Cointegration Analysis of Climate Change: An Exposition

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First version: July 31, 2007
this version: August 29, 2007

Note: Preliminary version.
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Abstract This paper describes in a non-technical manner the concept of cointegration and its implications in the modelling of climate data. It illustrates a number of inference procedures appropriate in integrated-cointegrated vector autoregressive processes (VARs). Particular attention is paid to the properties of VARs, to the modelling of deterministic terms, to the determination of the number of cointegration vectors, and to testing hypotheses on long-run relation and short-run adjustment. The analysis is illustrated by an empirical analysis of sea and land temperatures in the period 1500-2000 and how the long-run movements in these temperatures have been influenced by solar radiation, greenhouse gasses and volcanic erruptions.

1 Introduction

The aim of this paper is to introduce climatologists to some fairly recent methods in time series analysis, dubbed multivariate cointegration. The methods were originally developed to provide adequate tools for empirical analysis of economic time series. The recognition that economic time series are non-stationary profoundly altered the technology of econometrics, introducing the concepts and tools associated with integrated-cointegrated data. Even though the procedures so far have mostly been used in the analysis of economic data, their applicability is by no means restricted to such data. Unit root processes (variables containing a stochastic trend) can be found in many other fields, for example in climate data. Cointegration analyses of such data have also been reported in the literature (see, Kaufman and Stern, 2002 and references therein), but still in a very limited scale. We are convinced there exists a much larger potential for analyzing climate data with the cointegrated VAR model.

Why is it important to use cointegration methods rather than the usual regression methods when there are unit roots in the data? The simple answer is that standard regression estimates are derived under the assumption of stationarity, implying that the variables are not trending, or if they are, that the trend is a deterministic time trend. A simple example illustrates. Assume for simplicity that annual temperature C_t follows a random walk:

$$\begin{aligned} C_t &= C_{t-1} + \varepsilon_t, \ \text{t} = 1,...\text{T}. \\ &= \varepsilon_t(1 + L + L^2 + ... + L^t) + C_0, \\ &= \textstyle\sum_{i=1}^{t} \varepsilon_i + C_0 \end{aligned} \tag{1}$$

where $\varepsilon_t \sim N(0, \sigma^2)$ is the temperature change from $t-1$ to t, so that $\Delta C_t = \varepsilon_t$, C_0 is the initial value, and L is the lag operator such that $L^m x_t = x_{t-m}$. By assuming that the temperature changes have a zero mean we have from the outset stated that temperature cannot have a deterministic trend, but as long as the coefficient to C_{t-1} is equal to 1.0, it will have a stochastic trend, defined by the cumulated changes (ε_i). This is the reason why a variable containing a stochastic trend, defined as the cumulated sum of Independent Normal errors, in short, ε_t distributed as $\mathsf{IN}(0, \sigma_\varepsilon^2)$ is often called a unit root process, in short I(1). A stochastic variable, similarly generated from $\mathsf{IN}(0, \sigma_\varepsilon^2)$ errors, but with a coefficient less then 1.0 in absolute value is then called an I(0) process[1]. Regressing I(1) variables on each other has the unfortunate consequence that the usual χ^2, F, and t distributions are no longer valid. This means,

[1] For a mathematically precise definition, see Johansen (1996).

for example, that the t ratios in such regression models are no longer distributed as Student't statistics and one does not know whether a coefficient is significant or not.

However, the very rich structure of the cointegrated VAR model is the main reason why it is likely to be enormously useful for the analysis of climate data. As the subsequent analysis demonstrates, it is possible to analyze long-run co-movements between trending variables, as well as short-run dynamic adjustment and feed-back effects within the same model. Furthermore, the model allows us to investigate which climate shocks (temperature changes) had a long-run permanent effect on the variables of the system, and which had just a temporary effect. In this sense, the analysis may potentially provide results on causal mechanisms in the long run and in the short run.

The mathematics of the cointegrated VAR model may easily have an intidimating effect on a non-mathematician. The derivation of the asymptotic distributions in Johansen (1996) provides a good illustration. In order not to scare away any potential user, the discussion here is kept on a non-technical level. Thus, the idea at this stage is to provide an intuition for the analysis, to illustrate questions that can be easily addressed, and to show how the result can be interpreted. In so doing, I shall introduce the concepts and the interpretations using just one, empirically rich, data set. It consists of annual land and sea temperatures over the period 1500-2000 from Stendel, Mogensen, and Christensen (2005) and of the following forcing variables: solar radiation, three greenhouse gasses, and volcanic erruptions from Robertson et.al. (2001)[2].

The organization is built up based on similar principles as in my recent book, Juselius (2006). But, contrary to the book, I shall here leave out almost all formal econometrics and, instead, refer to the chapter where it can be found. All empirical results are obtained by applying the software package CATS in RATS (Dennis et. al. 2006). The graphs are produced with the package GiveWin (Doornik and Hendry, 2006).

Finally, I should stress that the data have been analyzed purely from a statistical/econometric point of view. My prior knowledge of climatological models is close to zero, something I consider an advantage at this stage. It means I have approached the data without having any prior ideas of what to find, which signs of coefficients, etc. Thus, if the analysis provides interesting results it can be seen as evidence of the strength of the methods.

The organization of the paper is as follows: Section 2 introduces the data and Section 3 the concept of cointegration in a bivariate model

[2]The data base was kindly given to me by Peter Thejll.

setting applied to land and sea temperatures and provides some first, still tentative, results. Section 4 discusses how to check the statistical adequacy of the model, how to account for outlier observations in the data analysis, and how to calculate the characteristic roots of the model. Section 5 discusses how to determine the cointegration rank (the number of cointegration relations in the data) using all available information. Section 6 reports some preliminary results on the common trends and how they have affected the data. Section 7 test the model for parameter constancy and discusses how to interpret some recursive tests. Section 8 adds the forcing variables to the model and reports a more realistic analysis of the Robertson climate data. The concept of exogeneity is discussed at some length. Section 9 provides a complete analysis of the pulling and pushing forces in this climate model and Section 10 concludes.

2 Introducing the data

The variables of interest, the model determined land and sea temperatures from Stendel et.al. (2005), will first be analyzed without considering the impact of the forcing variables from Robertson et.al. (2001). Even though the empirical results from this analysis are interesting as such, the main advantage of this is to introduce the concepts in a very simple context. Nonetheless, the long-run impact of the forcing variables is what climatologists usually care most about and the last part of the paper is concerned with this question.

Being a time-series econometrician, I always begin by taking logs of the data as a matter of routine. This is because the results are often easier to interpret in relative changes (percentages), but also because the parameters of a VectorAutoRegressive (hereafter VAR) model in logs are usually more constant than in absolute values. However, I recognize that climate models are based on physical laws that that might suggest other transformations. In this case, there is an argument in favor of relating asolute temperatures to the log of CO2. I have done the analyses of this paper for (1) absolute values of all variables, (2) log values of all variables (except for volcanic erruptions as this variable contains numerous zero observations), and (3) absolute values of temperatures and solar radiation, but log values of the greenhouse gasses. Since all basic conclusions are essentially unchanged for the three cases, I continue with case (2). Figure 1, left hand side, shows the movements of land and sea temperatures, as well as solar radiation in log levels and right hand side the first differences over the period 1500-2000. We note that the mean values of the log level of land temperature, denoted $C_{l,t}$, and sea temperature, denoted $C_{s,t}$, seem to vary over time in a non-deterministic manner typical

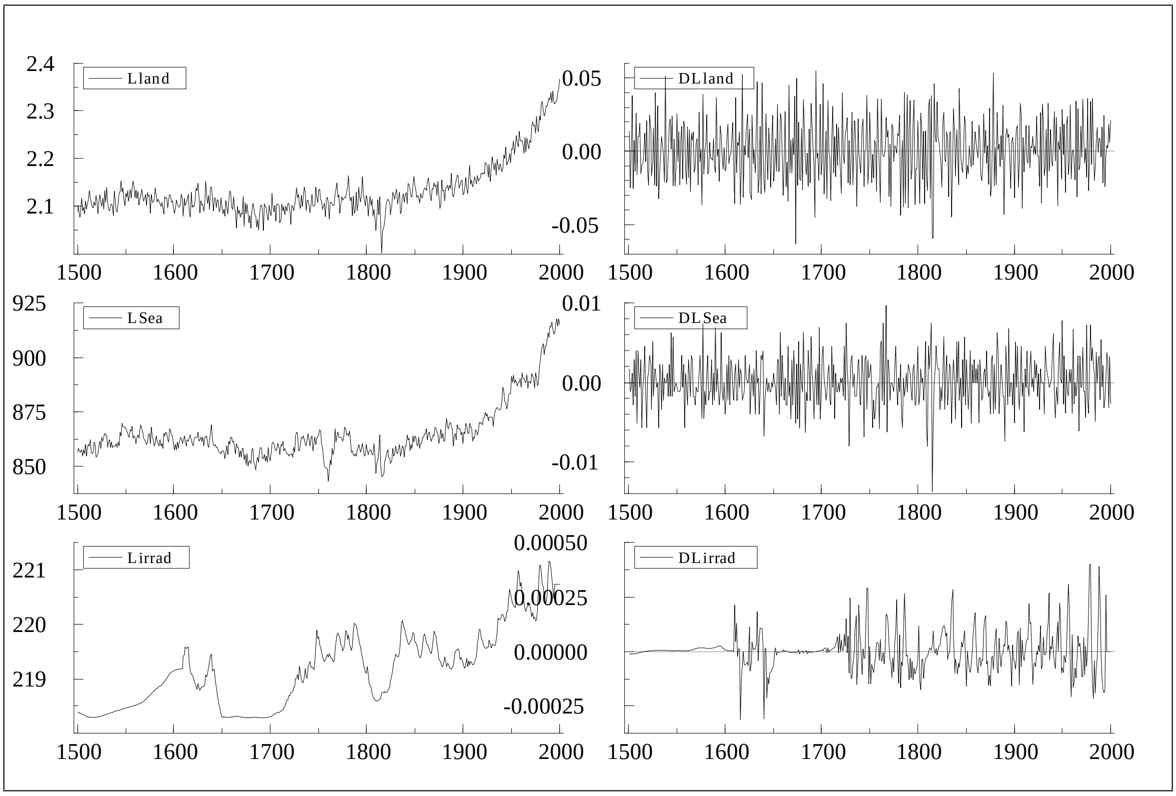

Figure 1: The graphs of land temperature, sea temperature, and solar radiation in log levels (left hand side) and log differences (right hand side).

of a non-stationary series. More interestingly, they seem to follow a common long-run path prompting the question whether the long-run trend is actually the same in the two temperatures. This is one of one of the basic questions one can answer with cointegration analysis. If the answer is yes, we say that the two temperatures share a common stochastic trend implying that they are cointegrated. Cointegration between the two temperature variables would then imply that both of them had been similarly affected by permanent temperature changes over time. Thus, a linear combination of the two temperatures would be stationary (i.e. do not exhibit any trending behavior) despite each of them evolving in a non-stationary manner. In this case, the common stochastic trend would be measured by (a linear combination of) cumulated temperature changes at land and sea over the sample period. Understanding what have caused these temperature changes would, of course, be the ultimate interest of the empirical analysis. To obtain some answers to this question the last part of the paper will add a number of potential

