Workshop on Orthogonal Polynomials, Hankel and Jacobi matrices

August 26–28, Copenhagen
Denmark

Preface

This document contains the program for the "Workshop on Orthogonal Polynomials, Hankel and Jacobi Matrices" held in Copenhagen, August 26–28.

The workshop is sponsored by the Danish Research Council and Nordforsk.

The organising committee consists of Professor Christian Berg, Assistant professor Jacob Stordal Christiansen both of the Department of Mathematical Sciences, Faculty of Science, University of Copenhagen and of Associate professor Henrik Laurberg Pedersen of the Department of Basis Sciences and Environment, Faculty of Life Sciences, University of Copenhagen. The workshop is hosted by the Department of Basic Sciences and Environment and it takes place at the Faculty of Life Sciences in central Copenhagen.

More information on the workshop can be found at `www.matdat.life.ku.dk/~henrikp/wop/`

Venue

The workshop takes place at the Faculty of Life Sciences, Thorvaldsensvej 40, Frederiksberg C.

The lecture room is Auditorium 3-20, which can be found in the north end of the main building between Thorvaldsensvej and Rolighedsvej. You should enter the building at Thorvaldsensvej 40, pass through the marble hall and continue through the main walking area, passing both the cafeteria and the bookstore. Auditorium 3-20 is located near the other end of the building, in the basement. There will also be signs directing you to the lecture room.

The lecture room is equipped with a computer and beamer, so that you need only bring your slides in preferably pdf format on an usb-disk. There is also an overhead projector and black boards. (It is possible to use both black board and e.g. beamer at the same time.)

Registration and practical information

The registration is open from 12.00 to 13.25. The opening of the workshop is at 13.25. Both registration and opening take place in Auditorium 3-20.

At the registration desk you will receive the book of abstracts (this document), your badge and (in case you have indicated so) a username and password for wireless access to the internet.

On Thursday we plan dinner for all registered participants of the workshop. The dinner will take place in "Væksthuset", which is a cafe on campus inside a former green house. The cafe can be found on the other side of the street Thorvaldsensvej (on the map it is between the blue sign BUS and the sign 2-02). There will be a fee of 20 Euro (150 DKK) for registered participants in order to take part in the common dinner. The fee for accompanying persons is 50 Euro (375 DKK). Payment of the fee is due at the registration and can be paid in cash only, either in Euros or in Danish Crowns.

Coffee will be served close to the lecture room.

Lunch can be bought in the cafeteria in the main walking area.

Program for the workshop

The program consists of 9 invited lectures of approximately 50 minutes and 7 contributed talks of 25 minutes. All lectures take place in Auditorium 3-20.

	Wednesday, August 26	Thursday, August 27	Friday, August 28
09.00–10.00		Remling	Marcellan
10.00–10.30		Coffee break	Coffee break
10.30–11.30		Yuditskii	Obermaier
11.30–12.00		Sanchez-Moreno	Zeng
12.00–13.30	Registration and opening	Lunch	Lunch
13.30–14.30	Van Assche	Koelink	Ismail
14.30–15.30	Szwarc	Levin	
15.30–16.00	Coffee break	Coffee break	
16.00–16.30	Anshelevich	Kajihara	
16.30–17.00	González-Vera	Sri Ranga	
17.00–17.30	Lasarow		

Chairmen

Wednesday afternoon:	Henrik L. Pedersen
Thursday morning:	Jacob Stordal Christiansen
Thursday afternoon:	Christian Berg
Friday morning:	Erik Koelink
Friday afternoon:	Christian Berg

The titles and abstracts of invited and of contributed talks can be found on the following pages.

