$$

where  $\varepsilon_t$  are i.i.d.  $N(0, 1)$ . Note that  $E(x_t) = 1/(1 - 0.9) = 10$ , and  $Var(x_t) = 1/(1 - 0.9^2) = 5.263$ . A sample of the data is shown in Figure 1.

Consider two different statistical models for conducting inference in the form of asymptotic confidence intervals on the mean of the process  $x_t$ . Both models give rise to a Gaussian likelihood, but inference based upon the two models is very different.

**Model 1:** We first model the data as

$$x_t = \mu + \varepsilon_t, \quad (2)$$

with  $\varepsilon_t$  i.i.d.  $N(0, \sigma^2)$ , so that  $E(x_t) = \mu$ . Let  $P_{\mu, \sigma^2}$  be the probability measure for  $(x_1, \dots, x_T)$  determined by (2). We want to make inference on  $\mu$  and find that

the maximum likelihood estimator is the average  $\hat{\mu}_1 = \bar{x} = T^{-1} \sum_{t=1}^T x_t = 9.138$ , which is distributed as  $N(\mu, \sigma^2/T)$ , and  $\hat{\sigma}_1^2 = T^{-1} \sum_{t=1}^T (x_t - \bar{x})^2 = 5.256$ , so that  $Var(\hat{\mu}_1) = T^{-1}\sigma^2$  is estimated by 0.0526. An asymptotic 95% confidence interval is given by

$$\hat{\mu}_1 \pm \frac{1.96}{\sqrt{T}} \hat{\sigma}_1 = 9.138 \pm 0.449,$$

so that if  $x_t$  were generated by (2), then

$$P_{\mu, \sigma^2} \left\{ \hat{\mu}_1 - \frac{1.96}{\sqrt{T}} \hat{\sigma}_1 < \mu < \hat{\mu}_1 + \frac{1.96}{\sqrt{T}} \hat{\sigma}_1 \right\} \rightarrow 95\%. \quad (3)$$

**Model 2:** We could also model the data as a stationary time series

$$x_t = \rho x_{t-1} + \mu(1 - \rho) + \varepsilon_t, \quad (4)$$

where again with  $\varepsilon_t$  i.i.d.  $N(0, \sigma^2)$  and  $E(x_t) = \mu$ , if  $|\rho| < 1$ . We let  $P_{\mu, \rho, \sigma^2}$  denote the probability measure for the observations determined by (4). We find the estimates  $\hat{\rho} = 0.923$  and

$$\hat{\mu}_2 = \bar{x} - \hat{\rho} T^{-1} (x_0 - x_T) / (1 - \hat{\rho}) = 9.123,$$

and  $\hat{\sigma}_2^2 = T^{-1} \sum_{t=1}^T (x_t - \hat{\rho} x_{t-1} - \hat{\mu}_2(1 - \hat{\rho}))^2 = 0.774$ . For  $|\rho| < 1$ ,  $\hat{\mu}_2$  has the same asymptotic distribution as the average  $\bar{x}$ , but now  $Var(\hat{\mu}_2) \approx \sigma^2 T^{-1} (1 - \rho)^{-2}$ , which is estimated as 1.314, so that we get a much wider asymptotic 95% confidence set for  $\mu$ :

$$\hat{\mu}_2 \pm \frac{1.96}{\sqrt{T}} \frac{\hat{\sigma}_2}{1 - \hat{\rho}} = 9.123 \pm 2.247,$$

so that if  $x_t$  were generated by (4), then

$$P_{\mu, \rho, \sigma^2} \left\{ \hat{\mu}_2 - \frac{1.96}{\sqrt{T}} \frac{\hat{\sigma}_2}{1 - \hat{\rho}} < \mu < \hat{\mu}_2 + \frac{1.96}{\sqrt{T}} \frac{\hat{\sigma}_2}{1 - \hat{\rho}} \right\} \rightarrow 95\%. \quad (5)$$

In order to conduct Gaussian likelihood inference we have to choose between these two models, and in practice of course many more. If we choose the first model we get a confidence interval which is smaller than if we choose the second model. Thus inference is different depending on the model we choose, even though in both cases we are interested in the same quantity, namely the mean of the process, and in both cases we base our inference on the Gaussian likelihood.

These simple examples demonstrate that using maximum likelihood does not in itself guarantee useful inference, as different methods can be derived this way depending on the model chosen. So one has to ask the question of which method leads to valid inference when applied to the data at hand.

