Comments to the proof of Theorem 4.3.

Page 162, line 6 from below and onwards: We can assume $K \geq 1$; this justifies $\varepsilon/K \leq \alpha$ in the book. Let $D_\alpha = [0, \infty[\, \times B(0, \alpha)$ (where we use the notation $B(0, r) = \{\boldsymbol{y} \in \mathbb{R}^n \mid |\boldsymbol{y}| < r\}$). For $\varepsilon < \alpha$ (observe that we take a sharp inequality here!), let $\delta = \varepsilon/K$, and consider the maximal solution $\boldsymbol{\psi}(t)$ in D_α with $\boldsymbol{\psi}(t_0) = \boldsymbol{y}_0$ for some $\boldsymbol{y}_0 \in B(0, \delta)$. To the right of t_0 it is defined on an interval $[t_0, t_1[$ with $t_1 \leq \infty$, and it has been proved in the text that $|\boldsymbol{\psi}(t)| < \varepsilon$ there. We want to show that $t_1 = \infty$.

Here we use Corollary S4.3 for solutions in D_α. We have that $A\boldsymbol{y} + \boldsymbol{f}(t, \boldsymbol{y})$ is bounded on D_α:

$$|A\boldsymbol{y} + \boldsymbol{f}(t, \boldsymbol{y})| \leq |A||\boldsymbol{y}| + \eta|\boldsymbol{y}| \leq (|A| + \eta)\alpha;$$

then (c) cannot happen (Lemma 3.3). Since $|\boldsymbol{\psi}(t)| < \varepsilon$ for all $t \in [t_0, t_1[$, so that $(t, \boldsymbol{\psi}(t))$ has distance $\geq \alpha - \varepsilon$ from the boundary points $(t, \boldsymbol{y})$ with $|\boldsymbol{y}| = \alpha$, (a) cannot happen. Then only (b) is possible, and here it is $t \to \infty$ that happens since $\boldsymbol{\psi}(t)$ is bounded, so $t_1 = \infty$.