Abstracts: Invited Talks

Multiple orthogonal polynomials, Hankel matrices and recurrence relations

Walter Van Assche

Katholieke Universiteit Leuven, Belgium

Email: walter@wis.kuleuven.be

Multiple orthogonal polynomials are polynomials of one variable which satisfy orthogonality relations with respect so $r \geq 1$ measures (multiple orthogonality). We show that they give rise to block Hankel matrices and the concept of normality is related to the non-singularity of these matrices. Multiple orthogonal polynomials satisfy a linear recurrence relations on a lattice in $\mathbb{Z}^r$. For $r > 1$ the recurrence coefficients satisfy a partial difference equations, which is a new aspect that does not appear for $r = 1$ (the usual orthogonal polynomials). Multiple orthogonal polynomials are bi-orthogonal with respect to another family of functions (Type I versus Type II multiple orthogonal polynomials). We give some examples with the corresponding recurrence relations. The multiple orthogonal polynomials near the diagonal (on the stepline) contain sufficient information to retrieve the r orthogonality measures.

Hypergeometric Origins of Diophantine properties associated with the Askey Scheme

Mourad Ismail

University of Central Florida, USA

Email: mourad.eh.ismail@gmail.com

The "Diophantine" properties of the zeros of certain polynomials in the Askey scheme, recently discovered by Calogero and his collaborators, are explained, with suitably chosen parameter values, in terms of the summation theorem of hypergeometric series. Here the Diophantine property refers to having integer valued zeros. It turns out that the same procedure can also be applied to polynomials arising from the basic hypergeometric series. We found, with suitably chosen parameters and certain q-analogue of the summation theorems, zeros of these polynomials explicitly, which are no longer integer valued.

The zeros are eigenvalues and generalized eigenvalues of certain tridiagonal matrices. Connection with small oscillations of systems about equilibrium positions will also be presented.

The indeterminate moment problem for the q-Meixner polynomials

Erik Koelink

Radboud Universiteit Nijmegen, the Netherlands

Email: e.koelink@math.ru.nl

In the scheme of indeterminate moment problems in the q-Askey scheme as given by Jacob Christiansen in his 2004 thesis, the indeterminate moment problem corresponding to the q-Meixner polynomials are on the one but highest level. A two-parameter solution for the moment problem is presented, and we describe an orthogonal system complementing the orthogonal polynomials which in a particular case is self-dual. We also discuss how this fits in with the general scheme of Christiansen and how some special cases arise from quantum group theoretic considerations. The results are joint work with Wolter Groenevelt (TU Delft, the Netherlands).

Universality limits and their relations to zeros of orthogonal polynomials

Eli Levin

The Open University of Israel, Israel

Email: elile@openu.ac.il

The local correlations between the eigenvalues of unitary ensembles of random matrices are intimately related with orthogonal polynomials. These correlations, appropriately scaled, have universal behavior when the size of the matrices tends to infinity – in the bulk of the spectrum the behavior is described in terms of the sinc kernel $\sin \pi x/\pi x$. We survey recent results on universality limits and show how these yield asymptotics of spacing of zeros of orthogonal polynomials, for weights with compact support and for exponential weights on the real line.

Spectral transformations for Jacobi and Hessenberg matrices

Francisco Marcellan

Universidad Carlos III de Madrid, Spain

Email: pacomarc@ing.uc3m.es

Let μ be a nontrivial probability measure supported on the real line. A Jacobi matrix appears in a natural way as the representation of the multiplication operator with respect to

the basis of orthonormal polynomials associated with the measure μ. Some linear spectral transformations of the measure have been studied in the literature as well as the connections of the corresponding Jacobi matrices using LU and UL factorizations. See [1], [2], [3], and [6] for different frameworks like bispectral problems, integrable systems, matrix theory, group theory, and Stieltjes functions.

More recently, in [4] a similar approach has been done for nontrivial probability measures supported on the unit circle. In this case, a special case of Hessenberg matrix appears as the representation of the multiplication operator with respect to the corresponding sequence of orthonormal polynomials. This is the so called GGT representation (see [5]) and the unitary character of the matrix is related to the fact that the measure does not belong to the Szegő class. We have proved the connections between linear spectral transformations of the measure and these Hessenberg matrices using QR factorizations.

In this talk we will present a survey of these results as well as we discuss some open problems related with the relations between the CMV representation (see [5]) of the multiplication operator with respect to a basis of orthonormal Laurent polynomials and some rational spectral transformations of the measure.