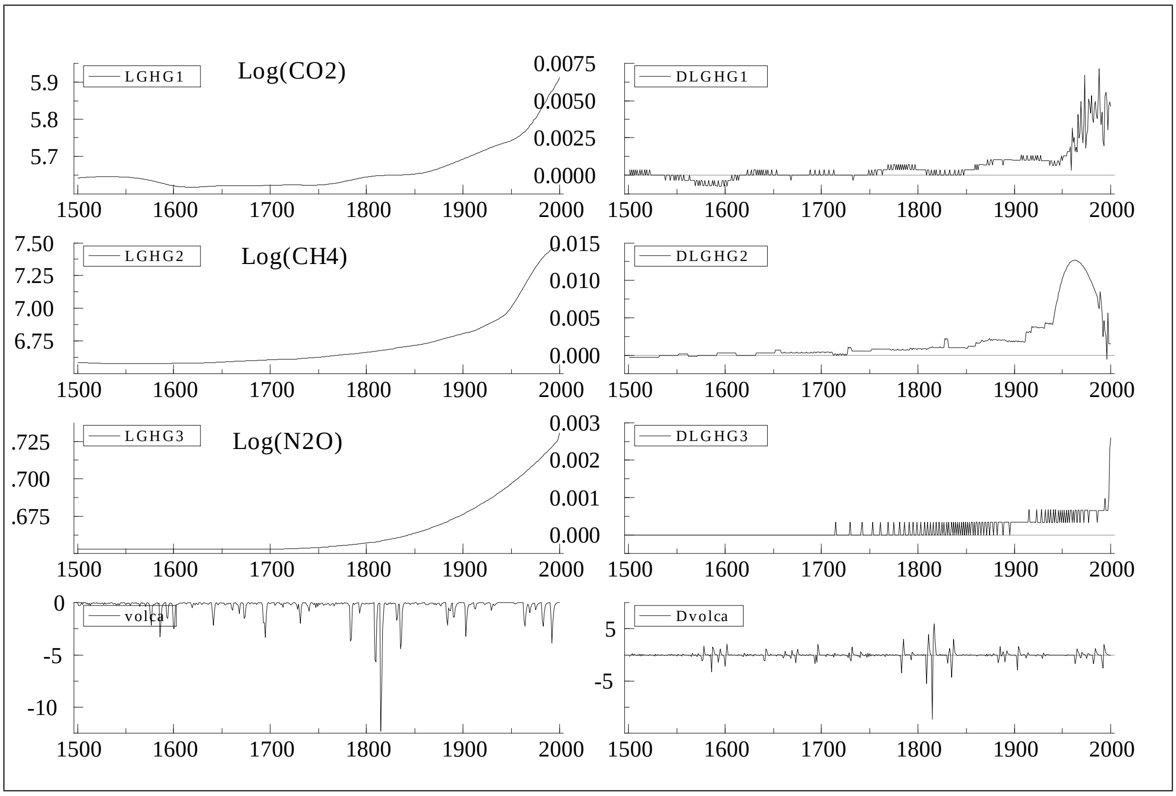

Figure 2: The graphs of GHG1, GHG2, GHG3, and volcanic erruptions in log levels (left hand side) and differences (right hand side).

driving variables, such as solar radiation, several greenhouse gasses, and volcanic emissions of aerosol, to the analysis.

In case the two temperature variables are not cointegrated, implying that the long-run stochastic path is not exactly the same, a linear combination between the two temperatures would still exhibit trending behavior. The interesting hypothesis would then be whether this left-out trend could be associated with any of the forcing variables, so that a linear combination of the two temperatures and the forcing variables produces a stationary variable. In this case, the forcing variables would have a different long-run impact on the land and sea temperature.

The bottom panels of Figure 1 show the graphs of solar radiation in log levels and differences and Figure 2 some potential forcing variables, here the greenhouse gasses GHG1 (CO_2), GHG2 (CH_4) and GHG3 (N_2O), and emissions of aerosol from volcanic erruptions.

The next section will introduce the concept of cointegration using exclusively land and sea temperatures. Sections 3-7 will introduce the relevant statistical concepts needed for a complete cointegration analysis using the temperature data as an illustration. Sections 8-9 will extend the analysis with the main forcing variables.

3 Introducing cointegration

This section introduces the basic concepts of cointegration using a first tentative analysis of the two temperature variables. At this stage all formal analysis will be avoided in order to slowly build up the intuition for what cointegration analysis is.

First it is important to realize that cointegration, as such, does not say anything about causality only about an association between variables. In this sense cointegration can be compared to a simple correlation coefficient (or a regression coefficient), which does not say anything about causal links. However, a correlation coefficient is a measure of a bivariate relation and it requires stationarity to be interpretable, whereas cointegration is defined for two or more variables and can be interpretable as a measure of association when data are non-stationary. Thus, a correlation coefficient (associating two stationary variables) can often be thought of as describing a short-run relationship, whereas a cointegration relation describes a relationship that holds over the long run. For example, the short-run relationship between changes in the land and sea temperature (which are stationary) can be completely different from the long-run relationship between temperature levels. In the present data set, the short-run association between changes in land and sea temperatures is 1 to 0.83 (obtained by regression analysis) whereas the long-run association between levels of temperature is 1 to 4.2 (obtained by cointegration analysis).

However, the cointegrated VAR model is not just modelling cointegration relations, it also involves the analysis of short-run adjustment effects in a system of equations. The latter are more informative on questions of causal links. For example, sea temperature could have changed and then caused a change in the land temperature. In this case we would find a significant adjustment coefficient in the equation for land temperature, but no such adjustment in the equation for the sea temperature. We say that sea temperature is exogenous to the land temperature. Or, it would have been the other way around and land temperature would have been exogenous to the sea temperature. In most cases, there is adjustment in both equations and we cannot statistically postulate which variable drives the other. We say that both variables exhibit feed-back effects as a consequence of some third forcing variable having pushed the system away from the equilibrium state.

3.1 A simple VAR formulation

Thus, to fully comprehend how to properly formulate a cointegrated VAR model, one has to understand that it contains two fundamentally different but interrelated analyses:

1. The analysis of long-run relations defined as stationary linear combinations between nonstationary variables. This implies that the cointegrated variables are co-moving in a stationary long-run relationship. When they are pushed apart some adjustment forces will be activated to bring them back toward the long-run equilibrium level.

2. The analysis of the equations which define how the short-run adjustment takes place that bring the variables back to the equilibrium level.

Thus, cointegration analysis is inherently multivariate both in the sense of finding cointegration relations between variables and learning about how the system react to exogenous shocks. All this can be analyzed at the same time in one model, the vector autoregressive (VAR) model, where each variable is 'explained' by its own lagged values, and the lagged values of all other variables in the system. A VAR(2) model (with two lags) would look like follows:

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + \mu + \varepsilon_t \tag{2}$$

where ε_t is assumed $\mathsf{IN}_2[0, \Omega_\varepsilon]$, and Ω_ε is the (positive-definite, symmetric) covariance matrix of the error process. Writing it out for the land and temperature variables it becomes:

$$\begin{bmatrix} C_{l,t} \\ C_{s,t} \end{bmatrix} = \begin{bmatrix} \pi_{1.11} & \pi_{1.12} \\ \pi_{1.21} & \pi_{1.22} \end{bmatrix} \begin{bmatrix} C_{l,t-1} \\ C_{s,t-1} \end{bmatrix} + \begin{bmatrix} \pi_{1.11} & \pi_{1.12} \\ \pi_{1.21} & \pi_{1.22} \end{bmatrix} \begin{bmatrix} C_{l,t-2} \\ C_{s,t-2} \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

The VAR model (2) can be given different parametrizations without imposing any binding restrictions on the parameters of the model (i.e., without changing the value of the likelihood function). When the aim is to discriminate between short-run adjustment effects and long-run relations, the following equilibrium error correction form (VecEcm) is the most useful one:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Pi x_{t-1} + \mu + \varepsilon_t \tag{3}$$

with $\Pi = I_p - \Pi_1 - \Pi_2$ and $\Gamma_1 = -\Pi_2$. Writing it out gives:

$$\begin{bmatrix} \Delta C_{l,t} \\ \Delta C_{s,t} \end{bmatrix} = \begin{bmatrix} \gamma_{11} \, \gamma_{12} \\ \gamma_{21} \, \gamma_{22} \end{bmatrix} \begin{bmatrix} \Delta C_{l,t-1} \\ \Delta C_{s,t-1} \end{bmatrix} + \begin{bmatrix} \pi_{11} \, \pi_{12} \\ \pi_{21} \, \pi_{22} \end{bmatrix} \begin{bmatrix} C_{l,t-2} \\ C_{s,t-2} \end{bmatrix} + \tag{4}$$

$$+ \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \tag{5}$$

The explanatory part of model (4) is now formulated so that one can directly discriminate between short-run transitory effects of changes in the lagged differences ($\Gamma_1 \Delta x_{t-1}$) and long-run effects between the lagged levels (Πx_{t-1}) and how they have affected temperature changes. However, the estimated coefficients (and their p-values) can vary considerably between (2) and (4), despite being identical in terms of errors $\{\varepsilon_t\}$ and, hence, explanatory power[3].

The above VAR models are at this stage formulated without considering the order of integration of the variables and can be estimated by OLS. However, some inference is not standard (χ^2, F, t) unless $x_t \sim \mathsf{I}(0)$. This is, in particular the case with the Π matrix, the estimated coefficients of which are not distributed as Student's t when x_t is non-stationary. This is exactly where cointegration offers a solution. When $x_t \sim \mathsf{I}(1)$, the Π matrix has reduced rank, r:

$$\Pi = \alpha\beta' \tag{6}$$

where α is a $p \times r$ matrix of adjustment coefficients describing which equations adjust and which do not, and β is a $p \times r$ matrix of coefficients describing r long-run relations $\beta' x_t$. To show which questions can be asked within this cointegrated VAR, we insert (6) in the VecEcm model for the two temperature variables $C_{l,t}$ and $C_{s,t}$:

$$\begin{bmatrix} \Delta C_{l,t} \\ \Delta C_{s,t} \end{bmatrix} = \begin{bmatrix} \gamma_{11} \, \gamma_{12} \\ \gamma_{21} \, \gamma_{22} \end{bmatrix} \begin{bmatrix} \Delta C_{l,t-1} \\ \Delta C_{s,t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} [C_l - b_1 C_s]_{t-1}$$

$$+ \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \tag{7}$$

where $\beta' = [1, \, b_1]$ so that $\beta' x_{t-1} = [C_l - b_1 C_s]_{t-1}$. Based on the cointegrated VAR model in (7), we could explain the two temperature changes from period $t-1$ (previous year) to t (this year) as a result of:

[3]This illustrates the increased difficulty of interpreting coefficients in dynamic models relative to static regression models: many significant coefficients need not imply high explanatory power, but could result from the parameterization of the model.

(i) an adjustment to previous temperature changes, with impacts γ_{ij} for the j^{th} lagged change in the i^{th} equation;

(ii) an adjustment to previous disequilibria between land and sea temperatures, $(C_{l,t} - b_1 C_{s,t})$, with impacts α_{i1} in the i^{th} equation;

(iv) a constant term μ_i; and

(v) random shocks, $\varepsilon_{i,t}$.

When the temperature variables are $\mathsf{I}(1)$, and land and sea temperature are cointegrated, the changes in temperatures as well as $(C_{l,t} - \omega_1 C_{s,t})$ are $\mathsf{I}(0)$. Thus, treating $(C_{l,t} - b_1 C_{s,t})$ as a variable measuring the disequilibrium error (temperature imbalance) at each point in time, means that the VecEcm model is now formulated in stationary variables and standard inference applies. Figure 3, upper panel, shows that the trending behavior of the two temperatures evolve in a very similar manner when adjusted for mean and range. The lower panel shows that the linear combination, $C_{l,t} - 4.2C_{s,t}$, annihilates the trend, which suggests that the stochastic long-run trend is indeed the same in the two temperature variables.

A constant term is not usually of great interest. However, in the cointegrated VAR model the constant term has a different interpretation if it is part of the equations or the cointegration relations. As a matter of fact, understanding the above distinction between equations and relations is crucial in order to understand the role of all deterministic components (trends, dummy variables as well as the constant term) in the cointegrated VAR model. This is a fairly complicated issue which will not be covered in any detail in this paper. For a more detailed account the reader is referred to Johansen et.al. (2000) and Juselius (2006, Chapter 6).

