Mathematics 3MI **Hans Plesner Jakobsen**
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Weekly notice #6

The lectures in week 10: We finished the integration of positive functions. The main result is Lebesgue's Monotone Convergence Theorem (Hovedssætning 4.3) and Fatou's Lemma (4.7).

The lectures in week 11: We will continue with §4.2 and after that §4.7.

Homework - to be handed in to the TA in week 12. : 3MI -exam January 1999 Problem 2: ii), iii), and iv) and June 1999 Problem 3. (The first problem can be found on the Weekly Notice #4, the second may be found below).

The problem sessions in week 12: 4.8*, 4.19, 4.23, 4.24*, = 4.41*, 4.42, W6.1, and W6.2.

Problem W6.1. Show that

$$\lim_{n\to\infty}\int_0^{2\pi} sin(x)^n dx = 0.$$

Problem W6.2. For $f \in \mathcal{L}(\mathbb{R}, \mathbb{B}, m)$, consider the Fourier Transform g of f given by

$$g(t) = \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} f(x)e^{-i\cdot xt}dx.$$

Prove that the right hand side makes sense and in fact defines a *continuous* function of t. Set $f_1(x) = -i \cdot xf(x)$. Prove that if $f_1 \in \mathcal{L}(\mathbb{R}, \mathbb{B}, m)$ then g is differentiable and

$$g'(t) = \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} f_1(x)e^{-i\cdot xt}dx.$$

June 1999 Problem 3: Set

$$F(t) = \int_0^1 \sin(x^2 + t^2)dx \ , \qquad t \in [-10, 10].$$

Give the reasons why F is *continuous* and *differentiable*, and show that

$$F'(t) = \int_0^1 2t\cos(x^2 + t^2)dx.$$