1 M. Adler and P. van Moerbeke, *Darboux Transforms on Band Matrices, Weights, and Associated Polynomials*, *IMRN*, **18** (2001), 935–984.

2 M. I. Bueno and F. Marcellan, *Darboux transformations and perturbation of linear functionals*, Linear Algebra and Appl., **384** (2004), 215–242.

3 F. A. Grunbaum and L. Haine, *Bispectral Darboux Transformations: An extension of the Krall polynomials*, IMRN, **8** (1997), 359–392.

4 L. Daruis, J. Hernandez, and F. Marcellan, *Spectral Transformations for Hermitian Toeplitz Matrices*, J. Comput. Appl. Math., **202** (2007), 155–176.

5 B. Simon, *Orthogonal Polynomials on the unit circle. Part 1: Classical Theory*, Amer. Math. Soc. Coll. Publ. Series, **54,** Amer. Math. Soc. Providence, RI. (2005).

6 A. Zhedanov, *Rational spectral transformations and orthogonal polynomials*, J. Comput. Appl. Math., **85** (1997), 67–83.

Fourier analysis with respect to orthogonal polynomials

Josef Obermaier

Helmholtz Zentrum München, Germany

Email: josef.obermaier@helmholtz-muenchen.de

Various aspects of Fourier analysis with respect to orthogonal polynomials will be discussed. The study of Fourier series can be done in the context of so-called harmonic Banach spaces, where a Riemann-Lebesgue lemma and a uniqueness theorem hold. Among other things some new summability methods for Fourier series will be presented.

Reflectionless Jacobi matrices

Christian Remling

University of Oklahoma, USA

Email: cremling@math.ou.edu

Reflectionless Jacobi matrices are of interest because they form the basic building blocks for arbitrary Jacobi matrices with some absolutely continuous spectrum. I'll review these results, and, if time permits, I'll also report on recent attempts, largely unsuccessful so far, to understand specific classes of reflectionless operators in more detail.

Jacobi matrices on trees

Ryszard Szwarc

University of Wroclaw, Poland

Email: szwarc2@gmail.com

Symmetric Jacobi matrices on one sided homogeneous trees are studied. Essential selfadjointness of these matrices turns out to depend on the structure of the tree. If a tree has one end and infinitely many origin points the matrix is always essentially selfadjoint independently of the growth of its coefficients. In case a tree has one origin and infinitely many ends, the essential selfadjointness is equivalent to that of an ordinary Jacobi matrix obtained by the restriction to the so called radial functions. For nonselfadjoint matrices the defect spaces are described in terms of the Poisson kernel associated with the boundary of the tree. (Based on joint work with Agnieszka Kazun.)

Orthogonal polynomials and polynomials of least deviation from zero on a system af intervals: similarities and differences

Peter Yuditskii

J. Kepler University, Austria

Email: petro.yudytskiy@jku.at

The classical Chebychev polynomials $T_n(z) = \zeta^n + \zeta^{-n}$, $z = \zeta^{-1} + \zeta$, are both of least deviation on a single interval $[-2, 2]$ and orthogonal with respect to the measure $dz/\sqrt{4 - z^2}$. Following the historical line of its development we describe the main features on the corresponding theories in the several interval case. In the end of our talk we present our new results with A. Eremenko and F. Peherstorfer on the best Chebychev kind approximation of $\mathrm{sgn}(x)$ on two intervals.

Abstracts: Contributed Talks

Multivariate coefficient stripping

Michael Anshelevich

Texas A&M University, USA

Email: manshel@math.tamu.edu

The notions of orthogonal polynomials of the first and second kind, and the related operation of coefficient stripping, go back to the work of Darboux and Stieltjes at the end of the 19th century. In this talk I will present a natural generalization of these notions to the multivariate context which involves *non-commuting* variables. Time permitting, I will mention the free Appell polynomials, which also could have been (but weren't) found by Darboux or Appell himself.