At this stage we shall only discuss how to specify the constant term or the trend in the model. To start with we need to decide whether the variables contain a deterministic linear time trend or not. This is essentially the question of whether the temperature changes have a mean which is different from zero or not. In the former case, the temperature has grown significantly over the sample period with an average growth rate equal to the average of the temperature change and we need to allow for a linear time trend in the model. In the latter case it has not grown deterministically and we should allow for a constant term, but no linear trend, in the model. The table below shows that the temperature

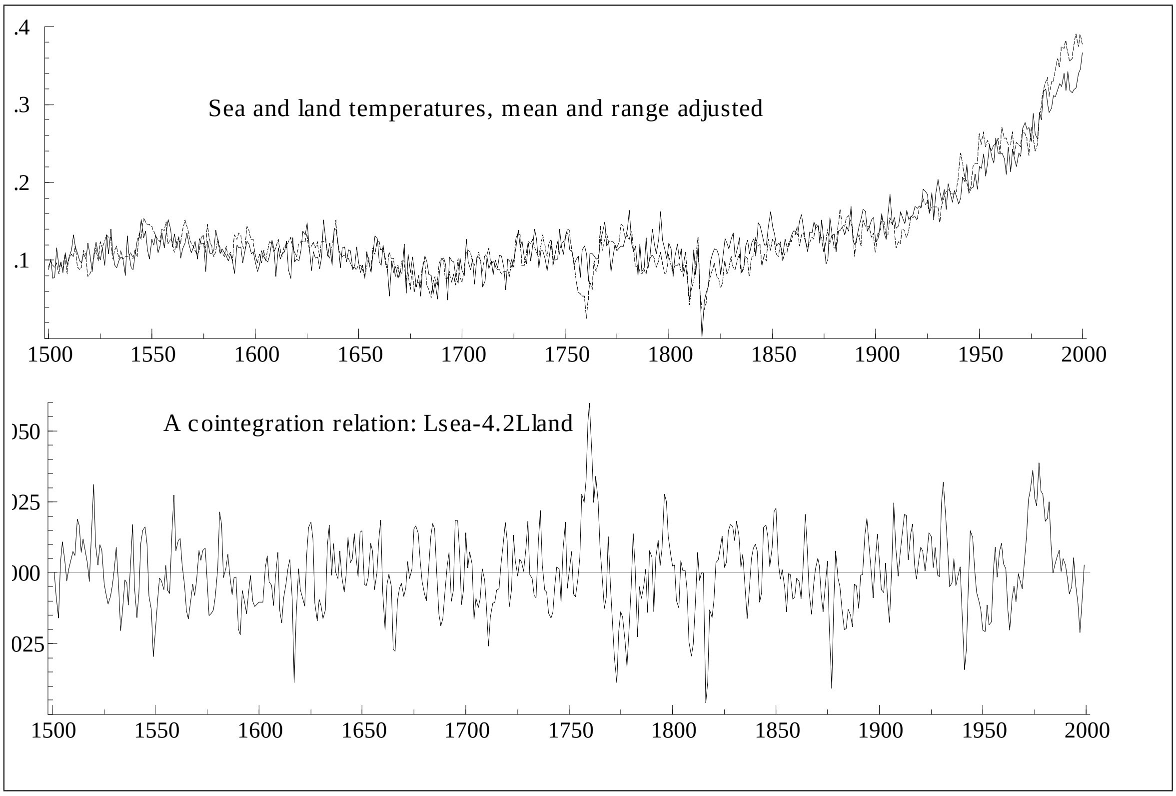

Figure 3: The graphs of land and sea temperatures adjusted for mean and range (upper panel) and of a stationary cointegration relation between the two (lower panel).

changes over this sample period are not significantly different from zero.

	$\Delta C_{l,t}$	$\Delta C_{s,t}$
Sample mean	0.00056	0.00012
Sample standard deviation	0.0210	0.0031

Thus our model needs a constant term, but no linear time trend. The constant term has to be restricted to exclusively enter the cointegration relations, as a constant term in the equations (defined for temperature changes) has the role of a linear growth rate. This means that the constant term will measure an intercept in the cointegration relations.

3.2 Some preliminary estimates

As already mentioned, the existence of cointegration by itself does not imply which temperatures 'equilibrium adjust' and which do not; nor does it entail whether any adjustment is fast or slow. Information about such features can be provided by the α_{ij} coefficients. To discuss the full model we reproduce some first estimates of (7):

$$\begin{bmatrix}\Delta C_{l,t}\\ \Delta C_{s,t}\end{bmatrix} = \begin{bmatrix}-0.26 & 1.08\\ -0.02 & -0.18\end{bmatrix}\begin{bmatrix}\Delta C_{l,t-1}\\ \Delta C_{s,t-1}\end{bmatrix} + \\ + \begin{bmatrix}-0.39\\ 0.03\end{bmatrix}[C_l - 4.20C_s + 9.90]_{t-1} + \begin{bmatrix}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{bmatrix}. \quad (8)$$

First we notice that $\alpha_{11} = -0.39[7.5]$[4] whereas $\alpha_{12} = 0.03[3.2]$, i.e. both temperatures adjust significantly to any 'deviant' behavior in land and sea temperatures, but the land temperature adjusts more quickly. When the product $\alpha_{1i}\beta_i < 0$, we say that the variables are equilibrium error correcting. As the product $-0.39 * 1.0 < 0$ and $0.03 * (-4.2) < 0$, both temperatures are equilibrium error correcting. This is the same as saying that the two variables adjust in a manner restoring an imbalance between land and sea temperatures once they have been pushed off the equilibrium state. However, the adjustment is not immediate, unless the product is -1.0. The smaller the coefficient the longer it takes.

Assume now that $\alpha_{12} = 0$. In this case, the sea temperature is not exhibiting any feed-back effects from 'deviant' behavior in land and sea temperatures, whereas land temperature is. In that case, one would be inclined to say that sea temperature has influenced land temperature, but not the other way around. This would certainly be the case if the residual correlation between land and sea temperatures were zero, i.e. the matrix Ω_ε were diagonal, so there were no contemporaneous links. In our example, the correlation coefficient is 0.41 and one would have to be more careful about such 'causal' interpretations. The interpretation of the parameter estimates is generally more straightforward when Ω_ε is diagonal, but unfortunately this is seldom the case; temperature shocks are often correlated, sometimes indicating an un-modeled causal link. For example, if an increase in land temperature affects the sea temperature within a few months (say), then this would show up as a residual correlation. The latter could then be accounted for by including the current change of the land temperature in the equation of the sea temperature .

4 The statistical adequacy of a VAR model

The cointegrated VAR model discussed here is based on the full-information maximum likelihood (FIML) approach. To derive FIML estimators and

[4] t values in brackets.

tests requires an explicit probability formulation of the model which according to (2) is the multivariate normality of the errors assumed to be uncorrelated and identically distributed. Suppose we estimate the model, and find that the residuals are not normally distributed, the residual variance is heteroscedastic instead of homoscedastic, or residuals exhibit significant autocorrelation, etc. The parameter estimates (based in this case on an incorrectly derived estimator) may not have any meaning, and since we do not know their 'true' properties, inference is likely to be hazardous. Therefore, to claim that conclusions are based on FIML inference is to claim that the empirical model is capable of accounting for all the systematic and random information in the data in a satisfactory way.

Thus, to understand when a VAR is an adequate description of reality, it is important to know the limitations as well as the possibilities of that model. The purpose of this section is, therefore, to demonstrate that a VAR model can be a convenient way of summarizing the information given by the autocovariances of the data under certain assumptions about the Data Generating Process (see Juselius, 2006, for details). However, the required assumptions may not hold in any given instance, so the first step in any empirical analysis of a VAR is to test if these assumptions are indeed appropriate.

There are essentially three crucial assumptions that need to be checked.

1. The stochastic properties given by $\varepsilon_t \sim \mathsf{IN}_p [0, \Omega_\varepsilon]$,
2. The order of integration of x_t,
3. The constancy of the parameters, $\Gamma_{1,}$ α, β

These conditions provide the model builder with testable hypotheses on the assumptions needed to justify the VAR.

4.1 Checking the stochastic specification

Unfortunately, the multivariate normality assumption is often not completely satisfied. This is potentially a serious problem, since, as already said, the statistical inference is only valid to the extent that the assumptions of the underlying model are correct. An important question is, therefore, whether it is possible to modify the standard VAR model to make it statistically more adequate when the assumptions fail. Simulation studies have demonstrated that statistical inference might be sensitive to the validity of some of the assumptions, such as parameter non-constancy, serially-correlated residuals and residual skewness (the more the worse), while moderately robust to others, such as excess kurtosis (fat-tailed distributions) and residual heteroscedasticity. Thus, it seems advisable to ensure that the first three are roughly valid. As a first check, it is often useful to calculate descriptive statistics combined with a graphical inspection of the residuals and then undertake formal mis-specification tests of each key assumption. Once the reason why a model fails to satisfy the assumption is understood, one can often modify it to end with a well-specified model. This will be illustrated below for the land-sea climate data.

The IN distributional assumption implies that all information about land and sea temperatures should have been modeled by the conditional mean, in the sense that the deviation between the actual outcome x_t and the model expectation $\mathsf{E}_{t-1}[x_t|\mathbf{X}^0_{t-1}]$ is a white-noise residual, not explicable by the past of the process and $\mathbf{X}^0_{t-1}$ contains all relevant past information. By way of contrast, a VAR with autocorrelated residuals would describe a model which has not used all information in the data as efficiently as possible. This is because we could do better by including the systematic variation left in the residuals, thereby improving the accuracy of the model's predictions about the temperature change. Checking the assumptions of the model, (i.e., checking the white-noise requirement of the residuals, and so on), is not only crucial for correct statistical inference, but also for the interpretation of the model as an adequate description of climate change.

As already mentioned, the white-noise assumption is often rejected for the first, tentatively estimated, model and one has to modify the specification of the VAR model accordingly. This can be done, for example, by:

- investigating parameter constancy (e.g., 'is there a structural shift in the model parameters'?);
- increasing the information set by adding new variables;
- changing the temporal or spacial aggregation;
- increasing the lag length;
- changing the sample period;
- adding dummies to account for significant meteorological events, volcanic errruptions, etc.;
- conditioning on weakly or strongly exogenous variables (forcing variables);
- checking the adequacy of the measurements of the chosen variables.

Table 1: Misspecification tests of the first VAR model

Multivariate normality test:					$\chi^2(4) = 3.49[0.48]^{*)}$
Multivariate ARCH(1) test:					$\chi^2(9) = 44.01[0.00]$
Multivariate autocorrelation LM(1) test:					$\chi^2(4) = 31.04[0.00]$
Univariate tests:					
	Skew.	*Kurt.*	*Norm.*	*ARCH*	R^2
$\Delta C_{l,t}$	-0.14	2.98	1.58[0.54]	14.24[0.00]	0.30
$\Delta C_{s,t}$	-0.10	3.47	5.61[0.06]	6.68[0.04]	0.09

$^{*)}$ p-values in brackets

- transforming the data, for example into logs.

