Algebra III/Introduction to Algebra III: Scheme Theory

Due: Please upload solutions to NUCT by Tuesday, July 12, 2022.

Problem 1. Let k be a field, and let $\pi: X = \mathbb{P}^1_k \to \operatorname{Spec}(k)$ be the projective line.

(1) Determine the sheaf cohomology of the line bundles $\mathcal{O}_X(n)$ on X for all $n \in \mathbb{Z}$. This means that you must specify an isomorphism of between the k-vector space $H^i(X, \mathcal{O}_X(n))$ and some other (known) k-vector space. It is not enough to say that an isomorphism exists. You have to specify one.

[Hint: Use the Mayer–Vietoris long exact sequence and use the maps therein to produce the desired specific isomorphism.]

(2) Calculate the Euler characteristic

$$\chi(X, \mathcal{O}_X(n)) = \sum_{i \ge 0} (-1)^i \dim_k H^i(X, \mathcal{O}_X(n))$$

for all $n \in \mathbb{Z}$.