On the computation of symmetric Szegő-type quadrature formulas

Pablo González-Vera

La Laguna University, Spain

Email: pglez@ull.es

Let $\sigma(x)$ be a weight function on $[-1,1]$ and $\omega(\theta)$ the symmetric weight function on $[-\pi,\pi]$, or more generally on the unit circle $\mathbb{T}=\{z\in\mathbb{C}:|z|=1\}$, defined by $\omega(\theta)=\sigma(\cos\theta)|\sin\theta|$. In this talk we shall be dealing with the numerical estimation of integrals of the form $I_\omega(f)=\int_{-\pi}^{\pi} f(e^{i\theta})\omega(\theta)d\theta$, by means of an n-point symmetric Szegő-type quadrature formula, that is an expression like

$$I_n^\omega(f) := \sum_{j=1}^{n}\lambda_j f(z_j), \quad \text{all } z_j\in\mathbb{T} \text{ distinct and } \lambda_j>0,$$

with nodes $\{z_j\}_{j=1}^n$ either real (i.e. ± 1) or in complex conjugate pairs and where at most two of them may be fixed in advance. Our purpose is to show how the symmetric character of ω can reduce to a half the computational work of such rules, from a matrix point of view. Indeed, the eigenproblem for isometric Hessenberg or unitary CMV matrices of dimension $2n$ or $2n+1$ will be transformed into an equivalent one for a new associated Jacobi matrix of dimension n or $n+1$ respectively, and whose entries are given in terms of the Verblunsky coefficients for ω. We will also show how these computations can be organized. As an application, Gauss-Radau quadrature formulas with a prefixed node anywhere on $[-1,1]$ will be characterized. The work is part of joint work in progress with A. Bultheel, R. Cruz-Barroso and F. Perdomo-Pío.

On classes of transformations for bilinear sum of (basic) hypergeometric series and multivariate generalizations

Yasushi Kajihara

Kobe University, Japan

Email: kajihara@math.kobe-u.ac.jp

In this talk, I will present classes of bilinear transformation formulas for basic hypergeometric series and Milne's multivariate basic hypergeometric series associated with the root system of type A. Our construction is very similar to one of elementary proof of Sears-Whipple transformation formula for terminating balanced ${}_4\phi_3$ series while we use multiple Euler transformation formula with different dimensions which has been obtained in our previous work.

On canonical solutions of the truncated trigonometric matrix moment problem

Andreas Lasarow

K.U. Leuven, Belgium

Email: Andreas.Lasarow@cs.kuleuven.be

The main theme of the talk is the discussion of some distinguished solutions of the truncated trigonometric matrix moment problem. Roughly speaking, these solutions are molecular nonnegative Hermitian matrix-valued Borel measures on the unit circle with a special structure. We give some general information on this type of solutions, but we will focus on the so-called nondegenerate situation. In that case, these molecular measures form a family of solutions which can be parametrized by the set of unitary matrices. In particular, we will show that each member of this family offers an extremal property within the solution set of the moment problem in question concerning the weight assigned to some point of the open unit disk. In doing so, an application of the theory of orthogonal matrix polynomials on the unit circle is used to get that insight.