Table 1 reports a number of mis-specification tests (for a detailed discussion see Juselius, 2006, Chapter 4) that can be used to assess the assumption that $\varepsilon_t \sim \mathsf{IN}_p [0, \Omega_\varepsilon]$. The multivariate normality assumption seems fully acceptable, whereas the presence of multivariate ARCH (AutoRegressive Conditional Heteroscedasticity), cannot be rejected. More seriously, the model rejects that the errors are uncorrelated, a fairly important condition, as all χ^2, F, and t tests are based on the assumption of independent normal errors. However, when the sample is long even tiny deviations from the null hypothesis can become significant. As the model's inference is likely to be fairly robust to the presence of tiny autocorrelations, it is often useful also to check the magnitude of these coefficients. Figures 4 and 5 illustrate. A few things stick out: first the autocorrelations are generally small, despite significant. However, for sea temperature they seem uncorrelated at smaller lags, but become significant after a very long lag of roughly 70 years. Thus, the sea temperature seems to have a built-in long time dependence. Second, even though the residual seem normally distributed, there are a few outliers sticking out: one in 1767 that affects both land and sea temperature, one in 1815 affecting the land and the sea temperature followed by another one in 1816 primarily affecting land temperature. The latter outliers coincide with a volcanic erruption and will no longer be an outlying residual when this variable is included as a forcing variable in Section 8. The effect of the 1767 outlier can also be seen in the cointegration relation in Figure 3 where the trained eye can detect a slight change in the mean of the cointegration relation. In the next subsection I'll discuss how to account for these effects.

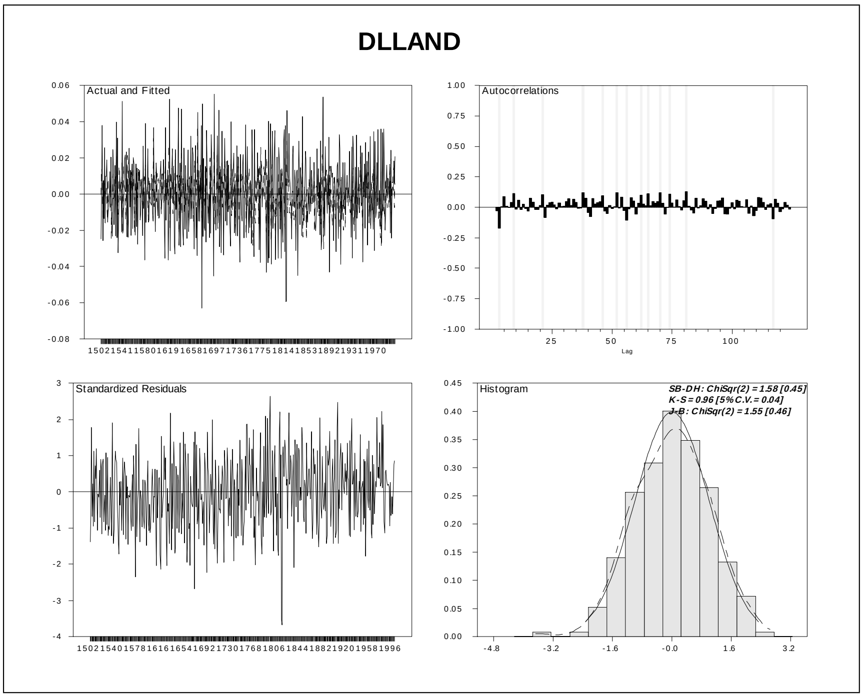

Figure 4: A graph of the land temperature residuals from model (7) (lower left hand side panel), the residual autocorrelogram (upper right hand panel) and the residual normality plot (lower right hand side panel).

4.2 Accounting for outlier observations

The above extraordinary events need to be properly accounted for as the model otherwise will suffer from specification failure. As mentioned in Section 3, to do it correctly it is crucial to understand how they affect the equations (defined in terms of differenced variables) and the cointegration relations, i.e. the equilibrium errors (defined in terms of the levels of the variables). For example, a large 'blip' outlier in the residuals corresponds to an extraordinary large change in the equations,

Table 2: Estimates of the outlier effects

	impulse dummies				step dummy
	1767	1815	1816		1767
$\Delta C_{l,t}$:	0.02[5] [1.4]	**−0.07** [−4.0]	**−0.07** [−4.3]	$\beta' x_t$:	**−0.01** [−5.7]
$\Delta C_{s,t}$:	**0.01** [3.4]	**−0.01** [−3.9]	**−0.01** [−2.9]		

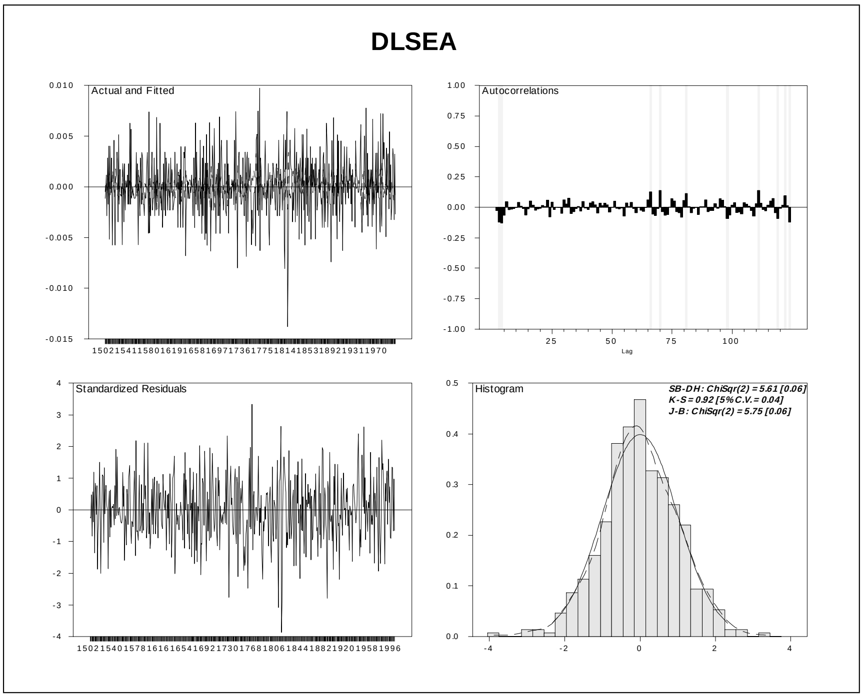

Figure 5: A graph of the sea temperature residuals from model (7) (lower left hand side panel), the residual autocorrelogram (upper right hand panel) and the residual normality plot (lower right hand side panel).

whereas it corresponds to a large shift in the level of the variables (see Juselius, 2006 Chapter 6, for a detailed discussion) which may or may not cancel in the cointegration relations. A large 'blip' outlier among the temperature changes is best accounted for by a blip (impulse) dummy ($D_{p,t} = \cdots$,0,0,0,1,0,0,$\cdots$) and a level shift in the cointegration relations by a shift (step) dummy ($D_{s,t} = \cdots$,0,0,0,1,1,1,$\cdots$). Including these effects in the present model means that we can test whether they are significant or not. It turned out that the volcanic erruption in 1815 did not change the equilibrium level significantly, whereas the one in 1767 did. Table 2 reports the estimates and shows that the 1767 event was causing a significant increase particularly in the sea temperature, which subsequently changed the relationship between land and sea temperature in the following way:

$$C_{l,t} = 3.93 C_{s,t} + 0.01 D_s 1767 + const.$$

The volcanic erruption in 1815 caused both the land and sea temperature to drop with an additional effect in the next year which was more significant for the land temperature than for the sea temperature.

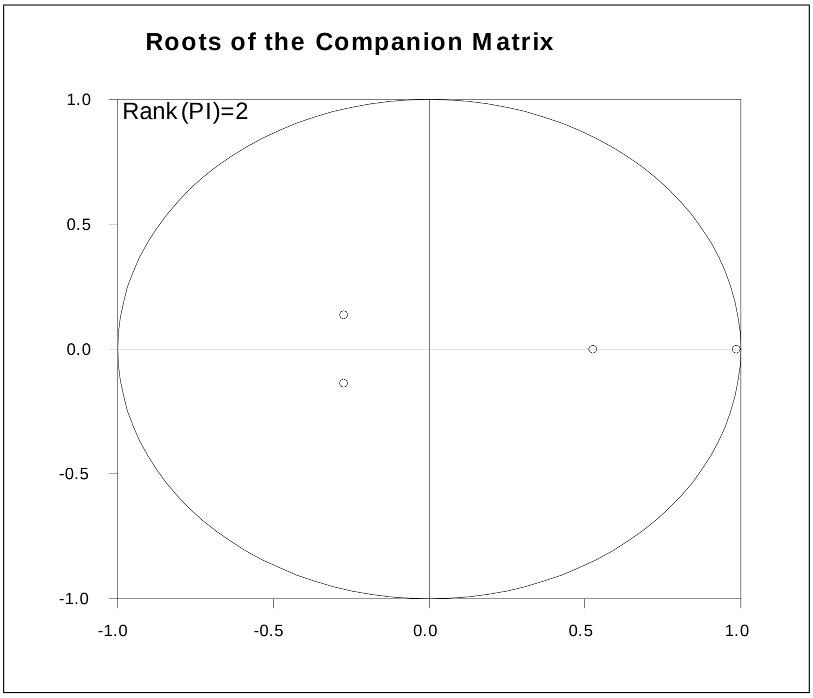

Figure 6: The roots of the model with dummies

One can ask whether it is important to take care of these effects? Adding the dummies has clearly improved the statistical model specification: the LM(1) test of autocorrelated errors is now reasonably acceptable with a test value of $\chi^2(4) = 7.16[0.13]$[6] (the long dependence in sea temperature is, however, still there); the test for ARCH is also acceptable with a test value of $\chi^2(9) = 12.5[0.19]$; the test for normality is improved with a test value of $\chi^2(4) = 2.84[0.58]$. In this sense the reliability of the statistical inferences is now improved. In terms of the estimated coefficients, most results were very robust, for example, the estimated coefficient in the cointegration relationship changed from 4.20 to 3.93 which does not seem much. It is important that the statistical model is correctly specified, as the subsequent analyses are based on this model.

4.3 Stability and unit-root properties

Up to this point, I have discussed the VAR model discussed as if it were stationary, i.e., without considering unit roots.[7] The dynamic stability

[6] p-values in the brackets.

[7] One can always estimate the unrestricted VAR with OLS, but if there are unit roots in the data, some inferences are no longer standard, as discussed in Hendry

of the process in (2) can be investigated by calculating the roots of:

$$\left(I_p - \Pi_1 L - \Pi_2 L^2\right) x_t = \Pi(L) x_t,$$

where $L^i x_t = x_{t-i}$. Define the characteristic polynomial:

$$\Pi(z) = \left(\mathbf{I}_p - \mathbf{\Pi}_1 z - \mathbf{\Pi}_2 z^2\right).$$

The roots of $|\mathbf{\Pi}(z)| = 0$ contain all necessary information about the stability of the process and, therefore, whether it is stationary or non-stationary. In econometrics, it is more usual to discuss stability in terms of the companion matrix of the system, obtained by stacking the variables such that a first-order system results. Ignoring deterministic terms, we have:

$$\begin{pmatrix} x_t \\ x_{t-1} \end{pmatrix} = \begin{pmatrix} \Pi_1 & \Pi_2 \\ I_p & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix}, \tag{9}$$

where the first row is the original VAR model, and the second merely an identity for x_{t-1}. Now, stability depends on the eigenvalues of the coefficient matrix in (9), and these are precisely the roots of $|\Pi(z^{-1})| = 0$. For a p-dimensional VAR with 2 lags, there are $2p$ eigenvalues. The following results apply:

(a) if all the eigenvalues of the companion matrix are inside the unit circle, then $\{x_t\}$ is stationary;

(b) if all the eigenvalues are inside or on the unit circle, then $\{x_t\}$ is non-stationary;

(c) if any of the eigenvalues are outside the unit circle, then $\{x_t\}$ is explosive.

For the bivariate land-sea temperature VAR(2) model, we have $2 \times 2 = 4$ roots, the moduli of which are:

$$1.0,\ 0.64,\ 0.32,\ 0.32$$

Figure 6 illustrates these in the unit circle.