Obviously the density of the data in Figure 1 corresponds to the parameter values  $\rho = 0.9$ ,  $\mu = 10$ ,  $\sigma = 1$  in the AR(1) model, and hence the likelihood assumption is satisfied for Model 2, but there are no parameters  $(\mu, \sigma^2)$  from the first model that correspond to the data density. Thus applying Model 2, we can use all the

results of maximum likelihood, but with Model 1 we get a serious underestimate of the variance of the estimator (0.0526 instead of 1.314). Thus we cannot rely on the standard error of the estimate or the confidence interval when applying Model 1 to the data in Figure 1. In the process of checking the likelihood assumption above we are able to show that the model chosen for the first likelihood analysis does not satisfy the assumption that for some values of the parameters the model captures the density generating the data. More precisely, the residuals  $\hat{\varepsilon}_t = x_t - \bar{x}$  based upon Model 1 will show autocorrelation, so that likelihood inference derived from Model 1, which assumes no autocorrelation, is useless for this particular data. In fact we find an estimate of the correlation between  $\hat{\varepsilon}_t$  and  $\hat{\varepsilon}_{t-1}$  to be 0.923, which is clearly different from zero, which is the value assumed by the first model. Thus we are warned by the misspecification test that the mathematical results of the likelihood based methods for Model 1 are not valid.

Note that the probability statement (3) involves a probability measure  $P_{\mu, \sigma^2}$  which is irrelevant in view of the way the data is generated from (1) using  $P_{10, 0.9, 1}$ .

Thus, there is nothing wrong with either of the two models analyzed, they are both standard models. The problem is that, only the second model describes the data, the first does not. Hence, for the data at hand, application of the asymptotic results of maximum likelihood of the second method is valid and application of the asymptotic results for the first is not, even though both are simple applications of Gaussian maximum likelihood. Thus, unfortunately, it is the wider confidence interval  $9.123 \pm 2.247$  which is the correct one for the simulated data.

### 3.2 Which statistical model to choose?

In practice there is obviously a serious problem with the method of maximum likelihood, because we can of course never know with certainty if we have the right model, and the likelihood assumption is difficult, not to say impossible, to check completely. But we can sometimes show that *it is not satisfied*, and this is a very useful piece of information, because it implies that *we cannot use the model considered for conducting inference*.

What we have to do in this case is of course to find a new and better statistical model by specifying a different and more flexible family of densities, which has a better chance of capturing the density of the data, while still embedding the economic model. Only then can we conduct valid inference on the parameters of the economic model. More constructively we should ask the question

- Which statistical model describes the data adequately?

If we want to analyse an economic model, it looks like a waste of time to spend too much effort on the above question, which is clearly a statistical question. But that depends on the purpose of the analysis.

If the idea is to test hypotheses on the parameters of the economic model and test the model itself, then we need to establish a sound statistical basis for doing so, that is, we need to embed the economic model in a statistical model that describes

the data. This is then used as a platform for making inference on coefficients and relations.

In case we find that the statistical model does not allow for the restrictions on the parameter space as defined by the economic model, the explanation can be that the economic model is inadequate, or of course that the data quality is not good enough. Whatever the case, the statistical model cannot be used for inference about the economic model, and we either have to rethink and extend the economic model, or check the quality and relevance of the data.

If the purpose of the analysis, however, is not to make inference on coefficients or model equations, but to apply an economic model that we believe firmly in with the purpose of making predictions, say, then perhaps it is enough to pick some likelihood method to get some estimates. But then, consistency of the estimators it is not guaranteed, and standard errors derived from the likelihood theory, let alone  $t$ -statistics, should not be reported.

If we are in the first situation, that we want to make inference about our economic model from the data, there is no way we can avoid checking the many assumptions behind the likelihood methods. Thus, after we have suggested a statistical model that embeds the economic model, we should always ask the question

- How can we prove that this statistical model is incorrect?

For example, are the residuals uncorrelated? If they are not, then  $t$ -tests,  $\chi^2$ -tests, and  $F$ -tests would no longer be valid. Are the parameters constant over the sample? If they are not we cannot define the economic parameters and conduct inference on them.

Only after having confronted the statistical model with all possible misspecification tests and not having been able to reject the underlying assumptions, is it possible to be more confident in applying the general theory of likelihood. Of course, we still run the risk that someone else will analyze the data, and show that the model we have chosen is incorrect, and hence that the conclusions we have reached might need to be modified.