Szegő polynomials and Szegő type polynomials from hypergeometric functions

A. Sri Ranga

DCCE/IBILCE, Universidade Estadual Paulista, Brazil

Email: ranga@ibilce.unesp.br

Szegő type polynomials coming from hypergeometric functions are looked at. In particular, Szegő polynomials with respect to the weight function $\omega(\theta) = e^{(\pi-\theta)\eta}[\sin(\theta/2)]^{2\lambda}$, where $\eta, \lambda \in \mathbb{R}$ and $\lambda > -1/2$, are considered. We show that these are the hypergeometric polynomials ${}_2F_1(-n, b+1; b+\bar{b}+1; 1-z)$ with $\mathfrak{Re}\, b = \lambda$ and $\mathfrak{Im}\, b = \eta$. Many of the basic relations associated with these polynomials are given explicitly. For example, the moments satisfy $\overline{\mu}_{-j} = \mu_j = (-b)_j/(\bar{b}+1)_j$, the zeros of ${}_2F_1(-n, b+1; b+\bar{b}+1; 1-z)$ lie within the ring $|b+n|^{-1}|b| \leq |z| < 1$ and the associated Szegő function is $(1-z)^b|\Gamma(b+1)|/\sqrt{\Gamma(b+\bar{b}+1)}$. We also show that $\{{}_2F_1(-n, b; b+\bar{b}; 1-z)\}$ and $\{{}_2F_1(-n, b+1; b+\bar{b}+2; 1-z)\}$ are sequences of para-orthogonal polynomials for these Szegő polynomials. Consequently, the zeros of ${}_2F_1(-n, b; b+\bar{b}; 1-z)$ are distinct and lie on the unit circle, provided also that $\mathfrak{Re}\, b > 0$.

Information-theoretic lengths of hypergeometric orthogonal polynomials

Pablo Sanchez-Moreno

University of Granada, Spain

Email: pablos@ugr.es

The Renyi, Shannon and Fisher spreading lengths of the hypergeometric orthogonal polynomials [1], which are quantifiers of their distribution all over the orthogonality interval, are defined and investigated. These quantities which are information-theoretic measures of their associated Rakhmanov probability density have parallel properties as the familiar standard deviation, what makes them particularly appropriate to discuss the polynomials spreading.

The determination of the Renyi length, as well as for other information measures such as the disequilibrium, and the Renyi and Tsallis entropies requires the efficient computation of some powerlike functionals of the classical orthogonal polynomials called frequency or entropic moments. The explicit expressions of these moments for the Hermite and Laguerre polynomials are obtained in terms of the polynomial degree and the involved parameters. The Bell polynomials [2], which characterize the finite power of an arbitrary polynomial, play a relevant role for this problem. In addition, for the Laguerre polynomials the result is given in terms of Lauricella functions by use of the Srivastava-Niukkanen linearization relation [3] for these polynomials.

Finally, the application to some physical prototype systems is discussed [4-5]. The harmonic oscillator, whose quantum-mechanical states are described by the Hermite polynomials, is analyzed by means of these informational spreading lengths. On the other hand, the stationary

states of the half-line Coulomb potential are described by quantum-mechanical wavefunctions which are controlled by the Laguerre polynomials. Later, sharp bounds to the information-theoretic quantities mentioned above are found for general states. Finally, a numerical study of these quantities for the ground state and various excited states of the system is performed.

[1] A.F. Nikiforov and V.B. Uvarov, *Special Functions in Mathematical Physics*, Birkäuser-Verlag, Basel, 1988.

[2] L. Comtet, *Advanced Combinatorics*, D. Reidel Publishing Company, Dordrecht, 1974.

[3] H.M. Srivastava and A.W. Niukkanen, *Some Clebsh-Gordan type linearization relations and associated families of Dirichlet integrals*, Math. Comput. Modelling **37**, 2003, 245-250.

[4] P. Sánchez-Moreno, J.J. Omiste and J.S. Dehesa, *Entropic moments of Laguerre polynomials and complexity properties of the half-line Coulomb potential.* Preprint, 2009.

[5] P. Sánchez-Moreno, J.S. Dehesa, R.J. Yáñez and D. Manzano, *Spreading lengths of classical orthogonal polynomials.* Preprint, 2009.

Two curious q-analogues of Hermite polynomials

Jiang Zeng

University Lyon, France

Email: zeng@math.univ-lyon1.fr

Two well-known q-Hermite polynomials are the continuous and discrete q-Hermite polynomials. In this talk we consider two new q-Hermite polynomials and prove several curious properties about these polynomials. One striking property is the connection with q-Fibonacci polynomials and the recent works on the combinatorics of the Matrix Ansatz of the PASEP.