We note that the system is stable (no explosive roots), that there is one unit root suggesting the presence of one common stochastic (unit root) trend, and three fairly small roots two of which come as a pair of complex roots.

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and Juselius (1999).

5 Determining cointegration rank

Since there is one root on the unit circle, $x_t \sim I(1)$. The first condition, needed to ensure that the data are cointegrated, is that Π has reduced rank $r < p$, so can be written as:

$$\Pi = \alpha\beta' \tag{10}$$

where α and β are $p \times r$ matrices, both of rank r. Substituting (10) into (3) delivers the cointegrated VAR model:

$$\Delta x_t = \Gamma \Delta x_{t-1} + \alpha \left(\beta' x_{t-1}\right) + \mu + \varepsilon_t. \tag{11}$$

We have the following cases:

1. If $r = p$, then x_t is stationary, so standard inference (based on t, F, and χ^2) applies.

2. If $r = 0$, then Δx_t is stationary. Each x_t has its own stochastic trend and it is not possible to obtain stationary relations between the levels of the variables by linear combinations. Such variables do not have any cointegration relations, and hence, cannot move together in the long run. In this case (11) becomes a VAR model in differences but, since $\Delta x_t \sim \mathsf{I}(0)$, standard inference still applies.

3. If $p > r > 0$, then $x_t \sim \mathsf{I}(1)$ and there exist r directions in which the process can be made stationary by linear combinations, $\beta' x_t$.

To find the value of $\widehat{\beta}$ that maximizes this likelihood function one has to solve an eigenvalue problem (Johansen, 1988). The solution delivers p eigenvalues λ_i where $0 \leq \lambda_i \leq 1$:

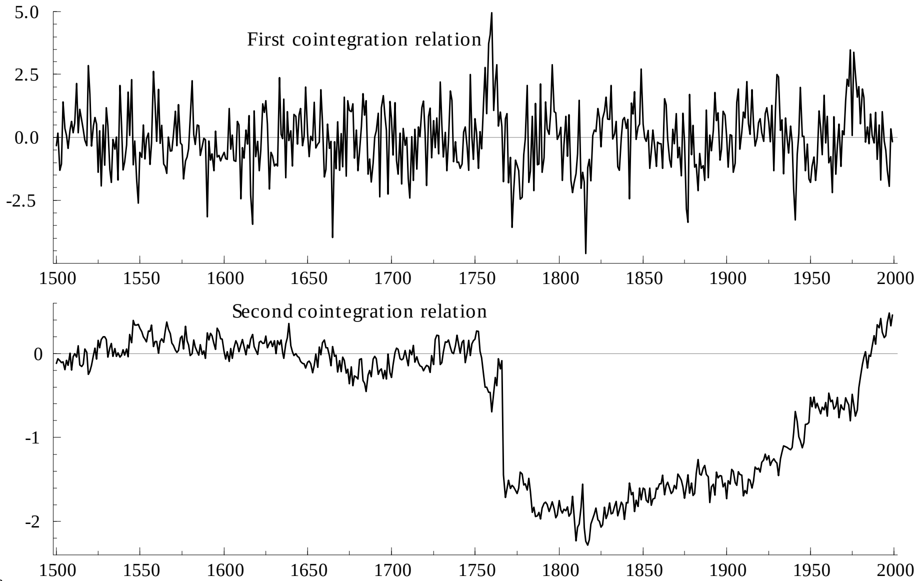
$$\boldsymbol{\lambda}' = (\lambda_1, \lambda_2, \ldots, \lambda_p), \tag{12}$$

which are ordered such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$. Each λ_i is associated with an eigenvector which gives the estimates of the cointegration relation β_i. Furthermore, each λ_i can be interpreted as the squared canonical correlation between a linear combination of the variables in levels, $\beta_i' x_{t-1}$, and a linear combination of the differenced variables, $\varphi_i' \Delta x_t$. In this sense, the magnitude of λ_i is an indication of how strongly the linear combination $\beta_i' x_{t-1}$ is correlated with the stationary part of the process $\varphi_i' \Delta x_t$. If $\lambda_i \approx 0$, the linear combination $\beta_i' x_{t-1}$ is not at all correlated with the stationary part of the process and, hence, is non-stationary. If $\lambda_i = 1.0$, the corresponding $\beta_i x_t$ is perfectly correlated with the stationary part of the model, and hence has to be stationary.

Table 3: Rank determination

Rank determination					The two largest roots			Adjustment	
λ_i	r	$p-r$	$Trace_i$	Q_{95}	$(r=2)$	$(r=1)$		$\mathbf{t}_{\alpha_{1.}}$	$\mathbf{t}_{\alpha_{2.}}$
0.22	0	2	126.4	26.5	0.99	1.00	$\Delta C_{l,t}$	9.7	1.3
0.01	1	1	5.2	12.9	0.54	0.54	$\Delta C_{s,t}$	2.4	2.2

rel



7.pdf

Figure 7: The estimated cointegration relations between land and sea temperature

Table 3 shows that λ_2 is essentially zero, implying that the second relation $\beta_2' x_t$ is a nonstationary process, whereas λ_1 is larger ($\sqrt{0.22} \simeq 0.47$) implying that the correlation of $\beta_1' x_t$ with the stationary part of the process is 0.47. This is illustrated in Figure 7 where we have graphed the two relations. Obviously, the first relation is stationary whereas the second one is not. Thus, the first one can be interpreted as a stationary equilibrium error, $\beta_1' x_t = e_t$, in the following relationship:

$$C_{l,t} = 3.93 C_{s,t} + 0.01 Ds1767 + const + e_t$$

The equilibrium error (i.e. the cointegration relation) in the upper panel is then:

$$e_t = C_{l,t} - 3.93 C_{s,t} - 0.01 Ds1767 - const = \beta_1' x_t.$$

A correct choice of the cointegration rank is crucial for the analysis, but unfortunately often difficult in practice. The following information is often useful when deciding on the choice of cointegration rank:

1. the trace test for cointegration rank;

2. the characteristic roots of the model: if the $r^{th}+1$ cointegration vector is non-stationary and is wrongly included in the model, then the largest characteristic root will be close to the unit circle;

3. the t-values of the α-coefficients for the $r^{th}+1$ cointegration vector; if these are all small, say less than 3.0, then one would not gain much by including that vector as a cointegrating relation in the model;

4. the graphs of the cointegrating relations: if the graphs reveal distinctly non-stationary behavior of a cointegration relation, which is supposedly stationary, one should reconsider the choice of r, or find out if the model specification is in fact incorrect;

5. the interpretability of the results.

5.1 The trace test

The likelihood ratio test, called the trace test (see Johansen, 1996, or Juselius, Chapter 8, for its derivation) is used to distinguish between those λ_i which corresponds to stationary relations (the cointegration relations) and those which corresponds to nonstationary relations. It has a nonstandard distribution which has been tabulated by simulations. However, there is not just one table to consult as the asymptotic distributions depend on whether there is a constant and/or a trend in the model; and whether these are restricted to the cointegration relations. Also other deterministic components, such as dummy variables, are likely to influence the shape of the test distributions. In particular, care should be taken when a deterministic component generates trending behavior in the levels of the data. A shift dummy $(\cdots,0,0,0,1,1,1,\cdots)$ restricted to the cointegration relations (as in our example) will also change the asymptotic distributions. In this case the asymptotic distributions need to be simulated for each case as they depend on the number and positions of the shift dummies in the model (see Johansen et.al. 2000 or Juselius, 2006 Chapter 8.3). Thus, before applying the trace test it is important to make sure that the empirical model is well-specified.

In the present example, the sample period is fairly long and the asymptotic distributions are likely to be very precise. But, when the size of the sample is small the tabulated asymptotic distributions can be rather poor approximations as has been demonstrated in many papers (Johansen, 2002). Another reason for concern is that when using correct small-sample distributions for the trace test, the size of the test is correct, but the power can be low, sometimes even of the same magnitude as the size when a stationary root is close to the unit circle. In such cases, a 5% test procedure will reject a unit root incorrectly 5% of the time, but accept a unit root incorrectly 95% of the time!

In our example, the trace test should be able to discriminate between the following alternatives: no unit roots, one unit root, or two unit roots. The first case corresponds to temperatures being stationary, the second to them being I(1) with one stationary cointegration relation, and the last case to them being I(1) but with no cointegrating relation between them. The trace test procedure starts from the top of the table and moves down until $Trace_i \leq Q_{95}$.

The first trace test statistics in Table 3 is larger than the 95% quantile ($126.4 > 26.5$), so λ_1 has to be considered different from zero, suggesting that there exists at least one stationary relation between the temperatures. The second trace statistics is smaller than the 95% quantile ($5.2 < 12.9$), so λ_2 cannot be considered different from zero. This tells us we cannot reject the presence of one unit root, and hence we accept one common stochastic trend and one cointegration relation. Thus, the trace test is consistent with the information in the graphs.

5.2 Other criteria

The roots of the characteristic polynomial (see Juselius, 2006, Chapter 3.6) shows that the choice of $r = 2$ would leave a very large root (0.99) in the model. This would make the statistical inference highly unreliable as standard χ^2, F, and t tests are derived for stationary data. For the choice of $r = 1$, the largest root in the model is 0.54[8] which is fully acceptable. Thus the characteristic roots also point to the choice of $r = 1$. Finally the t values of the adjustment coefficients α shows that for $r = 1$ there is very significant adjustment in the land temperature equation (and less significant in the sea temperature equation), whereas for $r = 2$ the t values are not large enough. This is because when the second relation is nonstationary we should not use Student's t values, but something like Dickey-Fuller τ values (which are higher, see Dickey and Fuller, 1979). Thus, also in this case the information points to $r = 1$. The interpretation of the results is that land temperature is primarily adjusting and that changes in sea temperatures have been pushing.

[8] Note that, the last relation, $\beta_2' x_t$, which is now classified as unit root nonstationary, will not be included in the model when $r = 1$.

However, this conclusion is without taking account of the current correlation coefficient of 0.39. Furthermore, the results are interpreted under the usual *ceteris paribus* (everything else constant) assumption. Adding new relevant variables might, of course, change the interpretation of the results to some extent. Sections 8-9 will discuss the results for a model with a number of forcing variables included.

Altogether, we were fortunate in this case, as the choice of rank was completely straightforward. This is far from always the case. In doubtful cases it is always advicable to do some sensitivity analyses to find out if important information is lost by leaving out the $r^{th}+1$ cointegration vector, or if anything is gained by including it.

6 The VAR model in moving-average form

For the initial values of the process, x_0, we can express x_t as:

$$x_t = \mathbf{C}\sum_{i=1}^{t}\boldsymbol{\varepsilon}_i + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t + x_0 + \text{ determ.comp.} \tag{13}$$

In (13), x_t is decomposed into a stochastic trend, $C\sum_{i=1}^{t}\varepsilon_i$, and a stationary stochastic component, $C^*(L)\varepsilon_t$.[9] See Johansen (1996) or Juselius (2006, Chapters 5 and 6).

An important feature of 'reduced rank' matrices like α and β is that they have orthogonal complements, which we denote by $\alpha_\perp$ and $\beta_\perp$: i.e., $\alpha_\perp$ and $\beta_\perp$ are $p\times(p-r)$ matrices orthogonal to α and β (so $\alpha_\perp'\alpha=\mathbf{0}$ and $\beta_\perp'\beta=\mathbf{0}$), where the $p\times p$ matrices $(\alpha\ \alpha_\perp)$ and $(\beta\ \beta_\perp)$ both have full rank p. These orthogonal matrices play a crucial role in understanding the relationship between cointegration and 'common trends' as we explain below (a simple algorithm for constructing $\alpha_\perp$ and $\beta_\perp$ from α and β is given in Hendry, 1995).