In the above illustration of the US data by means of the DSGE model, which assumes constant parameters, it is impossible to give a meaning to a confidence interval based upon full sample estimates, when misspecification tests show that the data generating process does not have constant parameters.

In the next section we assume that we have chosen the correct model and discuss the consequences of accepting that a very large (but less than one) root is indeed a stationary root. We conduct a simple simulation experiment and show that asymptotic inference can be seriously misleading, because the number of observations needed to make inference is excessively large.

## 4 How reliable are asymptotic results with a near unit root?

In this section we analyse, by simulation, the problem of making inference on the mean  $\mu$  in the autoregressive process (4), when  $|\rho| < 1$ . We know that standard asymptotic inference applies as long as  $|\rho| < 1$ , but it is to be expected that with  $\rho$  close to one, the asymptotic results offer poor approximations to the finite sample distribution, which is needed for making reliable inference.

### 4.1 Simulations

The equations determining the univariate process  $x_t$  are

$$x_t = \rho x_{t-1} + (1 - \rho)\mu + \varepsilon_t, t = 1, \dots, T \quad (6)$$

where  $\varepsilon_t$  are i.i.d.  $N(0, \sigma^2)$ . Under the assumption of stationarity, the asymptotic distribution of the maximum likelihood estimator  $\hat{\mu}$  is given by

$$T^{1/2}(\hat{\mu} - \mu) \xrightarrow{w} N\left(0, \frac{\sigma^2}{(1 - \rho)^2}\right).$$

The problem we want to discuss is how inference can be distorted when  $\rho$  is close to one. We see that, as the root  $\rho$  approaches one, the variance blows up and there is little information about the mean of the process. Not only does the variance blow up, the finite sample distribution deviates seriously from the asymptotic Gaussian distribution, so that inference of the type

$$\hat{\mu} \pm \frac{1.96}{\sqrt{T}} \frac{\hat{\sigma}}{1 - \hat{\rho}}$$

becomes highly misleading and tests on  $\mu$  become unreliable. In order to illustrate this, we choose as simulation experiment the equation

$$x_t = \rho x_{t-1} + \varepsilon_t, \quad (7)$$

that is, we choose  $\rho = 0.5, 0.9, 0.99$ , and  $0.999$ , take  $\sigma^2 = 1$ , and choose  $\mu = 0$ . We do 10,000 simulations for  $T = 50, 100$ , and  $500$ . As a statistical method we choose Gaussian maximum likelihood inference based on model (6), which in this case happens to be the correct one because the likelihood assumption is satisfied for the statistical model chosen.

We simulate the distribution of the estimator of  $\hat{\mu}(1 - \rho)\sqrt{T}$ , which has been normalized to have asymptotic variance one. Figure 2 shows some histograms for different values of  $\rho$ , compared with the Gaussian distribution ( $\rho = 0$ ), based upon 5,000 simulations of a time series with  $T = 50$ . Note that for  $\rho = 0.5$  and  $\rho = 0.9$  the asymptotic distribution describes the finite sample distribution quite well, but for larger  $\rho$  the asymptotic distribution is far from the finite sample distribution, and inference, assuming it is Gaussian, is highly misleading.

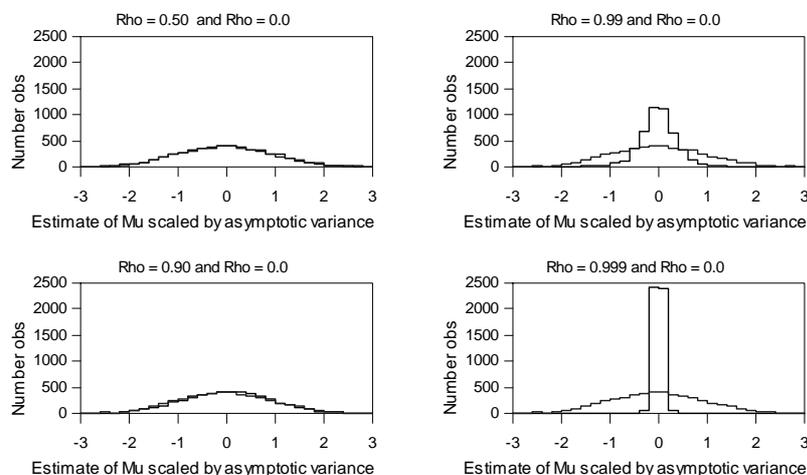
Histogram for 5000 estimates of  $\mu^*(1-\text{Rho})\sqrt{T}$ ,  $T = 50$ 

Figure 2: In the DGP (1) we simulate the distribution based upon 10,000 simulations of the estimator of  $\mu$  scaled by the asymptotic standard error for  $\rho = 0.0, 0.5, 0.9, 0.99, \text{ and } 0.999$ , and  $T = 50$ .

