

# Commutative Algebra



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Editors

# Commutative Algebra

Noetherian and Non-Noetherian Perspectives

 Springer

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# Preface

This volume contains a collection of invited survey articles by some of the leading experts in commutative algebra carefully selected for their impact on the field. Commutative algebra is growing very rapidly in many directions. The intent of this volume is to feature a wide range of these directions rather than focus on a narrow research trend. The articles represent various significant developments in both Noetherian and non-Noetherian commutative algebra, including such topics as generalizations of cyclic modules, zero divisor graphs, class semigroups, forcing algebras, syzygy bundles, tight closure, Gorenstein dimensions, tensor products of algebras over fields,  $v$ -domains, multiplicative ideal theory, direct-sum decompositions, defect, almost perfect domains, defects of field extensions, ultrafilters, ultraproducts, Rees valuations, overrings of Noetherian domains, weak normality, and seminormality.

The papers give a cross-section of what is happening and of what is influential in commutative algebra now. The target audience is the researchers in the area, with the aim that the papers serve both as a reference and as a source for further investigations.

We thank the contributors for their wonderful papers. We have learned much from their expertise, and we hope that these papers are as inspirational for the readers as they have been for us. We also thank the referees for their constructive criticism, and the Springer editorial staff, especially Elizabeth Loew and Nathan Brothers, for their patience and assistance in getting this volume into print.

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