Weekly notice #14

There will be no more notices. Consult the course home page for further and forthcoming information about the exam.

The lectures in week 19: The main results covered were Hölder's and Minkowski's Inequalities and the Fisher Completeness Theorem.

The lectures in week 20: A little time will spent on $\mathcal{L}_\infty$ and L_∞, but the main topic from now on will be the Fourier Transform (§8.1 and §8.2).

The problem sessions in week 21: 8.1, 8.2, 8.3*, 8.4, 8.6 †, W14.1, W14.2*, W14.3*.

(†) [Information to exercise 8.6: The absolute value of the Jacobian determinant for the change of variables is $r^2 \sin\theta$, where θ is the angle between the position vector (x, y, z) and the z-axis.]

Problem W14.1. Prove that the equation $\frac{\partial^2}{\partial x^2}\psi = \frac{\partial^2}{\partial y^2}\psi$ has no solutions in the Schwartz space $\mathcal{S}(\mathbb{R}^2)$. [Hint: Look at the Fourier transform.]

Problem W14.2. (Assumes complex function theory as in Matematik 2KF.) Prove that if $f(x) = e^{-\frac{1}{2}x^2}$ then $\hat{f}(\xi) = \sqrt{(2\pi)}e^{-\frac{1}{2}\xi^2} = \sqrt{(2\pi)}f(\xi)$. [Hint: Write

$$\hat{f}(\xi) = e^{-\frac{1}{2}\xi^2} \int_{\mathbb{R}} e^{-\frac{1}{2}(x+i\xi)^2} dx,$$

and view the integral as a complex curve integral of the entire function $e^{-\frac{1}{2}z^2}$ along a line parallel to the x-axis and going through $i\xi$. Such an integral may be seen to be independent of ξ and may hence be computed for $\xi = 0$.]

Problem W14.2. Granted the fact that the Schwartz space $\mathcal{S}(\mathbb{R}^k)$ is dense in $\mathcal{L}_1(\mathbb{R}^k)$, do exercise 8.10.