## Holomorphic Day, November 15, 2013 Department of Mathematical Sciences University of Copenhagen

## Abstract

The purpose of the holomorphic day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant 10-083122 from The Danish Council for Independent Research | Natural Sciences

## 1 Schedule

The lectures take place at the premises of the Department of Mathematical Sciences, Universitetsparken 5, Copenhagen.

Arrival, coffee, tea: 9.45-10.15 in E 419 (Fourth floor)

Walter Bergweiler: 10.15-11.05 in Aud. 10

Gol'dberg's constants

**Christian Berg:** 11.15-12.05 in Aud. 10

On the growth of entire functions associated with indeterminate moment problems

Lunch: 12.15-13.15

**Eero Saksman:** 13.15-14.05 in Aud. 10

On quasiconformal homogenization

Carsten Lunde Petersen: 14.15-15.05 in Aud. 10

Rescaling limits of quadratic rational maps

Coffee break: 15.05-15.30

Eric Bedford: 15.30-16.20 in Aud. 10

Basins in holomorphic dynamics

Haakan Hedenmalm: 16.30-17.20 in Aud. 10

Coulomb qas in 2D

**Dinner:** 18.30-

## 2 Abstracts

Walter Bergweiler, Universität Kiel: Gol'dberg's constants

A. A. Gol'dberg showed in 1973 that there exists an absolute constant  $A_0$  such that if f is meromorphic in the unit disk and if the numbers of zeros, poles and 1-points of f are all different, then at least one zero, pole or 1-point has modulus greater than  $A_0$ . We determine the exact value of  $A_0$ .

We also study this problem for holomorphic functions, i.e., the special case where the number of poles is 0. In this case the corresponding constant is denoted by  $A_2$ . Gol'dberg had shown that  $A_2 > A_0$ . We improve the previously known lower bounds for  $A_2$ . We also study some other related constants.

The results are joint work with Alexandre Eremenko.

Christian Berg, University of Copenhagen: On the growth of entire functions associated with indeterminate moment problems

To a probability distribution  $\mu$  on  $\mathbb{R}$  with moments of any order

$$s_n = \int_{-\infty}^{\infty} x^n d\mu(x), \quad n = 0, 1, \dots,$$
 (1)

we consider the orthonormal polynomials  $P_n$ , i.e.,

$$\int_{-\infty}^{\infty} P_n(x) P_m(x) \, d\mu(x) = \delta_{nm}.$$

They satisfy  $P^2(z) := \sum |P_n(z)|^2 < \infty$  for all complex z precisely in the indeterminate case, where there are different probability measures with the same moments (1). This leads to a study of entire functions like

$$K(z, w) = \sum_{n=0}^{\infty} P_n(z) P_n(w), \quad z, w \in \mathbb{C}$$

and

$$L(z) = \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{s_{2n}}}.$$

During the last 20 years there has been a general study of these entire function as well as of many concrete examples, often related to q-series. We will give a review of some of these results together with new results about the relation between the growth of P and summability properties of the sequence  $(P_n(z))$ .

The new results are based on joint work with Ryszard Szwarc, Wrocław.

Eero Saksman, University of Helsinki: On quasiconformal homogenization

We study a homogenization limit problem for random quasiconformal mappings in the plane. Talk is based on joint work with Kari Astala (Helsinki), Steffen Rohde (Seattle) and Terence Tao (UCLA).

Carsten Lunde Petersen, University of Roskilde: Rescaling limits of quadratic rational maps

Let  $f_k$  be a sequence of quadratic rational maps diverging in moduli space. Under certain circumstances, there may exist a sequence of conjugate maps  $F_k$  and some  $q \geq 2$  such that the iterates  $F^{kq}$  converge algebraically to a rational map of degree 2 or more. It has been conjectured that, up to a suitable notion of equivalence, there can be at most two such rescaling limits, the first a quadratic rational map with a fixed point of multiplier 1, the second a quadratic polynomial. I shall give the appropriate definitions and state precise theorems. The theorems will be accompanied by illustrations and if time allows some of the elements of the proofs. This is joint work with Adam Epstein.

Eric Bedford, Indiana University: Basins in holomorphic dynamics

We will discuss the structure of attracting basins and parabolic basins as they arise in the dynamics of holomorphic mappings in two complex dimensions.

Haakan Hedenmalm, KTH, Stockholm: Coulomb gas in 2D

We survey recent developments in two-dimensional Coulomb gas ensembles.

Organized by Christian Berg, Christian Henriksen, Henrik Laurberg Pedersen, Carsten Lunde Petersen