There are $(p-r)$ linear combinations between the cumulated residuals, $\alpha_\perp'\sum_{i=1}^{t}\widehat{\varepsilon}_i$ which define the common stochastic trends that affect the variables x_t with weights $B=\boldsymbol{\beta}_\perp(\boldsymbol{\alpha}_\perp'\boldsymbol{\Phi}\boldsymbol{\beta}_\perp)^{-1}$ so that $C=B\alpha_\perp'$. In this sense, there exists a beautiful duality between cointegration and common trends. The following example illustrates.

[9] It can be shown that:

$$C=\beta_\perp(\alpha_\perp'(I-\Gamma_1)\beta_\perp)^{-1}\alpha_\perp', \tag{14}$$

so the C matrix can be calculated from estimates of α, β, and Γ_1 see e.g., Johansen (1992). Letting $B=\beta_\perp(\alpha_\perp'(I-\Gamma_1)\beta_\perp)^{-1}$, then $C=B\alpha_\perp'$, so the common stochastic trends have a reduced-rank representation similar to the stationary cointegration relations.

Assume that there exists one common trend between the two temperature series, and hence one cointegration relation. Then, $r = 1$ and $p - r = 1$, and we can write the moving-average (common-trends) representation as:

$$\begin{bmatrix} C_{l,t} \\ C_{s,t} \end{bmatrix} = \underbrace{\begin{bmatrix} 2.72 \\ 0.69 \end{bmatrix}}_{B} \sum_{i=1}^{t} \hat{u}_i + C^*(L) \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \tag{15}$$

where $B' = (b_{11}, b_{21})$ are the weights of the estimated common trend given by $\widehat{u}_i = \alpha'_{\perp}\widehat{\varepsilon}_i = \varepsilon_{s,i} + 0.04\varepsilon_{l,i}$. Thus, the common stochastic trend seems primarily to derive from permanent sea temperature shocks (changes) over this period. The significance of the two shocks in the expression for $\alpha'_{\perp}\widehat{\varepsilon}_i$ can be measured by their t ratio, which are 2.4 and 15.8 for land and sea temperature, respectively.

7 Parameter constancy

Constancy of the model parameters is fundamental for inference, theory, and simulations. Many tests exist for that hypothesis, focussing on different aspects of the model (see, for example, Hendry 1996). To check the present model for parameter constancy, I have applied the various tests in Hansen and Johansen (1999) (described in Juselius, 2006, Chapter 9 and implemented in CATS for RATS) developed specifically for the cointegrated VAR model.

Ideally, the recursive tests help to diagnose problems in the model that can be remedied. In other cases the recursive tests continue to signal non-constancies in the model. In the latter case, it might still be useful to continue the empirical analysis, albeit keeping in mind that the estimated parameters measure average effects. In particular, it is important to remember that applied tests might produce unreliable results as the underlying assumptions of the model are not satisfied. One useful way of thinking of the recursive tests is that they can provide a general assessment of the confidence we place on the conclusions from the model.

All the recursively calculated tests in CATS for RATS start from a baseline model estimated for a sub-sample period, $1, ..., T_1$, where $T_1 < T$, and then recursively extending the end point of the recursive sample, t_1, until the full sample is covered, i.e. $t_1 = T_1, T_1 + 1, ..., T$.

As it would be excessive to present all available tests here, I'll focus on the recursively calculated Max loglikelihood function and the recursively calculated coefficient between land and sea temperature as well as the adjustment coefficients in respective equations. They summarize

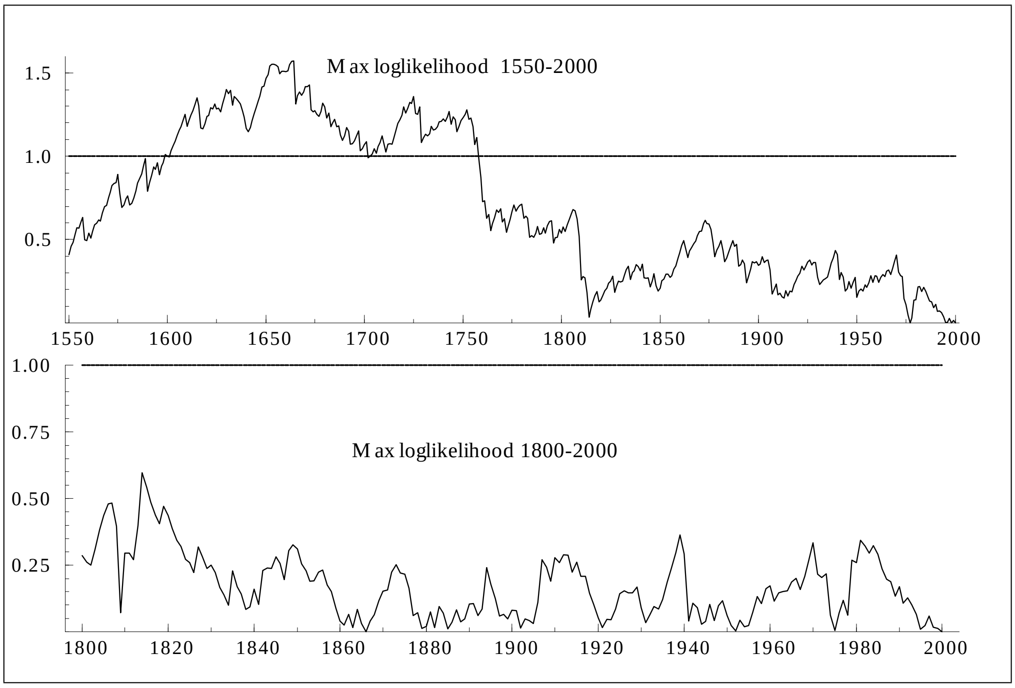


Figure 8: The recursively calculated likelihood function divided by the 95% quantile.

the essential information on the stability (and the lack thereof) of the parameters of the model.

The 'Max likelihood function' test measures the overall constancy of the model and can be compared with the Chow tests of parameter constancy (Chow, 1960) in the single regression model. The recursive graph in Figure 8, upper panel is based on the sample 1500-2000 ($T = 500$), starting from the baseline sample 1500-1550 ($T_1 = 50$). The test statistic has been divided by the 95% quantile so that constancy is rejected at the 5% level when the graph is above the unit line.

It appears that the model shows some evidence of instability in the first period. To check whether the model is more stable in 1767-2000 (which is the sample period to be used in the last part of this paper), the same function was calculated for the latter part and reported in the lower panel of Figure 8. Here the sample period is 1770-2000 ($T = 230$) with baseline sample 1770-1800 ($T_1 = 30$). No failure of model constancy can be detected in the second part of the sample period.

Figure 9 shows a similar picture. The β coefficient was reasonably stable (roughly 2.8) until approximately 1650, after which it increased and stayed at a new level (roughly 3.2) until 1750, after which it returned

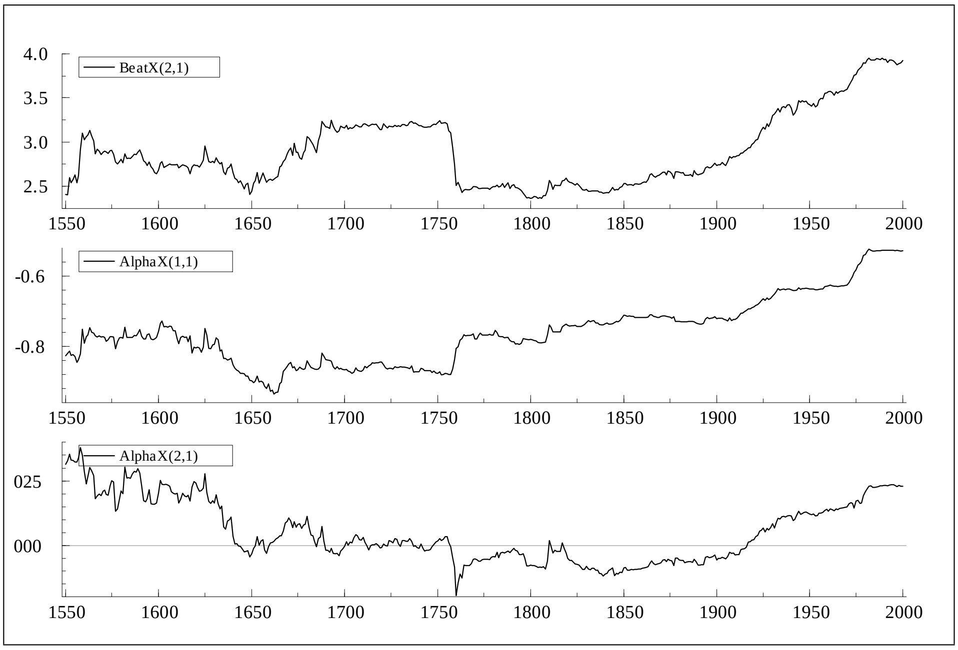


Figure 9: The graphs of the recursively calculated β coefficient between land and sea temperature (upper panel) and the adjustment coefficients, α, in the land temperature equation (middle panel) and the sea temperature equation (lower panel).

to the previous level. From the beginning of twentieth century the coefficient has started to increase again, ending a the full sample value of roughly 4.0. Similar results can be seen for the α coefficients. The α_{11} coefficient (middle panel) shows that the adjustment in land temperature became faster in the period 1650-1750, but has declined since then. The α_{12} coefficient (middle panel) shows that the adjustment in sea temperature was not different from zero in the period 1650-1900, but has gradually become so ending with the full sample estimate of roughly 0.02 (still very tiny).

How should we interpret the finding that the model parameters are not completely stable? First a word of caution: after the first rejection of constancy, all subsequent tests may cease to have a meaning. This is because the tests are derived under the null of constant parameters up to time $T_1 + t_1$. If there is a structural break at $T_1 + t_1$, then the remaining tests are derived under an incorrect null hypothesis. In practice, it is not uncommon to see a rejection of constancy for a short period, for example in connection with a special event or a new regime (cycle), but then the graphs return back to the acceptance region. This is more or less what seems to be the case here. One way of interpreting the results is that there are some long cycles in the land and sea temperatures during which the relationships changes to some extent. However, there might also be a new cycle at the end of the sample, the magnitude and length of which would of course be of extreme interest.

To summarize: The recursive analysis suggests that the basic relationships are relatively robust, but it also suggests that there is some need for fine-tuning the VAR model to properly account for long cyclical variations. This, however, is outside the aim of this paper. I will, however, continue the analysis focussing on the period after 1767 as this seemed to be more stable than the full period.

8 The VAR model with forcing variables

In the last part of the paper I'll include a number of forcing variables in model with the aim of investigating which (if any) can be associated with the stochastic long-run trend in land and sea temperatures. The forcing variables introduced in Section 2 and graphed in Figures 1 and 2 consist of solar radiation, three greenhouse gasses (CO2, CH4, and N20), and aerosols from volcanic erruptions.

Among these variables only solar radiation can potentially qualify as a stochastic I(1) variable as its first difference looks reasonably close to a stationary variable with fixed mean and constant variance. However, this is the case only from the middle of the 1700 century. As we already found evidence of a structural break in 1767 in the mean of the cointegration relation between land and sea temperature in Section 4.2 as well as of parameters nonconstancy in the period after 1767, I decided to restrict the analysis to the sample 1767-2000. For this period, the stochastic variation of the changes in solar radiation looks almost normally distributed and the VAR model was able to describe its variation with acceptable precision. For this reason I decided to include solar radiation in the VAR as a system variable. The advantage is that one can then test whether it is a proper forcing variable instead of just assuming it. For example, if we find that the sea and land temperature are causing solar radiation, it would imply that either the VAR model is not appropriate for analyzing climate data or that some important variables are missing. In this sense including solar radiation as a VAR variable in its own right is a way of validating the results.

