OUTLINE OF GRADUATE COURSE SPRING 05: COHOMOLOGY OF GROUPS

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Abstract. The aim of this course is to give a modern introduction to group cohomology, somewhat from the viewpoint of homotopy theory, starting from the basics. The methods we discuss will sometimes work best when the group is finite, but part of the goal of the course to see which of the methods generalize to classes of infinite groups, such as arithmetic groups, mapping class groups, and automorphisms of free groups. We'll start by discussing the basic theory as described in Brown: "Cohomology of Groups", Benson: "Representations and Cohomology I+II", or Evens: "Cohomology of Groups", including a bunch of examples. We'll then embark on understanding how to use the action of the group on various "buildings" to get information back about the group and its cohomology. Here we'll use parts of the very readable papers: Quillen:"Homotopy properties of the poset of non-trivial p-subgroups of a group" and Dwyer:"Classifying spaces and homology decompositions", among others. We also plan a part on the spectrum of an equivariant cohomology ring and the variety of groups and modules, as well as other topics according to the interests of the participants.

The course should be accessible to all graduate students in their second year and higher, and should be of interest to students in geometry/topology and group theory.

Textbooks: Brown: "Cohomology of Groups", Benson: "Representations and Cohomology I+II", or Evens: "Cohomology of Groups", Adem+Milgram: Cohomology of finite groups. as well as research papers: Quillen:"Homotopy properties of the poset of non-trivial p-subgroups of a group", Quillen: "Cohomology and K-theory of general linear groups over a finite field", Dwyer:"Classifying spaces and homology decompositions", Grodal: "Higher limits via subgroups complexes", etc.

The topics below are topics that we will probably touch upon. They do not necessarily all need to be treated in the order listed.

1. Group cohomology: elementary results and definitions (1-2 weeks?)

- def via classifying spaces and via resolutions. products. cohomology of cyclic groups. elementary abelian groups.
- bar resolution
- transfer and consequences for order of cohom group elts.
- relations ship to topology: fibrations etc. Classification of group extensions.

2. Spectral sequences and calculations (2 weeks?)

- intro to spectral sequences/ steenrod operations.
- calculation of cohomology rings. cohom less than or equal to 15.

Date: March 28, 2005.

- cohomology ring noetherian.
- cohomology of general linear groups following Quillen.

3. Groups acting on posets. Homology decompositions (3 weeks?)

- Technical tools: Quillen's theorem A, Grothendieck construction, homotopy colimits.
- Elementary results on contractibility of $S_p(G)$. Products, equivalence to other collections.
- Equivalence of $S_p(G)$ to the building in the case of finite groups of Lie type.
- Euler characteristic of $S_p(G)$
- Homology decompositions. higher limits. sharp homology decompositions.
- More calculations using new tools!

4. Varieties (2 weeks?)

- Quillen's theorem on stratification cohom ring. (proof by lannes/henn)
-version for modules? (following Evens? Avrunin-scott)
- rank varieties?

5. p-local finite groups, special classes of infinite groups, stability theorems, and/or other special topics (remaining time?)

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