List of Registered Participants

1. **Ali, S Ahmad** BBD National Institute of Technolgy, India
   ahmad67@rediffmail.com

2. **Anshelevich, Michael** Texas A& M University, USA
   manshel@math.tamu.edu

3. **Arvesu, Jorge** University Carlos III de Madrid, Spain
   jarvesu@math.uc3m.es

4. **Van Assche, Walter** Katholieke Universiteit Leuven, Belgium
   walter@wis.kuleuven.be
5. **Berg, Christian** University of Copenhagen, Denmark
   berg@math.ku.dk
6. **Christiansen, Jacob Stordal** University of Copenhagen, Denmark
   stordal@math.ku.dk
7. **Deaño, Alfredo** University of Cambridge, United Kingdom
   ad495@cam.ac.uk
8. **Ghrissi, Abdallah** Institu dínformatique mahdia, Tunisia
   Abdallah.Ghrissi@fsg.rnu.tn
9. **González-Vera, Pablo** La Laguna University, Spain
   pglez@ull.es
10. **Hennessy, Aoife** Waterford Institiute of Technology, Ireland
    aoife.hennessy@gmail.com
11. **Hussain, Mohammad Azhar** Veer Kunwar Singh University, India
    azharhu@gmail.com
12. **Ismail, Mourad** University of Central Florida, USA
    mourad.eh.ismail@gmail.com
13. **Kajihara, Yasushi** Graduate School Of Science, Kobe University, Japan
    kajihara@math.kobe-u.ac.jp
14. **Karlsson, Per W** DTU, Denmark
    P.W.Karlsson@mat.dtu.dk
15. **Koelink, Erik** Radboud Universiteit Nijmegen, the Netherlands
    e.koelink@math.ru.nl
16. **Lasarow, Andreas** K.U. Leuven, Belgium
    Andreas.Lasarow@cs.kuleuven.be
17. **Levin, Eli** The Open University of Israel, Israel
    elile@openu.ac.il
18. **Lopez Lagomasino, Guillermo** Universidad Carlos III de Madrid, Spain
    lago@math.uc3m.es
19. **Marcellan, Francisco** Universidad Carlos III de Madrid, Spain
    pacomarc@ing.uc3m.es
20. **Maslouhi, Mostafa** , Morocco
    mostafa.maslouhi@gmail.com

21. **Medem, Juan Carlos** Universidad de Sevilla, Spain
jcmedem@us.es

22. **Obermaier, Josef** Helmholtz Zentrum München, Germany
josef.obermaier@helmholtz-muenchen.de

23. **Pedersen, Henrik Laurberg** University of Copenhagen, Denmark
henrikp@dina.kvl.dk

24. **Petersen, Helle Bjerg** University of Copenhagen, Denmark
hellebp@gmail.com

25. **Ranga, A. Sri** Universidade Estadual Paulista / Campus of Sao Jose do Rio Preto, Brazil
ranga@ibilce.unesp.br

26. **Remling, Christian** University of Oklahoma, USA
cremling@math.ou.edu

27. **Rosengren, Hjalmar** Chalmers University, Sweden
hjalmar@chalmers.se

28. **Sanchez-Moreno, Pablo** University of Granada, Spain
pablos@ugr.es

29. **Shina, Ashish** bhaktapur, liberty771@hotmail.com
liberty771@hotmail.com

30. **Singh, Sree Ram** Veer Kunwar Singh University, India
alazharhussain@yahoo.co.in

31. **Szafraniec, Franciszek Hugon** Jagiellonian University, Poland
umszafra@cyf-kr.edu.pl

32. **Szwarc, Ryszard** Wroclaw University, Poland
szwarc2@gmail.com

33. **Tookos, Ferenc** Helmholtz Zentrum München, Germany
ferenc.tookos@helmholtz-muenchen.de

34. **Vignat, Christophe** Université de Marne la Vallée, France
vignat@univ-mlv.fr

35. **Yuditskii, Peter** J. Kepler University, Austria
petro.yudytskiy@jku.at

36. **Zeng, Jiang** Institut Camille Jordan, University Lyon, France
zeng@math.univ-lyon1.fr