The greenhouse gasses, on the other hand, are too far from being normally distributed variables to qualify as a VAR variable in their own right. This means that they have to enter the VAR model in a similar manner as a time trend. Thus, the forcing variables should be allowed to enter the cointegration relations, and their differences should be allowed to enter the equations as explained below. The aerosols from volcanic erruptions exhibit no trending behavior over time and look similar to a series of impulse dummies. From a statistical point of view, such a series would best be treated as a dummy variable with the erruptions as extraordinary impulses.[10] In the present analysis they will have the role of a (series of) dummy variable(s).

8.1 Specification and rank determination

The extended VAR model is now specified as:

$$\Delta x_{1,t} = A\Delta x_{2,t} + \Gamma_1 \Delta x_{t-1} + \alpha\beta' x_{t-1} + \alpha\beta_0 + \Phi V_t + \varepsilon_t \qquad (16)$$

where $x'_t = [x'_{1,t}, x'_{2,t}]$, $x'_{1,t} = [C_{l,t}, C_{s,t}, Sr_t]$, $x'_{2,t} = [G1_t, G2_t, G3_t]$, and Sr_t measures solar radiation, $G1_t, G2_t, G3_t$ measure greenhouse gasses; $G1_t$ is CO_2, $G2_t$ is CH_4, $G3_t$ is N_2O, and V_t measures aerosoles from volcanic erruptions. A first general test of significance of the forcing variables showed that CO_2 was individually long-run excludable with a p-value of 0.40, CH_4, with a p-value of 0.25 and N_2O with a p-value of 0.56. The joint test of the long-run exclusion of CO_2 and N_2O was also accepted with a p-value of 0.69, but the joint exclusion of CH_4 with the other gasses was rejected. Exactly the same result (but with slightly lower p-values) was obtained when relating absolute temperature values to log transformed greenhouse gasses. We continue the analysis with CH_4 as the only greenhouse gas in the model. Solar radiation was strongly rejected as long-run excludable and, thus, is kept as a system variable in the VAR.

The model, reformulated with $x_{2,t} = G2_t$, is reasonably well specified: the multivariate LM(1) test of autocorrelated residuals suggested independent errors based on $\chi^2(9) = 12.66[0.18]$; the multivariate normality test was borderline rejected based on $\chi^2(6) = 13.66[0.03]$ as was the ARCH LM(1) test based on $\chi^2(36) = 60.93[0.01]$. However, as Table 4 shows, the borderline rejection was exclusively due to the equation for solar radiation, both normality and ARCH were fully acceptable (with high p-values) in the equations for land and sea temperatures. As solar radiation subsequently will be shown to be a forcing variable, this minor deviation from the assumptions should be of no importance. It was notable that the long time dependence in the residuals of the sea temperature evident in Figure 5 was no longer visible. Thus, it seems likely

[10] If, on the other hand one would like to test the hypothesis that volcanic outbreaks have had a significant long-run effect on the climate, one would first have to cumulate the volcanic aerosols over time, and then allow them to enter as a forcing variable in the cointegration relations.

Table 4: Specification tests

Rank determination				The three largest roots		Specification tests			
λ_i	r	$p-r$	$Trace_i$	$r=2$	$r=3$		$t_{\hat{\alpha}_{3.}}$	$ARCH$	$Norm.$
0.35	0	3	145.8 [0.00]	1.00	0.83	$\Delta C_{l,t}$:	-0.7	0.88 [0.64]	1.05 [0.59]
0.12	1	2	46.1 [0.00]	0.78	0.76	$\Delta C_{s,t}$:	0.2	2.35 [0.31]	2.11 [0.35]
0.07	2	1	16.4 [0.01]	0.36	0.54	ΔSr_t:	-4.1	10.72 [0.00]	10.55] [0.01

that the long dependence in the sea temperature is some way related to solar radiation.

The smallest eigenvalue in the first column of Table 4, 0.07, indicates that the third cointegration relation is only weakly correlated with the stationary process. Nonetheless, it is significantly different from a unit root process based on a p-value of 0.01 (implying that the largest unrestricted characteristic root, 0.83, for $r = 3$, is significantly different from one). Why does the trace test find that the small value of $\lambda_3 = 0.07$ is still significantly different from zero? The simple explanation is that the sample size is quite large, 221 annual observations. Because the trace test is calculated as $T \ln(1-\boldsymbol{\lambda})$, even a small deviation from zero can be found to be significant when T is large enough. However, when there is a near unit root in the model, inference is much closer to the so called Dicky–Fuller distributions than to standard t-, F-, and χ^2-distributions. Hence, to make inference more robust, it is often a good idea to approximate a near unit root by a unit root even when it is found to be statistically different from one (see Hendry and Juselius, 1999).

Before finally deciding about the rank, we first check the interpretability of the third cointegration relation to see if it contains valuable information for the analysis. It appears from Table 4, the column of $t_{\hat{\alpha}_{3.}}$, that the land and sea temperatures do not adjust significantly to the third β relation, only solar radiation does. Also, solar radiation does not adjust to the first two β relations (the t values of the third α row are -0.8, -0.5, -4.1). Thus the third relation, probably a spurious relation between solar radiation and CH4, is only relevant in the equation for solar radiation. Thus, the choice of $r = 2$ will include all relevant information on how land and sea temperatures have adjusted to the forcing variables and we shall continue with this choice.

8.2 Weak exogeneity and partial models

Based on the first two cointegration relations, it is relevant to test whether solar radiation is a weakly exogenous variable in this system.

The hypothesis of weak exogeneity is that a variable (solar radiation) is influencing the long-run development of the other variables of the system (land and sea temperatures), but is not influenced by them through the long-run relations. If, in addition, a weakly exogenous variable is not affected in the short run by the other variables it is called a strongly exogenous variable. The latter is the statistical counterpart of a forcing variable. Weak exogeneity is often called the hypothesis of 'no levels feedback', or long-run weak exogeneity. It can be formulated as the following hypothesis on α:

$$\mathcal{H}_\alpha(r) : \alpha = A\widetilde{\alpha} \tag{17}$$

where α is $p \times r$, A is a $p \times s$ matrix, $\widetilde{\alpha}$ is an $s \times r$ matrix of nonzero α-coefficients and $s \geq r$. The condition $s \geq r$ implies that the number of non-zero rows in α must not be greater than r. This is because a variable that has a zero row in α is not adjusting to the long-run relations and, hence, its cumulated errors can be considered as a driving trend in the system, i.e., as a common stochastic trend. Since there can at most be $(p - r)$ common trends, the number of zero-row restrictions can at most be equal to $(p - r)$. In our case it means that at least two of the α rows have to be nonzero, or equivalently, at most one of the α rows can be zero.

The hypothesis (17) can be expressed as:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \widetilde{\alpha} \\ 0 \end{bmatrix}.$$

where α_1 contains the nonzero α rows, and α_2 the zero rows. The weak exogeneity hypothesis can be tested with a LR test procedure described in Johansen and Juselius (1990) or in Juselius (2006, Chapter 11). It is asymptotically distributed as χ^2 with the degrees of freedom $\upsilon = s \times r$, i.e., equal to the number of zero restrictions on the α-coefficients.

Because $p = 3$ and $r = 2$ in our model, there can at most be one weakly exogenous variable. The hypothesis that the α row of solar radiation is zero was accepted with a p-value of 0.75. Thus, solar radiation has been shown to be a proper forcing variable in this model. A different result would of course have raised serious doubts about the model's ability to adequately explain climate data.

When a zero-row restriction on α is accepted, we can partition the three variables into two variables (land and sea temperature) which exhibit long-run feedback, and one variable (solar radiation) with does not. Because an exogenous variable does not contain information about the long-run β parameters, we can obtain fully-efficient estimates of β from the two adjustment equations, conditional on the marginal models of the weakly-exogenous variable (see Engle, Hendry and Richard, 1983, Johansen and Juselius, 1992, and Juselius 2006, Chapter 11.2). This gives the condition for when partial models can be used to estimate β without losing information. Note that we did not test this condition for the greenhouse gasses as it would not have been possible to obtain a well-specified model for these variables.

In order not to loose any information (short-run as well as long-run) in a partial system, the conditioning variable(s) needs to be strongly exogenous. For example, weak exogeneity of solar radiation does not, as such, exclude the possibility that changes in land and sea temperatures have had a short-run impact on solar radiation, though such a result would be physically impossible. The estimates of the parameters in (16) are given in (18) where coefficients with a t-value > 2.0 are in bold face. The last row of Γ_1 matrix shows that there are no such significant effects. Thus, solar radiation is a valid forcing variable, theoretically as well as empirically.

$$\begin{bmatrix} \Delta C_{l,t} \\ \Delta C_{s,t} \\ \Delta S_t \end{bmatrix} = \underbrace{\begin{bmatrix} \underset{[-0.5]}{-0.03} & \underset{[2.7]}{\mathbf{0.93}} & \underset{[-1.3]}{-12.3} & \underset{[-0.5]}{-0.71} \\ \underset{[0.8]}{0.01} & \underset{[-2.4]}{\mathbf{-0.15}} & \underset{[0.6]}{1.00} & \underset{[-0.4]}{-0.11} \\ \underset{[-0.8]}{-0.0} & \underset{[-0.1]}{-.00} & \underset{[7.6]}{\mathbf{0.46}} & \underset{[-0.8]}{-0.01} \end{bmatrix}}_{\Gamma_1} \begin{bmatrix} \Delta C_{l,t-1} \\ \Delta C_{s,t-1} \\ \Delta S_{t-1} \\ \Delta G2_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \underset{[-0.0]}{-0.04} & \underset{[7.6]}{\mathbf{0.01}} \\ \underset{[-0.1]}{-0.04} & \underset{[6.4]}{\mathbf{0.00}} \\ \underset{[-0.9]}{0.01} & \underset{[-0.8]}{-0.00} \end{bmatrix}}_{A+\Phi} \begin{bmatrix} \Delta G2_t \\ V_t \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} \underset{[-10.3]}{\mathbf{-0.79}} & \underset{[3.5]}{\mathbf{1.09}} & \underset{[6.6]}{\mathbf{11.36}} & \underset{[7.7]}{\mathbf{0.13}} & \underset{[-7.0]}{\mathbf{-84.3}} \\ \underset{[-1.64]}{-0.02} & \underset{[-4.01]}{\mathbf{-0.23}} & \underset{[6.5]}{\mathbf{2.02}} & \underset{[6.4]}{\mathbf{0.02}} & \underset{[-6.5]}{\mathbf{-14.0}} \\ \underset{[0.9]}{0.00} & \underset{[-0.7]}{-0.00} & \underset{[-0.1]}{-0.00} & \underset{[-0.2]}{0.00} & \underset{[0.2]}{0.01} \end{bmatrix}}_{\Pi=\alpha\beta'} \begin{bmatrix} C_{l,t-1} \\ C_{s,t-1} \\ S_{t-1} \\ G2_{t-1} \\ const \end{bmatrix}_{t-1} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}, \qquad (18)$$

Most of the coefficients in Γ_1 are insignificant. The only exception is the lagged change in sea temperature, for which the coefficients in the land and sea temperature equations are significant. Thus, the statistical analysis tell us that there is both an immediate and delayed effect on land temperature from a change in the sea temperature. The coefficients in the last column of Γ_1 and the first column of the matrix A are all insignificant, implying that there are no strong immediate and lagged effects from changes in the methane gasses. Thus they only seem to have a long-run impact on the temperature. The coefficients in the matrix Φ measuring the impact from volcanic errruptions, shows a negative and highly significant effect on land and sea temperature.