In Table 1 we calculate the rejection probabilities of the likelihood ratio test that  $\mu = 0$ , using the asymptotic critical value of  $\chi_{0.95}^2(1) = 3.84$ . We see that even 500 observations are not enough for getting anywhere near a 5% test when  $\rho$  is 0.99 or 0.999. Thus one can insist that a root of 0.99 is a highly persistent stationary root so that variables have a well defined steady state value, but the price paid is that one cannot make inference on this steady state values using usual sample sizes. Thus when Ireland (2004) finds  $\hat{\rho} = 0.9987$  and takes that to support that the variables are stationary, the consequence is that many more observations would be needed to make inference on the assumed steady state values of the variables.

Rejection probability in % for test of $\mu = 0$			
$\rho \backslash T$	50	100	500
0.5	6.1	5.8	5.4
0.9	11.9	9.3	6.1
0.99	24.6	22.4	12.4
0.999	27.6	27.4	25.1

Table 1: In the DGP (7) we simulate the rejection probability of the likelihood ratio test statistic for the hypothesis  $\mu = 0$ , for various values of  $\rho$  and  $T$  using the quantile  $\chi_{0.95}^2(1) = 3.84$ . The results are based on 10,000 simulations.

## 5 Concluding discussion

If the approaches being described in this paper were limited to a few papers, one could perhaps see them as outliers. But, unfortunately, they are not. In efforts to bring models to the data researchers all too often fail to heed the two simple methodological rules given at the beginning of the paper. These rules place limits on which specifications can be reasonably used.

To summarize: When confronting an economic model with data in order to make inference on the parameters using likelihood methods it is not enough to use the Gaussian maximum likelihood methods. One has to specify a statistical model that not only contains the economic model but also the data generating process, so that the likelihood assumption is satisfied. This can be checked by misspecification tests. If the model is misspecified, one cannot rely on the results of the likelihood methods. Even if the assumptions are satisfied and a correct statistical model has been chosen, it may not be enough to rely on asymptotic methods for conducting inference. One must also check, by simulation or small sample corrections, how reliable the asymptotic results are. This is particularly the case for processes close to nonstationarity.

The failure to adequately confront the models with the data means that the development of economic theories cannot be given guidance from the data. The consequence is that the profession is likely to stick to theories which are no longer useful or even have become harmful. Even worse, the profession will fail to recognize new features in the data, which go against mainstream beliefs, which could signal a change in the underlying mechanism.

We should emphasize that the principles we have discussed in some detail have nothing specifically to do with the DSGE model, which was taken as an example of a mismatch between theory and evidence. We could consider many other examples, see for example the discussion in Juselius (2005).

What do these technical issues mean for policy makers? They mean that the macro data available are highly unlikely to provide support for a model in the manner often attempted by macro econometricians. If analysed correctly, however, the data can provide indications that certain assumptions do not work and hence guidance in choosing among alternative hypotheses, along the lines of the approach specified in Juselius and Johansen (2006). Therefore the frequently observed mismatch of the models and the empirical evidence should be of serious concern for any scientist and policy maker.

## 6 References

Altuğ, S. (1989), Time-to-build and aggregate fluctuations: some new evidence, *International Economic Review* 30, 889-920.

Haavelmo, T. (1943), Statistical implications of a system of simultaneous equations, *Econometrica* 11, 1-12.

Ireland, P. (2001), Technology shocks and the business cycle: An empirical investigation, *Journal of Economic Dynamics and Control* 25, 703-719.

Ireland, P. (2004), A method for taking models to the data, *Journal of Economic Dynamics and Control* 28, 1205-1226.

Juselius. K. (2005), *The cointegrated VAR Model, Methodology and Applications*, forthcoming Oxford University Press.

Kapetanios, G., Pagan, A. and Scott, A. (2004), Making a match: combining theory and evidence in policy oriented macro economic modeling. Discussion paper.

Söderlind, P. (1999), Solution and estimation of RE macro models with optimal policy, *European Economic Review* 43, 813-823.