Finally, the coefficients of the matrix $\Pi = \alpha\beta'$ are interesting as they measure the implicit long-run relation for land and sea temperature as a weighted average of the two cointegration relations. So why do we not present the α and the β estimates separately? Though the unrestricted β is uniquely determined based on the chosen normalization, they are not necessarily meaningful without (over-) identifying restrictions. The discussion of how to impose (over)identifying restrictions on β will be estimated and discussed in the next section. As $\Pi = \alpha\beta'$ is a unique representation of the combined long-run effects I prefer to discuss them at this stage.

The first row is interpreted as a relation between the land temperature and the other variables. It is useful to present it in the following form:

$$-0.79\{C_{l,t} - 1.4C_{s,t} - 14.4S_t - 0.16G2_t + 106.8\}$$

which tells us that land temperature adjusts fairly quickly (with an adjustment coefficient of -0.79) to a deviation from the equilibrium value. The latter is given by:

$$C_{l,t} = 1.4C_{s,t} + 14.4S_t + 0.16G2_t - 106.8 \tag{19}$$

We note that the coefficient to sea temperature is smaller when allowing for the effect of the forcing variables. I'll return to this result in the next section.

The second row in the Π matrix will be interpreted as a relation between sea temperature and the other variables. We express it similarly as:

$$-0.23\{C_{s,t} + 0.08C_{l,t} - 9.6S_t - 0.08G2_t + 61.0\}$$

and note that sea temperature adjusts more sluggishly to a deviation from the equilibrium value:

$$C_{s,t} = -0.08C_{l,t} + 9.6S_t + 0.08G2_t - 61.0. \tag{20}$$

We note that the negative coefficient to land temperature is insignificant. Thus, the result suggests that land temperature is significantly affected by the sea temperature and the adjustment is fast, whereas sea temperature is not adjusting to land temperature, but quite strongly to the forcing variables. In the next section, I'll use this information to impose identifying restrictions on the two β relations.

9 Identification of the pulling and pushing forces in the climate model

An important part of a long-run cointegration analysis is to impose (over)identifying restrictions on β to improve interpretability. Given the choice of cointegration rank (here $r = 2$), the Maximum Likelihood procedure gives the maximum likelihood estimates of the unrestricted cointegrating relations $\beta' x_t$. However, it is always possible to impose $r-1$ restrictions (here one) on each cointegration relation without changing the value of the likelihood function– no testing is involved for such 'restrictions'. See Juselius (2006, Chapter 12). Additional restrictions change the value and, thus, are testable.

9.1 The pulling forces

Hypotheses on the cointegration vectors can be formulated in the following way by specifying the s_i free parameters in each β vector,:

$$\beta = (\mathbf{H}_1\boldsymbol{\kappa}_1, ..., \mathbf{H}_r\boldsymbol{\kappa}_r),$$

where β is $(p_1 \times r)$, $\boldsymbol{\kappa}_i$ are $(s_i \times 1)$ coefficient vectors, and $\mathbf{H}_i$ are $(p_1 \times s_i)$ design matrices where p_1 is the dimension of x_{t-1} in (16). Thus, we use the design matrices to determine the s_i free parameters in each cointegration vector. Note that $m_i = p_1 - s_i$ is the number of imposed restrictions. Identifying and non-identifying restrictions can be tested by a likelihood-ratio procedure described in detail in Johansen and Juselius (1994) or Juselius (2006, Chapters 10, 12).

Our climate model has only two cointegration relations, so the number of interesting hypotheses to test is limited. Here, I shall test the hypothesis that sea temperature is a function exclusively of the forcing variables and that land temperature is a function exclusively of sea temperature (i.e. the latter is the same relation that already was found to be stationary in the first part of the paper[11]). The two design matrices are as follows

$$\mathbf{H}_1 = \begin{pmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,1 \end{pmatrix}, \quad \boldsymbol{\kappa}_1 = \begin{pmatrix} \kappa_{11} \\ \kappa_{12} \\ \kappa_{13} \end{pmatrix}, \quad \mathbf{H}_2 = \begin{pmatrix} 0\,0\,0\,0 \\ 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{pmatrix} \quad \boldsymbol{\kappa}_2 = \begin{pmatrix} \kappa_{21} \\ \kappa_{22} \\ \kappa_{23} \\ \kappa_{24} \end{pmatrix}$$

Thus, the first relation contains three parameters (of which the first one will be used for normalization). The second relation contains four parameters (of which the first will be used for normalization). Thus we

[11] Note that the cointegration property is invariant to changes in the information set.

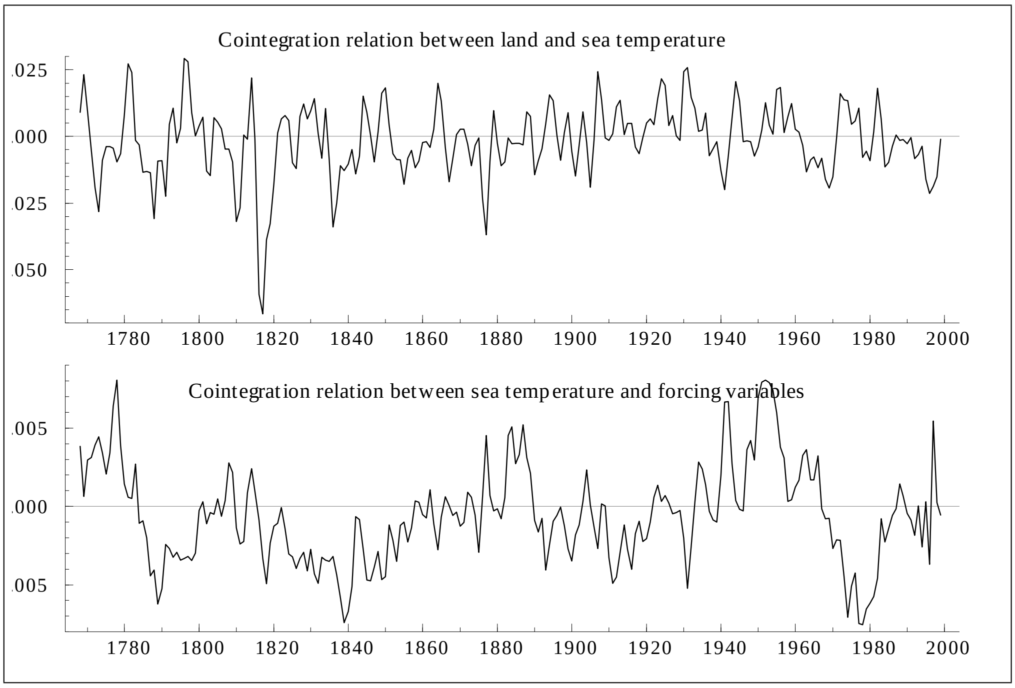


Figure 10: The graphs of the two identified cointegration relations

have imposed two restrictions on the first relation corresponding to the zero coefficients to the forcing variables, and one on the second relation, corresponding to a zero restriction on the land temperature. The hypothesis is tested by the LR test procedure described in Johansen and Juselius (1994). The test statistic value was 0.10, distributed as $\chi^2(1)$, and hence accepted with a p-value of 0.75. The two restricted cointegration relations and their adjustment coefficients are graphed in Figure 10 and reported in Table 5.

The first β relation is similar to the one being estimated in the first

Table 5: An identified system

	$C_{l,t}$	$C_{s,t}$	S_t	$G2_t$	$const$
β_1'	1.0	-3.97	0	0	9.24
		$[-24.5]$			[19.9]
β_2'	0	1.0	-5.81	-0.06	39.5
		[]	$[-4.8]$	[16.2]	[4.5]
	$\Delta C_{l,t}$	$\Delta C_{s,t}$	ΔS_t		
$a_{1,i}'$	-0.79	-0.02	0.00		
	$[-10.3]$	$[-1.7]$	[1.0]		
$a_{2,i}'$	-2.07	-0.31	0.00		
	[7.6]	$[-6.4]$	[0.1]		

Table 6: The estimated common trends representation

	ε_{C_l}	ε_{Cs}	ε_S
$\alpha'_{\perp,1}$	0.00 [0.9]	-0.00 [-0.4]	**1.0**
B'_1	**42.51** [8.5]	**11.94** [8.5]	**1.82** [8.5]
The estimated C matrix			
	$C_{l,t}$	$C_{s,t}$	S_t
$C_{l,t}$	0.03 [0.9]	-0.12 [-0.4]	42.51 [8.5]
$C_{s,t}$	0.01 [0.9]	-0.03 [-0.4]	11.94 [8.5]
S_t	0.00 [0.9]	-0.01 [-0.4]	1.82 [8.5]

part of this paper. There is a very significant adjustment (-0.79) in land temperature, but no significant adjustment in the sea temperature, nor in the solar radiation. The second relation shows that sea temperature is significantly related to solar radiation and methane. There is a very significant adjustment to this relation in land and sea temperature, whereas not in solar radiation. As the graphs in Figure 10 shows, the two cointegration relations are definitely stationary. Of course, if they were not it would be hard to interpret them as equilibrium errors. The fact that land temperature adjusts to both relations explains why the combined relation in the Π matrix (19) was different from the identified $\beta_{1.}$. The first row in the Π matrix is given by the linear combination of the two β relations, $-0.79\beta'_1 - 2.07\beta'_2$. The coefficients in (19) can be interpreted to mean that the long-run movements in land temperature are directly associated with the long-run movements of solar radiation and methane through its dependence on the sea temperature. The fact that the coefficient to land temperature is insignificant in (20), but the coefficient to sea temperature is significant in (19) suggests that solar radiation is first affecting sea temperatures, which is then affecting land temperatures. The combined effects in (19) suggests that some of the long-run movements in land temperature are indirectly related to solar radiation and methane, whereas some of it are directly related to the sea temperature.

9.2 The pushing forces

Based on the estimates on α and β it is possible to derive the estimates of the common trends matrices B, $\alpha_{\perp}$, and C, defined in Section 6. They are reported in Table 6:

Consistent with the weak exogeneity results, only the cumulated shocks to solar radiation explain significantly the long-run movements in land and sea temperatures. The total effect on land temperature is, however, larger than on sea temperature. As the variables are in logarithmic form, this is potentially quite interesting. In addition to the solar radiation, methane also acts as a forcing variable in this system. Because the fairly insignificant effects of short-run changes in methane on this system I shall, at this stage, refrain from providing some common trends effect of this variable.

10 Conclusions

This paper has tried to demonstrate that the cointegrated VAR approach can potentially be a useful method for the analysis of climate data. It is a rich model: the β_{ij} coefficients characterize long-run relationships between levels of variables; the α_{ij} coefficients describe changes that help restore an equilibrium position; the γ_{ij} coefficients describe short-term changes resulting from previous temperature changes; ϕ_i, describe extraordinary events, like a volcano erruption. Here I have analyzed the properties of this model in every detail, demonstrated the changes needed to validate inference procedures, and illustrated the powerful new modeling procedures based on land and sea temperature series and some of the most important forcing variables.

The results of the analysis suggested that sea temperatures drive land temperatures; the long-run trend in sea temperatures is strongly associated with solar radiation and to some extent to the level of methane. It was not possible to obtain significant effects from CO2 and nitrous oxide gasses. Whether these findings are believable or not is for the climatologists to discuss.

Modeling cointegrated series is difficult because of the need to model systems of equations in which one has to simultaneously specify the forcing variables and how they enter, determine the lag length, and ensure a well-specified representation. Nevertheless, powerful software facilitates the task for those wishing to undertake their own analyses, including the programs CATS in RATS and *PcGive* that I have utilized here.

11 Acknowledgments

All the computations reported in this paper were carried out using the software CATS in RATS and the graphs were produced with the OxMetrics package. I am grateful to Peter Thejll and Torben Schmith for asking me to analyze the present climate data.

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